A brief summary of topics that we touched in the last class (Apr. 15)

1. Behavioral traps.
   a) Anchoring (dirty experiment with a question is population of Turkey more than 20 million, clean experiment with last three digits of the social security number.)
   b) Foot in the door.
   c) Probability of 60 boys out of 100 vs 6000 our of a 10000, most people misjudge the ratio of the two probabilities. Easy to use the central limit theorem to obtain the probabilities. (You must know how to do it.)
   d) Understanding diversification over time vs over many stocks. Consider N stocks with independently and identically distributed returns. Consider an investor who holds a portfolio that equally divides the funds among N, as N→∞ the variance of the portfolio returns converges to zero (you must understand why). Now consider an investor who lives for N periods, in period one he puts all his money in one stock and holds it for N periods. Suppose that the returns across periods are independently and identically distributed. Note that in this case the variance of the portfolio does not converge to zero as N converges to infinity (not even close).

   In the first case the return of the portfolio is $r_1 + r_2 + ... + r_N$ in the second case the return is $\rho_1 \cdot \rho_2 \cdot ... \cdot \rho_N$, where $r_i$ is the return of stock $i$ and $\rho_i$ is the return in year $i$.

   You must know and understand all of the above well. Do not hesitate to talk to me if any of this is confusing. All of the above is likely to appear on the final.

   We argued that $U(w) = \ln w$ has some special properties, namely, investors with this utility function are likely to dominate the market on the long run with very high probability (this utility function plays a special role in evolutionary theory if $w$ is a number of offsprings there is a good argument that this utility function give an animal a kind of long run evolutionary advantage). If this is confusing—fine, do not worry about it. (Outside my office I have a few copies of a paper that makes this point. It is very difficult, if you would like a real challenge you can pick up a copy, goes without saying it is well beyond the scope of this class.)

   Other small points. That may appear on the final (again talk to me if something is not clear).

   We can always say that
   
   $r_t = \mathbb{E}[r_t | \text{all information available at time } t-1] + \epsilon_t$ where $\mathbb{E}[\epsilon_t] = 0$
   
   $r_{t+1} = \mathbb{E}[r_{t+1} | \text{all information available at time } t] + \epsilon_{t+1}$

   Note that the value of $\mathbb{E}[\epsilon_{t+1} | \epsilon_t] = 0$. You must understand why. Note that the above argument does not rely on market efficiency. It only hinge on the fact
that $\varepsilon_t$ is known at time $t+1$ and hence all the information from $\varepsilon_t$ is already incorporated in $E[r_{t+1}]$. I did not mention above that $r_t$ is the market return in period $t$ because the same equality holds for any variable, $r_t$ may be the rainfall in Goorambia in year $t$, all that matters is that $r_t$ is known in period $t+1$. At this point it is worth mentioning that $E[\varepsilon_{t+1}|\varepsilon_t] = 0$ does not mean that $\varepsilon_t$ does not contain any useful information about $\varepsilon_{t+1}$. For instance, $E[(\varepsilon_{t+1})^2|\varepsilon_t]$ need not be equal to $E[(\varepsilon_{t+1})^2]$. In the context of the stock market there is a significant persistence of volatility, i.e. a day with a large movement is likely to precede a day with a large stock movement (of course the moves need not be in the same direction).