Consider a 40 wt% Sn-60 wt% Pb alloy on the lead-tin phase diagram.

**Part 1:**

At 183.1 degrees C, just above the eutectic line,

a) what phase(s) is (are) present?

b) what is (are) the compositions of the phase(s)?

c) what is the relative amount of each phase present, in mass fraction?

a)

Find the point corresponding to 40 wt% Sn and 183.1 degrees C on the diagram. It lies in an alpha + liquid region. Therefore both the alpha phase and the liquid phase are present.

b)

Extend a horizontal line from this point to the closest phase boundaries. Drop a vertical line down from the phase boundary between the alpha + liquid two phase region and the liquid region to find the composition of the liquid phase. Drop a vertical line down from the other intersection to find the composition of the alpha
phase. In this case the composition of the liquid phase is 61.9 wt% Sn and the composition of the alpha phase is 19.2 wt% Sn.

c)
This step requires the use of the lever rule since the alloy consists of two phases.

\[
W_a = \frac{C_l - C_x}{C_x - C_a} = \frac{61.9 - 40}{61.9 - 19.2} = 0.513
\]

Thus the alpha phase makes up 51.3% of the alloy. There are only two phases so the sum of their mass fractions must equal 1. Therefore the liquid phase makes up .487, or 48.7%, of the alloy. This number could also have been calculated using the lever rule.

\[
W_l = \frac{C_x - C_e}{C_e - C_a} = \frac{40 - 19.2}{61.9 - 19.2} = 0.487
\]

**Part 2:**

Now, the temperature is lowered slightly to right below the eutectic line at 182.9%.

a) what is the composition of each phase?
b) what is the relative amount of each phase present, in mass fraction?

a)
Drop the lines from the intersections as before to discover that the composition of the alpha phase is 19.2 wt% Sn and the composition of the beta phase is 97.5% wt% Sn.

b)
The lever rule is used again.

\[
W_a = \frac{C_x - C_a}{C_x - C_a} = \frac{97.5 - 40}{97.5 - 19.2} = 0.734
\]

\[
W_b = \frac{C_e - C_e}{C_e - C_a} = \frac{40 - 19.2}{97.5 - 19.2} = 0.266
\]

Thus the mass fraction of the alpha phase is .734, or 73.4%, and that of the beta phase is .266, of 26.6%.
The alpha phase can be divided into two identifiable parts. The alpha phase described in Part 1 of this problem does not change in any way as the temperature is lowered. It is termed the proeutectoid alpha. However, as the temperature decreases, the liquid phase at the eutectic point solidifies into a standard eutectic structure consisting of alternating layers of the alpha and beta phases. The alpha phase found in this structure is termed the eutectic alpha. The only difference between the two types of the alpha phase is their place in the structure.

Using these definitions, what is the mass fraction of these phases at the conditions of Part 2?

a) proeutectic alpha
b) eutectic alpha
c) beta

The mass fraction of alpha compared to the mass fraction of liquid was determined in Part 1. This is the proeutectic alpha and is 0.513 of the whole. The total mass fraction of alpha was determined in Part 2 as being 0.734. Since the total amount of alloy did not change, the amount of eutectic alpha must be 0.734 - 0.513 = 0.221 of the whole. The mass fraction of beta was also determined in Part 2 to be .266.

a) 0.513
b) 0.221
c) 0.266

http://www.sv.vt.edu/classes/MSE2094_NoteBook/96ClassProj/examples/bethcon.html
Example 2 – from a previous Final Exam

2. Copper (Cu) and Lead (Pb) are fcc ("α") when pure and do not form any solid structure other than α when pure or mixed. The melting points of pure Cu and pure Pb are 1356.5 K and 600.6 K, respectively. Consider the phase diagram of the Cu-Pb system at $P = 100$ kPa. It will help you to read the entire problem carefully before starting on the solution to any particular part.

![Phase diagram of Cu-Pb system](image)

a) Label each area of the phase diagram above with the phase(s) present in equilibrium. A space is provided at the end of the exam; make sure you indicate which is your "final answer". **Hint: the width of one or more phase regions is thinner than the line of ink on the page, and therefore appears as a thin black line; you do not have to label those.**

b) Determine how many of the intensive potentials (P, T, $\mu_{Cu}$, $\mu_{Pb}$) can be varied independently (i.e. the "number of degrees of freedom") while remaining in equilibrium in the topmost area (i.e. containing the point (0.9, 1300 K)) of this phase diagram. Also, find the number of degrees of freedom as you move along the curved line that has a peak at 1253 K and the number of degrees of freedom as you move along the horizontal line labeled "1227°". In all cases the pressure is fixed at 100 kPa and you may not vary it.

c) In your blue book draw neatly and carefully G(X) curves, where $X = X_{Pb}$, for all phases at $T_1 = 1300$ K, $T_2 = 1230$ K, $T_3 = 1225$ K, $T_4 = 601$ K, $T_5 = 600$ K, and $T_6 = 598$ K (a separate diagram for each temperature), including common tangent lines whenever they occur. **Hint: because the width of one or more phase regions is thinner than the line of ink on the page, to make your free energy curves clear you will have to draw them with the horizontal scale "not to scale".**

d) Superposed on the diagram below, draw what the phase diagram would look like if the α phase did not exist (neither the α1 nor the α2) and label each area of the resulting phase diagram. Justify your answer. A space is provided at the end of the exam.
Solution:
(2) See part (c) for explanation.
(b) Gibbs Phase Rule \( F = C - P + 2 \); \( C = 2 \) (Cu, Pb) if pressure held fixed.

\[ \text{Upper-right: Liquid, } P=1 \Rightarrow F = 2-1+1 \tag{2} \]

Example: \( T \) and \( \mu_a \) may be varied independently,

but \( \mu_b \) is determined by these (e.g. by tangent construction).

\[ \text{Curved line: } L_1 + L_2 \text{ in equilibrium along this line} \Rightarrow P = 2 \]

\[ F = 2 - 2 + 1 = 1 \tag{1} \]

Example: \( T \) may be varied but \( \mu_a \) and \( \mu_b \) are then determined by this (e.g. by the common tangent construction).

\[ \text{Horizontal line: } 0_1, L_1, \text{ and } L_2 \text{ in equilibrium (one tangent is common to } 2 \text{ } G(x) \text{ "lobes"}) \Rightarrow P = 3 \]

\[ F = 2 - 3 + 1 = 0 \tag{0} \]

Example: You can't change \( T \), or \( \mu_a \), or \( \mu_b \) while remaining in 3-phase equilibrium (at \( P = 100 \text{kPa} \)).
(c) \( T_1 \) 
\[ G(1300K) \]

\( T_2 = 1230K \)
\[ G(T_2) \]

\( T_3 = 1228K \)
\[ G(T_3) \]

\( T_4 = 1091K \)

\( T_5 = 600K \)

\( T_6 = 598K \)

Note how when \( T \) changes only slightly, the \( G(X) \) curves change only slightly - both in their shape and in their relative position.

(d) Now eliminate the \( G_a(X) \) curve from all of the diagrams above. What's left is a single-phase liquid at all \( X \) at \( T_1 \), and \( L_1 \), \( L_2 \) at all other \( T \)'s, just like methanol and cyclohexane in demo.
part (b)
the only areas are L and L₁ + L₂

Note: Solubility of Cu in Pb-rich liquid (L₂) must increase because common tangent is now with metastable L₁ instead of with stable α₁.