Problem Set 3
Due: 11:59pm, Sunday, March 3rd

See homework submission instructions at
http://sites.fas.harvard.edu/~cs124/cs124/problem_sets.html

Problem 4 is worth 40% of this problem set; problems 1-3 are worth 20% each.

For each problem where you are asked to give an algorithm, more points are given for asymptotically faster algorithms. In judging the number of points to award a correct solution, we only consider the running time in asymptotic notation, i.e. a writeup of an algorithm taking $1000n^2$ steps versus one taking $.01n^2$ steps would receive the same number of points — both would be simply treated as $\Theta(n^2)$-time solutions.

1 Problem 1

Design an efficient algorithm to find the longest path in a directed acyclic graph given in adjacency list representation. (Partial credit will be given for a solution where each edge has weight 1; full credit for solutions that handle general real-valued weights on the edges, including negative values.)

2 Problem 2

Consider the shortest paths problem in the special case where all edge costs are non-negative integers. Describe a modification of Dijkstra’s algorithm that works in time $O(|E| + |V| \cdot L)$, where $L$ is the maximum cost of any edge in the graph.

3 Problem 3

Given a weighted graph with $n$ vertices and $m \leq n + 10$ edges, show how to compute a minimum spanning tree in $O(n)$ time.

4 Programming Problem