

Name:

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TTH 10 Peter Smillie
TTH 11:30 Aukosh Jagannath
TTH 11:30 Sebastian Vasey

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work.** Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

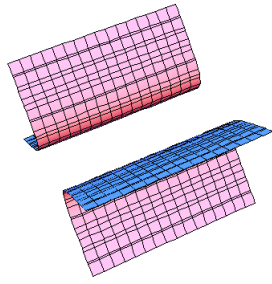
Mark for each of the 20 questions the correct letter. No justifications are needed.)

T  F

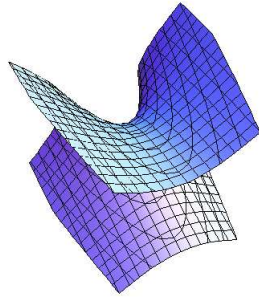
- 2)  T  F The length of the unit tangent vector  $\vec{T}$  for a curve  $\vec{r}(t)$  is independent of  $t$ .
- 3)  T  F For all vectors  $\vec{v}$  and  $\vec{w}$  the vector  $\vec{w} \times (\vec{w} \times \vec{v})$  is perpendicular to  $\vec{v}$ .
- 4)  T  F There is a point  $(x, y, z)$  in space, for which the cylindrical coordinates  $(r, \theta, z)$  and spherical coordinates  $(\rho, \theta, \phi)$  satisfy  $(r, \theta, z) = (\rho, \theta, \phi - \pi/2)$ .
- 5)  T  F The two planes  $x + y - z = 1$  and  $-x - y + z = 2$  intersect in a line.
- 6)  T  F  $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = 0$  implies  $|u| = |v|$ .
- 7)  T  F The contour curves  $\sin(x) + y = 1$  and  $\sin(x) + y = 2$  do not intersect.
- 8)  T  F There is a vector  $\vec{v}$  for which the vector projection  $\text{proj}_{\vec{v}}(\vec{j})$  is equal to  $2\vec{j}$ .
- 9)  T  F  $(\vec{k} \times \vec{i}) \times \vec{i} = \vec{j} \times (\vec{i} \times \vec{k})$
- 10)  T  F If a curve  $\vec{r}(t)$  lies in a plane, goes through the point  $(1, 1, 1)$ , and has the binormal vector  $\vec{B}(t) = \langle 3, 4, 5 \rangle$ , then the plane is  $3x + 4y + 5z = 12$ .
- 11)  T  F The angle between  $\vec{r}'(t)$  and  $\vec{r}''(t)$  is always 90 degrees.
- 12)  T  F A line intersects a hyperbolic paraboloid always in 2 distinct points.
- 13)  T  F There is a quadric surface, each of whose intersections with the coordinate planes is either an ellipse or a parabola.
- 14)  T  F The equation  $x^2 - y^2 - z^2 = 1$  defines a one-sheeted hyperboloid.
- 15)  T  F The function  $f(x, y) = 1/(1 + x^2 + y^2)$  is continuous everywhere.
- 16)  T  F If the number  $\vec{u} \cdot (\vec{v} \times \vec{w})$  is positive, then  $(\vec{w} \times \vec{v}) \cdot \vec{u}$  is positive.
- 17)  T  F The number  $|\vec{u} \times (\vec{v} \times \vec{w})|$  is the volume of the parallelepiped spanned by  $\vec{u}, \vec{v}$  and  $\vec{w}$ .
- 18)  T  F The set of points  $P$  for which the distance of  $P$  to the point  $(0, 0, 0)$  is 1 less than the distance to the point  $(0, 0, 2)$  is a paraboloid.
- 19)  T  F If  $\vec{v}, \vec{w}$  are two nonzero vectors, then the projection vector  $\text{proj}_{\vec{w}}(\vec{v})$  can be longer than  $\vec{v}$ .
- 20)  T  F The number  $|\vec{v} \times \vec{w}|$  is the area of the parallelogram spanned by  $\vec{v}$  and  $\vec{w}$ .

Problem 2a) (5 points)

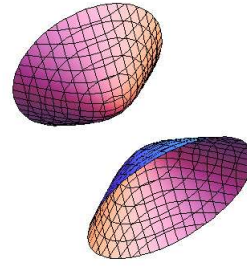
Match the equations with the pictures. No justifications are necessary in this problem.



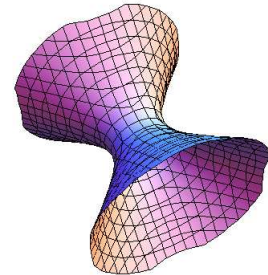
I



II

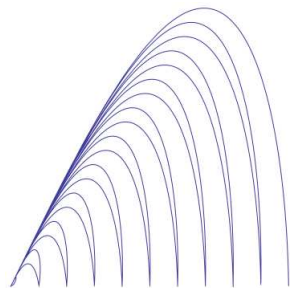


III

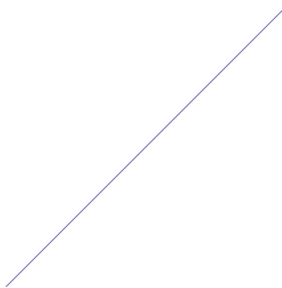


IV

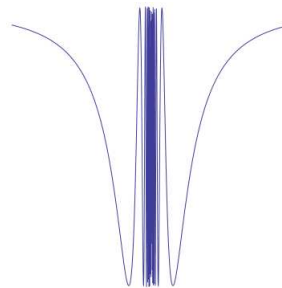
Enter I,II,III,IV here	Equation
	$x^2 + y - z^2 - 1 = 0$
	$y^2 - 2z^2 - 1 = 0$
	$x^2 - y^2 + z^2 + 1 = 0$
	$x^2 - y^2 + z^2 - 1 = 0$



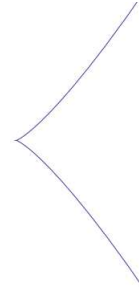
1



2

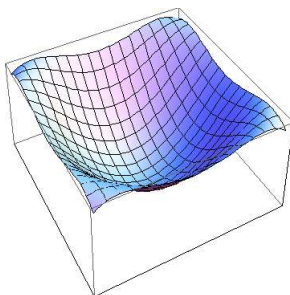


3

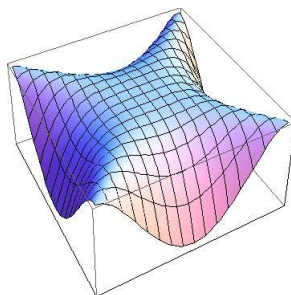


4

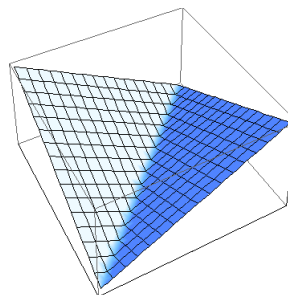
Enter 1,2,3,4 here	Equation
	$\langle t^4, 1 + t^5 \rangle$
	$\langle t \cos(5t), t \cos(5t) \rangle$
	$\langle  t \cos(5t) ,  t \sin(10t)  \rangle$
	$\langle 3 + 2t, \cos(1/t) \rangle$



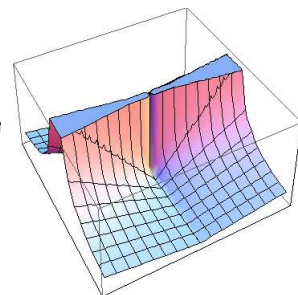
A



B



C

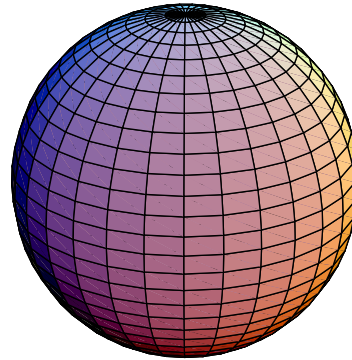
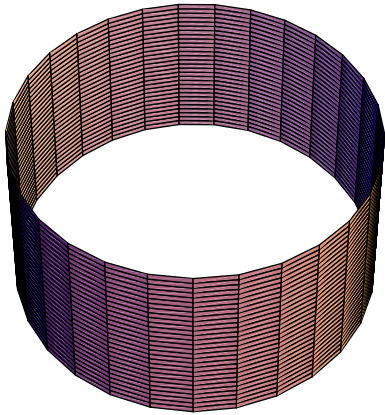


D

Enter A,B,C,D here	Equation
	$f(x, y) =  x/y $
	$f(x, y) = \sin(x^2 + y^2)$
	$f(x, y) =  x - y $
	$f(x, y) = \cos(x^2 - y^2)$

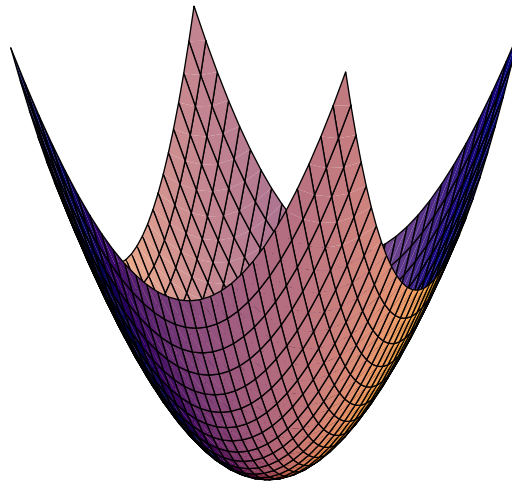
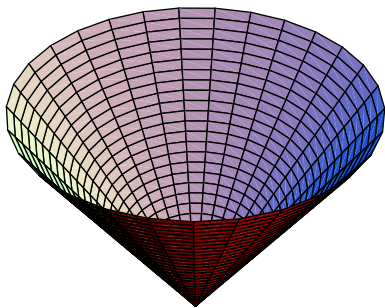
Problem 2b) (5 points)

Match the surfaces with their parametrizations as well as with the description either in cylindrical coordinates  $(r, \theta, z)$  or in spherical coordinates  $(\rho, \phi, \theta)$ .



I

II



III

IV

Enter I,II,III,IV here	Parametrization of the surface
	$\langle 3 \cos(\theta), 3 \sin(\theta), 2z \rangle$ .
	$\langle x, y, 3x^2 + 3y^2 \rangle$
	$\langle 3 \sin(\phi) \cos(\theta), 3 \sin(\phi) \sin(\theta), 3 \cos(\phi) \rangle$
	$\langle 3z \cos(\theta), 3z \sin(\theta), 2z \rangle$
Enter I,II,III,IV here	Description in cylindrical or spherical coordinates
	$z = 3r^2$
	$3z = 2r$
	$\rho = 3$
	$r = 3$

Problem 3) (10 points)

A tetrahedron has the vertices  $A = (1, 1, 0)$ ,  $B = (3, 2, 0)$ ,  $C = (2, 1, 1)$ ,  $D = (3, 2, 1)$  with base triangle  $A, B, C$ .

a) (5 points) Find the height of the tetrahedron.

b) (5 points) The volume of a tetrahedron is the base area times height divided by 3. What is the volume of the tetrahedron with vertices  $A, B, C, D$ .

Problem 4) (10 points)

Find a parametrization of the line containing the two planes

$$2x + y + z = 4$$

and

$$x - y + 2z = 5 .$$

Problem 5) (10 points)

What is the distance between the two cylinders  $x^2 + y^2 = 1$  and  $(z - 2)^2 + (x - 5)^2 = 4$ ?

Problem 6) (10 points)

Find the arc length of the parameterized curve

$$\vec{r}(t) = \left\langle 2 \sin(t), \frac{t^4}{4} + \frac{1}{2t^2}, 2 \cos(t) \right\rangle$$

from  $t = 1$  to  $t = 2$ .

Problem 7) (10 points)

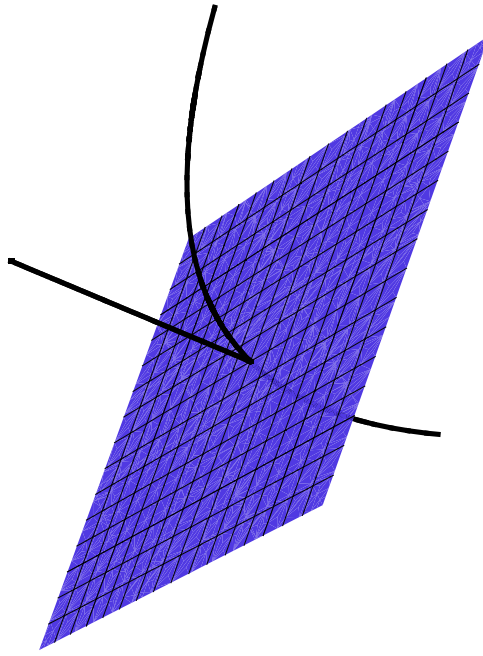
At time  $t = 0$  two trapeze artists have positions  $\vec{r}(0) = \langle 0, 0, 25 \rangle$  and  $\vec{s}(0) = \langle 10, 0, 23 \rangle$  and velocities  $\vec{r}'(0) = \langle 2, 0, 1 \rangle$  and  $\vec{s}'(0) = \langle -3, 0, 2 \rangle$ . They both experience a constant gravitational acceleration  $\langle 0, 0, -10 \rangle$ . Find the paths  $\vec{r}(t)$ ,  $\vec{s}(t)$  and determine at which point the artists meet.



Problem 8) (10 points)
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The angle between a curve and a plane is defined as  $\pi/2 - \alpha$ , where  $\alpha$  is the angle between the normal vector to the plane and the velocity vector of the curve at the point of intersection.

- a) (3 points) Find a normal vector to the plane  $x + y - z/2 = 1$ .
- b) (3 points) What is the velocity vector to the curve  $C : \vec{r}(t) = \langle 1, 0, 0 \rangle + t\langle 1, 1, 3 \rangle + t^2\langle 1, 1, 1 \rangle$  at time  $t = 0$ ?
- c) (4 points) Find the angle (in radians) between the plane  $x + y - z/2 = 1$  and the curve  $C$  at the point of intersection  $\vec{r}(0)$ .



Problem 9) (10 points)

The intersection of the paraboloid

$$x^2 + y^2 - z = 5$$

with the plane

$$x + y = 5$$

is a curve. Find the parametrization of this curve.

Problem 10) (10 points)

- (3 points) Parametrize the plane containing the three points  $A = (1, 1, 1)$ ,  $B = (1, 3, 2)$  and  $C = (3, 4, 5)$ .
- (4 points) Parametrize the sphere which is centered at  $(1, 1, 1)$  and has radius 3.
- (3 points) Parametrize the surface which is given in spherical coordinates as  $\rho = 3 + \sin(\phi) \sin(\theta)$ .