02-S3) Paulo Mancosu, "The Quaestio de Certitudine Mathematicarum"

Aristotle had formulated in Posterior Analytics an articulate theory of scientific knowledge. Such theory was based on the assumption that in order to possess scientific knowledge we need to know the cause of the results of which we possess knowledge. The opening passages of the Posterior Analytics are explicit:

We suppose ourselves to possess unqualified scientific knowledge of a thing, as opposed to knowing it in the accidental way in which the sophist knows, when we think that we know the cause on which the fact depends as the cause of the fact and of no other, and, further, that the fact could not be other than it is.7

In this context 'cause' means any of the four Aristotelian causes: formal, material, efficient, and final.2 Scientific knowledge, continues Aristotle, is obtained by demonstration. In order to guarantee a scientific transition from premisses to conclusion, a syllogism must not only be valid but premisses and conclusion must stand in a specified relation.

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2 It is the translation of the Greek aitia and the reader should be careful not to superimpose on it a Humean notion of causation. This is why many commentators prefer to translate aitia as explanation.8 I prefer 'cause' as a translation of aitia because it simplifies translations from the Latin causa.
What I now assert is that at all events we do know by demonstration. By demonstration I mean a syllogism productive of scientific knowledge, a syllogism, that is, the grasp of which is *eo ipso* such knowledge. Assuming then that my thesis as to the nature of scientific knowledge is correct, the premisses of scientific knowledge must be true, primary, immediate, better known than and prior to the conclusion, which is further related to them as effect to causes.⁹

It is immediately evident that the requirements set down on the premisses-conclusion relation are much stronger than simple logical consequence. In particular, there are several valid forms of inference that do not yield, in Aristotle’s theory, scientific knowledge. In *Posterior Analytics* 1.13 Aristotle introduced an important distinction between two types of demonstrations; demonstration ‘of the fact’ [demonstratio quia] and demonstration ‘of the reasoned fact’ [demonstratio propter quid]. The former proceeds from effects to their causes, whereas the latter explains effects through their causes. Aristotle gives the following examples. Suppose one wants to prove that the planets are near the earth. One could argue as follows:

The planets do not twinkle.
What does not twinkle is near the earth.
Therefore, the planets are near the earth.

This demonstration, says Aristotle, is a demonstration of the fact but not of the reasoned fact. Indeed, he explains, the planets are not near the earth because they do not twinkle, but they do not twinkle because they are near the earth. In this case we can reverse the major and the middle {terms} of the proof so as to obtain a proof of the reasoned fact.

What is near the earth does not twinkle.
The planets are near the earth.
Therefore the planets do not twinkle.

The second type of syllogism is superior to the first, according to Aristotle, because in it a property (not twinkling) is predicated of a subject (the planets) through a middle term (being near the earth) which is the proximate cause of the effect. Of all the syllogistic figures, Aristotle thought the first figure was the most adequate for demonstration of the reasoned fact.

Of all the figures the most scientific is the first. Thus, it is the vehicle of the demonstrations of all the mathematical sciences, such as arithmetic, geometry, and optics, and practically of all sciences that investigate causes: for the syllogism of the reasoned fact is either exclusively or generally speaking and in most cases in this figure.\(^3\)

The distinction between demonstration of the fact and of the reasoned fact was maintained by the Aristotelian commentators and was further developed by Averroes in his prohemium to Aristotle’s *Physics*, where demonstrations are partitioned into three genders: *quia*, *propter quid* and *potissima*. The *potissima* demonstration was considered to be the most certain type of proof (so a scientific syllogism)\(^3\). \{ . . . \}

The tripartite classification of demonstration is proposed by Alessandro Piccolomini (1508—1578) in a treatise published in 1547 entitled *Commentarium de Certitudine Mathematicarum Disciplinarum*. In this work Piccolomini challenged the traditional argument that mathematical sciences possess the highest degree of certainty because they make use of the highest type of demonstration, the *potissima* demonstration. He defined the *potissima* demonstration as that which gives at once the cause and the effect. The exact features of a *potissima* demonstration were widely debated. Suffice it to say they had to embody the properties that Aristotle ascribed to the scientific syllogism. Among other things, Piccolomini required a *potissima* demonstration to be a syllogism in the first figure with universal premisses that are prior and better known than the conclusion. Its middle must have the form of the definition of

\[^3\] For an explanation of the *potissima* demonstration, see p. ?? below.
a property, it must be unique, and it must function as the proximate cause of the conclusion.

In chapter 11 of his treatise, Piccolomini argued that demonstrations in mathematics do not fit and cannot possibly fit the definition for *potissima* demonstration. However, he argued for the certainty of mathematics by appealing to the conceptual nature of mathematical objects which, being created by the human mind, posses the greatest degree of clarity and certainty. { . . . }

The ensuing debate focused on the more general issue of whether mathematical demonstrations could be causal. As a *potissima* demonstration had to be causal, denying that mathematical demonstrations could be causal was {the same as saying} that mathematical demonstrations could not fit the definition for scientific demonstration { . . . } nor could mathematics be a science, in the Aristotelian sense. This position was held, for example, by Piccolomini, by Pereyra\(^4\), and later by Gassendi\(^5\). In a publication from 1562, Pereyra claimed:

> that the mathematical disciplines are not proper sciences.... To have science is to acquire knowledge of a thing through the cause on account of which the thing is; and science is the effect of demonstration. However, the most perfect kind of demonstration must depend upon those things which are ‘per se’ and proper to that which is demonstrated; indeed, those things which are accidental and in common are excluded from perfect demonstrations. But the mathematician neither considers the essence of quantity, nor treats of its affections as they flow from such essence, nor declares them by the proper causes on account of which they are in quantity, nor makes his demonstrations from proper and ‘per se’ but from

\(^4\) Bio-note  
\(^5\) Bio-note, cross ref. to G.
Pereyra believed, however, that in {Aristotelian} physics one could achieve the perfection of *potissimae* demonstrations, and thus included physics in the realm of Aristotelian science. By contrast, Gassendi, writing in 1624, went as far as to claim that no science {i.e., no true and certain knowledge} exists, and in particular {part of Aristotelian natural philosopher counted as a} science. In connection with mathematics, he simply referred to Pereyra’s opinion and concluded that “whatever certainty and evidence there is in mathematics is related to appearance, and in no way related the genuine causes of things.”

Understandably, these denials of the scientificity of mathematics {i.e., that it lacked the certainty of science in the strictest sense of the term} generated a host of reactions. Several scholars, such as Barozzi, Biancani, Barrow, and Wallis, addressed the issues raised by the Quaestio in an attempt to reinstate mathematics to the realm of causal sciences. Others, such as Clavius, worked at the institutional level to insure mathematics was given its due share and respect in the school curriculum of the Jesuits. The mathematization of physics posed a serious threat posed to the classical Aristotelian approach in natural philosophy. Thus it is not by chance that many of the Aristotelian scholars who attacked the scientificity of mathematics were professors of natural philosophy. Pereyra, for example, was a colleague of Clavius in the Collegium Romanum and for many years he taught the course in natural philosophy.

But what exactly were the arguments raised against the scientificity of mathematics? {To take but one example, Pereyra’s} strategy was to point to theorems

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6 *De communibus omnium rerum naturalium principiis et affectionibus* ([1562] 1576):

7 in the second part of his *Exercitationes Paradoxicae adversum Aristoteleos*, written in 1624 but published posthumously in 1658

8 Clavius, among the finest mathematicians of the early seventeenth century, taught mathematics in one of the most prestigious learned institutions of the late Renaissance and early seventeenth century, the Collegium Romanum.—PM.
in Euclid that could not easily be interpreted causally. Pereyra considered proposition 1.32 in Euclid’s *Elements*: i.e., the sum of the internal angles in a triangle equals two right angles.⁹

The geometer proves that the triangle has three angles equal to two right ones on account of the fact that the external angle which results from extending the side of that triangle is equal to two angles of the same triangle which are opposed to it. Who does not see that this middle term is not the cause of the property which is demonstrated? . . . Besides, such a {middle term} is related in an altogether accidental way to that property. Indeed, whether the side is produced and the external angle is formed or not, or rather even if we imagine that the production of the one side and the bringing about of the external angle is impossible, nonetheless that property will belong to the triangle; but, what else is the definition of an accident than what may or may not belong to the thing without its corruption?⁷

For Pereyra the middle term was the appeal to the auxiliary segments and to the external angle. This appeal to auxiliary segments shows how the proof is not truly causal, since the result holds even without consideration of the external angle and of the external segments. In other words, the external angle and the auxiliary segments cannot be the true cause of the equality.¹⁰

{Among those who agreed with Piccolomini’s and Pereyra’s denial of “causal explanation” in mathematical demonstration, and thus their denial of the scientificity of mathematics, were Catena¹¹ and the highly respected authors of the *Coimbran Commentaries*. The *Commentaries* were written by several Jesuit professors of

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⁹ Ref. to Euclid. “Recall how the proof goes. Let ABC be a triangle (see Fig. 1). Let BC be produced to D. Draw through C a parallel to BA, say CE. Then by appealing to previous theorems we have BAC = ACE and ECD = ABC. Thus ABC + ACB ± CAB = ACB + ACE ±ECD = two right angles.”—PM.

¹⁰ The {medieval} scholastic tradition would have assumed this to be a causal proof by maintaining that the triangle must have an essence (given by the definition of a triangle) that determines, as in a formal cause, the rest of its properties; i.e., the sum of the internal angles is equal to two right angles. However, a closer look at the theorem makes it hard to isolate the cause from which the property is derived, or in syllogistic terms, to isolate what plays the role of the middle term.—PM.

¹¹ Bio-note
philosophy who taught at the University of Coimbra, Portugal, in the last decade of the sixteenth century. The Commentaries, which were published between 1594 and 16?? ran to ?? volumes and consisted of the Greek texts of Aristotle’s several treatises, their translation into Latin, and extensive glosses and commentaries. The Commentaries were among the most scholarly, thorough, and influential compendia of Aristotelian natural philosophy in the seventeenth century.

Biancani’s *De Mathematicarum Natura* (1615)

The first text in the seventeenth century to address extensively the issues of the Quaestio was by Giuseppe Biancani (1566—1624), a Jesuit professor of mathematics at the University of Parma.19 Biancani had studied mathematics in the Collegium Romanum under Clavius and was very well informed on the status of the Quaestio because Pereyra had taught natural philosophy in the same institution for many years. (In 1615 he published) a lengthy treatise, *Aristotelis Loca Mathematica*, which combined a collection of Aristotle’s passages related to mathematics with and his commentary. His *De Mathematicarum Natura* appeared as an appendix to this work. . . . Biancani’s treatise embodies the reaction of the traditional Aristotelian scholastic against the innovative thesis of Piccolomini, Pereyra, and the Coimbran Commentaries.12 . . .

The second chapter is explicitly aimed at Piccolomini, Pereyra, and the Coimbran Commentaries, and is designed to show there are mathematical demonstrations that proceed from formal and material causes. Biancani begins with an erudite list of quotations from Aristotle, Plato, Proclus, Averroes, Toletus, Themistius, and Zabarella to remark that the tradition has always attributed to geometrical proofs the features of potissimae demonstrations. He then presents some arguments. First of all, Biancani limits the causes employed in geometry and arithmetic to formal and material causes {or intrinsic causes} and excludes the use of efficient and final causes {extrinsic causes}. . .

12 Note on the complexity of this statement: Pereyra et al. are the innovators, while Biancani, as champion of the traditional certainty of mathematical demonstration is the conversative!!
Biancani proceeds to argue that formal and material causes can be found in pure mathematics. Proposition I.1 from Euclid’s *Elements* shows how to construct an equilateral triangle over any given segment. The proof is causal, argues Biancani, since it shows that the cause of the equality of the sides of the triangle ABC is that they are radii of equal circles. And the argument ultimately rests on the definition of the circle, which thus acts as the formal cause of the proof. Biancani then deals with material causes by arguing that proposition 1.32, which we have already analyzed, proceeds by material causes when it infers the equality of the wholes from the equality of the parts.

Although Biancani’s arguments are often unsatisfactory, they do make sense within the Aristotelian framework. Moreover, his work is representative of an effort on the part of these late Aristotelians to proceed to a more careful analysis of mathematical practice (in order to) verify how far the causal model could be applied to mathematics. As we will shortly see, this led the Aristotelians to maintain the causal classification for the majority of demonstrations but to deny it to certain types of proof.

Biancani’s work was well received. His text was followed, often verbatim, by Hugh Simple in *De Disciplinis Mathematicarum* (1635) and by Bettini in *Aerarium Philosophiae Mathematicae* (1648), and is quoted with respect by mathematicians and philosophers such as Barrow, Mersenne, and Guldin. Moreover, Bayle refers

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13 “Its argument goes as follows. Let AB be a segment (see Fig. 2). Draw two circles with radii of equal length AB and centers in A and B, respectively. Let C be one of the points where the circles intersect. Connect A and C and C and B. Then ABC is an equilateral triangle, since its sides are equal to the radius of the same circle, and thus are equal to each other.—PM” Ref. to Euclid, supra.

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14 Bio-note. A Jesuit, no?
15 Bio-note
16 Bio-note
constantly to Biancani in his *Dictionnaire Historique et Critique* in the entry on Zeno of Sidon. {. . .}

**Barrow and Gassendi**

One might think these developments merely peripheral to the main movement of ideas that shaped the seventeenth century. This is not the case. Peter Dear has {argued that} Mersenne’s philosophical outlook owed much to the debate on mathematical certitude:

Mersenne’s scholastic apologia for mathematics shows both a determined and a selective use of argumentative and conceptual resources. All of his positions, equipped with standard arguments, conspired to justify a high estimation of the mathematical disciplines and the kind of knowledge they produced. Mathematical demonstrations were certain; they were also causal and thus scientific; their objects existed archetypically in the mind of God like those of physics; and their objects were necessarily concomitants of God’s creative power. Physics could be made to suffer by comparison, as Clavius had shown: if mathematics fulfilled all these desiderata while physical demonstrations typically failed to achieve that of certainty, a good foundation seemed established for an alternative mathematical natural philosophy to replace essentialist physics.20

The *Quaestio*, {. . .} in its seventeenth-century developments, also involv{ed} scholars such as Smiglecius, Barrow, Gassendi, Hobbes, and Wallis. {. . .} Hobbes dealt with the *Quaestio* in his *Exaininatio et Emendatio Matheniaticae odiernae* (1660), written to confute Wallis’s *Mathesis Universalis* (1657). Hobbes defended the thesis that all mathematical proofs are causal and scientific whereas physics, once thought to be the science that could best display causal reasonings, relies on *hoti* reasonings, which he interprets as fallible reasonings because they rely on induction.

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17 Bio-note  
18 Bio-note  
19 Bio-note  
Now I want to {turn} to Gassendi and Barrow, { . . . since they gave} a broader, and ultimately more interesting, interpretation of the Quaestio as an important event in the history of seventeenth-century skepticism. Let us start with Barrow.\textsuperscript{21} Barrow {wishes} to show that mathematics is a real science against those who “both have been, and still are so subtle as to deny that the Mathematics are truly Sciences, and that they afford true Demonstrations.”\textsuperscript{26} { . . . }

For {these detractors} attempt to prove that Mathematical Ratiocinations are not Scientific, Causal and Perfect, because the Science of a Thing signifies to know it by its Cause; according to that Saying of Aristotle; “We are supposed to know by Science, when we know the Cause.” And to use the Words of Pererius, who was no mean Peripatetic; “A Mathematician neither considers the Essence of Quantity, nor treats of its Affections, as they flow from such Essence, nor declares them by the proper causes by which they are in Quantity, nor forms their Demonstrations from proper and essential, but from common and accidental Predicates.”\textsuperscript{22, 29}

To which I answer, that those scientific Conditions, which Aristotle prefixest to Demonstration, who was most observant of its Laws, do most fitly agree with Mathematical Ratiocinations {whose premisses are universal, necessary, primary, and immediate, and thus they are } More Known and More Evident than the Conclusion inferred.”

{Moreover,}

Mathematical Demonstrations are eminently Causal, from whence, because they only fetch their Conclusions from Axioms which exhibit the principal and most universal Affections of all quantities, and from Definitions which declare the constitutive Generations and essential

\begin{footnotesize}
\begin{enumerate}
\item bio-note. “Barrow’s contribution to the Quaestio occurred in his Lectiones,\textsuperscript{25} especially the fifth and the sixth, which were entitled respectively “Containing answers to the objections which are usually brought against mathematical demonstrations,” and “Of the causality of mathematical demonstrations.”
\item Insert M.‘s fn 29
\end{enumerate}
\end{footnotesize}
Passions of particular Magnitudes. From whence the Propositions that arise from such Principles supposed, must needs flow from the intimate **Essences** and **Causes** of the Things.\textsuperscript{32}

\{ . . . \}

Barrow \{then\} proceeded to discuss \{Euclid’s Proposition\} 1.32, which had played the role of paradigm examples throughout the debate: “\textit{Pererius, and others, do produce another Instance, also blaming that celebrated Proposition which is the thirty-second of the first Element, as not scientifically demonstrated.}”\textsuperscript{38} He then went on to summarize the main criticisms by Pereyra and made four replies. In the first reply he invoked the authority of Aristotle who, he claimed, quoted this proposition as an example of causal demonstration; in his second remark he argued that since a triangle is constituted by straight lines then what is essential to lines also pertains to the triangle. “But it is the Property of a Right Line that it may be produced, therefore this Production is not altogether accidental or external to a Triangle.” The third point argued that division of the external angle is the most natural way to obtain the sought result. His fourth and last point was that one can give a step-by-step analysis showing how Euclid’s proof conforms to his schema for mathematical demonstrations, which as he has already argued, embodies the form of causal, and hence scientific, demonstrations. Barrow could at last conclude by boasting \{of\} the superiority of mathematical demonstrations:

\begin{quote}
It seems to me . . . that Demonstrations, though some do outdo others in Brevity, Elegance, Proximity to their first Principles, and the like Excellencies, yet are all alike in Evidence, Certitude, Necessity, and the essential Connection and mutual Dependence of the Terms one with another. Lastly, that Mathematical Ratiocinations are the most perfect Demonstrations.\textsuperscript{39}
\end{quote}

\{ . . . \} There does not seem to be any reason why Barrow would want to spend so many pages arguing against an author like Pereyra, who was far from being an authority. Barrow himself gives us a clue when he says that “some both have been, and \textit{still are so subtle} as to deny that the mathematics are truly sciences, and that they
afford true demonstrations." I believe that Barrow is addressing not Pereyra but Gassendi. {Barrow delivered these lectures in 1665}. Only a few years before, in 1658, Gassendi’s Opera Omnia had been published. The third volume contained a work written by Gassendi in 1624 but never published, the second part of the Exercitationes Paradoxicae adversus Aristoteleos. In the sixth Exercitatio, “That no science exists, and especially no Aristotelian science”, Gassendi argued that none of the so-called sciences could be said to provide Aristotelian knowledge, that is, causal knowledge from the essences of the subjects. { . . . } Gassendi quoted at length from Pereyra {and used his} to support his general attempt to show { . . . } “that whatever certainty and evidence there is in mathematics is related to appearances, and in no way related to the genuine causes of things.”\textsuperscript{41} Thus, Barrow had in mind not an obscure Jesuit from the previous century but an adversary of the caliber of Gassendi, whose influence on the philosophical world had already proved to be immense and therefore deserved an extensive {re}futation.\textsuperscript{42}

Gassendi’s appeal to the Quaestio to support his skeptical position as to the nature of our knowledge raises the further issue of the relationship between the Quaestio de Certitudine Mathematicarum of the sixteenth century and skepticism in the seventeenth century.