Name:

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MWF 9 Dennis Tseng
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MWF 10 Koji Shimizu
MWF 11 Oliver Knill
MWF 11 Chenglong Yu
MWF 12 Stepan Paul
TTH 10 Matt Demers
TTH 10 Jun-Hou Fung
TTH 10 Peter Smillie
TTH 11:30 Aukosh Jagannath
TTH 11:30 Sebastian Vasey

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False (TF) questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed. For

1) T F

2) T F There are unit vectors $\vec{v}$ and $\vec{w}$ in space for which $|\vec{v} \times \vec{w}| = 2$.

3) T F The vector $\langle 4,5,0 \rangle$ is perpendicular to the plane $-5x + 4y + z = 2$.

4) T F The distance between the cylinders $x^2 + z^2 = 1$ and $x^2 + (z-3)^2 = 1$ is 3.

5) T F The vector projection of $\langle 2,3,1 \rangle$ onto $\langle 1,1,1 \rangle$ is parallel to $\langle 1,1,1 \rangle$.

6) T F The equation $\rho \sin(\theta) \sin(\phi) = 2$ in spherical coordinates defines a plane.

7) T F There is a planar curve for which the arc length is 2π and the curvature is constant 1.

8) T F If we know the intersection of a graph $z = f(x,y)$ with the coordinate planes $x = 0, y = 0$ and $z = 0$, the function f is determined uniquely.

9) T F The surface given in cylindrical coordinates as $r = z^2$ is a paraboloid.

10) T F If the curvature of a space curve is constant 2 everywhere along the curve then the curve is a circle.

11) T F If $\vec{u}, \vec{v}$, and $\vec{w}$ are unit vectors then the volume of the parallelepiped spanned by $\vec{u}, \vec{v}$, and $\vec{w}$ is largest when the parallelepiped is a cube.

12) T F If a point is moving along a straight line parametrized by $\vec{r}(t)$ then the velocity $\vec{r}\,'(t)$ vector and acceleration vector $\vec{r}\,''(t)$ must be parallel.

13) T F The parametrization $\vec{r}(u,v) = \langle v\cos(u), v\sin(u), v \rangle$ with $0 \leq u < 2\pi$ and $v \in \mathbf{R}$ is a cylinder.

14) T F If two lines in space are not parallel, then they must intersect.

15) T F If two planes do not intersect, then their normal vectors are parallel.

16) T F $(\vec{i} \times \vec{j})$ and $(\vec{i} \times (\vec{i} \times (\vec{i} \times \vec{j})))$ are parallel.

17) T F The surface parametrized by $\vec{r}(u,v) = \langle \sin(u)\sin(v), \sin(u)\cos(v), \cos(u) \rangle$ with $0 \leq v < 2\pi, 0 \leq u \leq \pi$ is a sphere.

18) T F The unit tangent vector $\vec{T}$ to a curve at a given point is independent of the parametrization up to a factor of -1.

19) T F $z^2 = r^2(\cos^2(\theta) - \sin^2(\theta)) + 1$ is a one-sheeted hyperboloid.

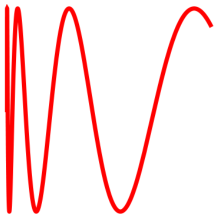
20) T F If $\vec{a} \cdot \vec{b} > 0$ and $\vec{b} \cdot \vec{c} > 0$, then $\vec{a} \cdot \vec{c} > 0$.

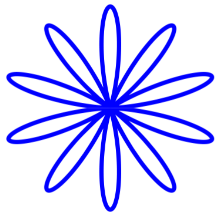
	Total

Problem 2) (10 points)

No explanations needed. I,II,III,O appear all once in each box.

a) (2 points) Match curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.


I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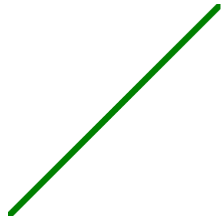
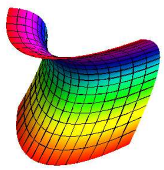
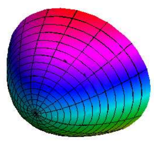

III

Parametrization $\vec{r}(t) =$	O, I,II,III
$\sin^2(5t)\langle\cos(t),\sin(t)\rangle$	
$\langle t^3,\sin(7t)\rangle$	
$\langle t^5, 1+t^5\rangle$	
$\langle\sin(t),\cos(t)\rangle$	

b) (2 points) Match the parametrization. Enter O, where no match.


I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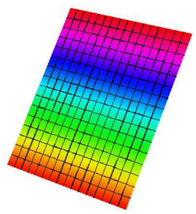
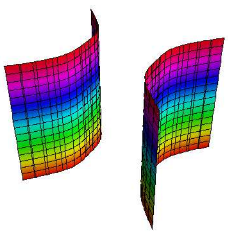
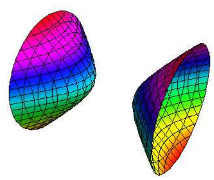

III

$\vec{r}(s,t)$	O,I,II,III
$\langle 1-t, 1+s, 2+s\rangle$	
$\langle s, t^2-s^2, t\rangle$	
$\langle t\cos(s), t\sin(s), s\rangle$	
$\langle s\cos(t), s^2, s\sin(t)\rangle$	

c) (2 points) The pictures show contour surfaces. Enter O, where no match.


I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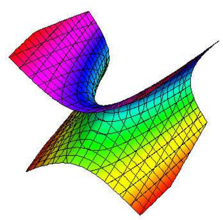
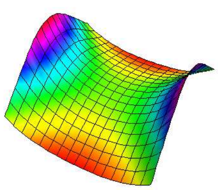
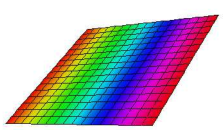

III

$g(x,y,z) =$	O, I,II,III
$x^2-y^2+z^2=-1$	
$x^2-y^2=1$	
$x^4+z=1$	
$x^2+y-z^2=1$	

d) (2 points) Match the graphs $z=f(x,y)$ with the functions. Enter O, where no match.


I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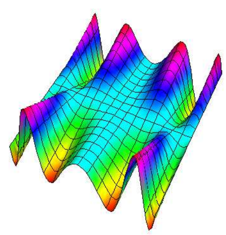
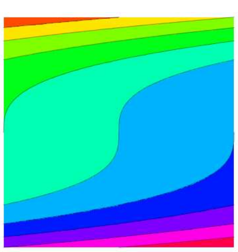

III

function $f(x,y) =$	O,I,II,III
$2x$	
$e^{-2x^2-2y^2}$	
$e^{x^2-y^2}$	
$y\sin(x^2)$	

e) (2 points) Match the family of level curves with $f(x,y)$. Enter O, where no match.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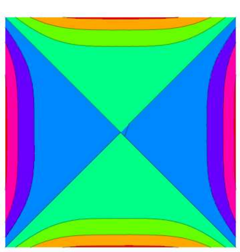
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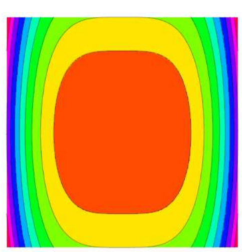

III

Function $f(x,y) =$	O,I,II,III
x^4+y^2	
x^4-y^4	
$x-y$	
$x-y^3$	

Problem 3) (10 points)

No explanations needed. In 3a), in each row check only one box.

a) (4 points) The intersection of a plane with a cone $S: x^2 + y^2 - z^2 = 0$ is called a **conic section**. What curve do we get?

Intersect S with	hyperbola	parabola	circle	line
$z = 1$ gives a				
$z = x$ gives a				
$z = x + 1$ gives a				
$x = 1$ gives a				

b) (3 points) By intersecting the upper half sphere $x^2 + y^2 + z^2 = 5, z > 0$ with the hyperboloid $x^2 + y^2 - z^2 = -3$ we get a curve. Which one? Check exactly one box.

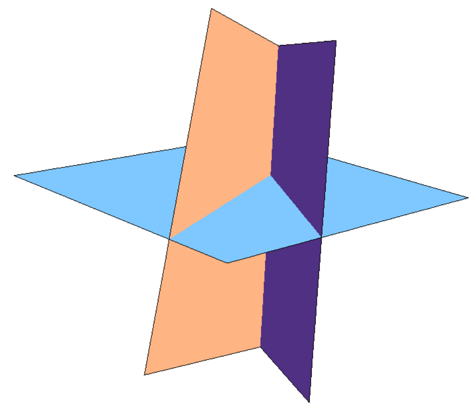
$\vec{r}(t) = \langle \cos(t), \sin(t), 2 \rangle$	
$\vec{r}(t) = \langle 0, 0, t \rangle$	
$\vec{r}(t) = \langle \cos(t), \sin(t), 2t \rangle$	

c) (3 points) Which of the following surface parametrizations gives a one sheeted hyperboloid? Check exactly one box.

$\vec{r}(t,s) = \langle s, t, s^2 - t^2 \rangle$	
$\vec{r}(t,s) = \langle \sqrt{1+s^2}\cos(t), \sqrt{1+s^2}\sin(t), s \rangle$	
$\vec{r}(t,s) = \langle \sqrt{1-s^2}\cos(t), \sqrt{1-s^2}\sin(t), s \rangle$	

Problem 4) (10 points)

We are given two planes $x+y+z=1$ and $x-y-z=2$. Find a third plane which contains the point $(1,0,0)$ and which is perpendicular to both.

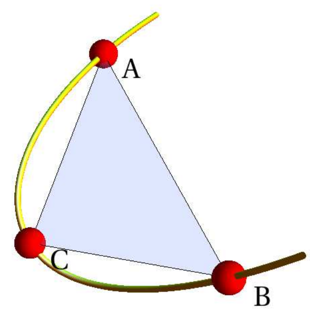


Problem 5) (10 points)

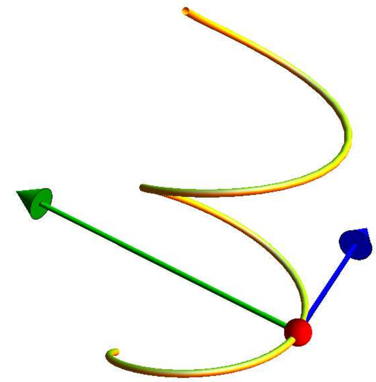
We are given a curve $\vec{r}(t) = \langle 1+t, t^2, t^3 \rangle$.

a) (5 points) Find the area of the triangle with vertices $A = \vec{r}(-1), B = \vec{r}(1)$ and $C = \vec{r}(0)$.

b) (5 points) Find an equation $ax+by+cz=d$ for the plane through A, B, C.



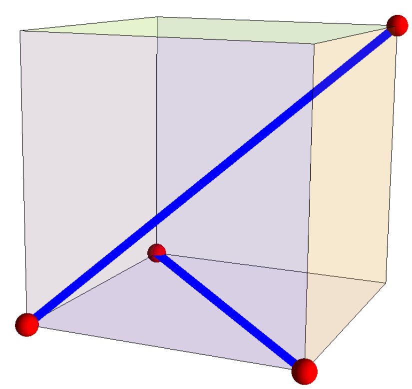
Problem 6) (10 points)



a) (3 points) Find the unit tangent vector $\vec{T}(t)$ of the curve $\vec{r}(t) = \langle t^2, \cos(t^2\pi), \sin(t^2\pi)\rangle$ at $t = 1$.
b) (3 points) What is the acceleration vector $\vec{r}\,''(t)$ at $t = 1$?
c) (4 points) Find the curvature at the time $t = 1$. You may use the formula

$$\kappa = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3} \; .$$

Problem 7) (10 points)



What's the closest that the long diagonal of the unit cube connecting the corners $(0,0,0)$ to $(1,1,1)$, comes to the diagonal of a face connecting the corners $(1,0,0)$ and $(0,1,0)$?

Problem 8) (10 points)

In a parallel universe of ours, the inhabitants live under a "Newton's law" of gravity in which the "jerk" $\vec{r}\,'''(t)$ rather than the acceleration is constant. Suppose that $\vec{r}\,'''(t) = \langle 0, 0, -10\rangle$ for all t.

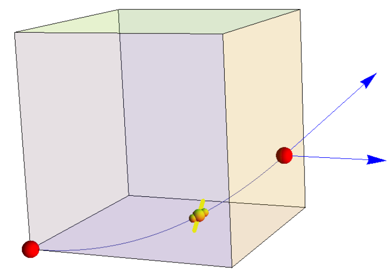
a) (3 points) Find $\vec{r}\,''(t)$ if you know $\vec{r}\,''(0) = \langle 0,0,0\rangle$.
b) (3 points) Now find $\vec{r}\,'(t)$ if we know also $\vec{r}\,'(0) = \langle 1,0,0\rangle$.
c) (4 points) Finally find $\vec{r}(t)$ if we know additionally $\vec{r}\,(0) = \langle 0,0,10\rangle$.

Problem 9) (10 points)

a) (2 points) A fly is trapped inside a unit cubicle made of planar glass panes. It flies, starting at $t = 0$ at the origin $(0, 0, 0)$ along the curve

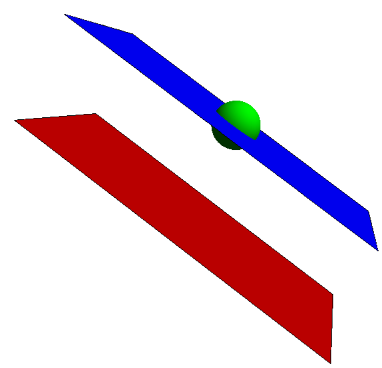
$$\vec{r}(t) = \langle t, \frac{t^2}{\sqrt{2}}, \frac{t^3}{3} \rangle \ .$$

At what time does it bump into the glass wall $x = 1$?
b) (4 points) Find the impact angle (= the angle between the normal vector of the plane and the velocity vector).
c) (4 points) How long is the path it has followed from $t = 0$ to the impact point?



Problem 10) (10 points)

When two uncharged metallic parallel plates are put close together, there is an attractive force between them which can be explained by quantum field theory only. In May 14, 2013, an article suggested to use this Casimir effect for microchip designs. (Source Nature: http://www.nature.com/ncomms/journal/v4/n5/full/nc



a) (3 points) Locate a point P on the plane $x + 2y + 2z = 4$.

b) (7 points) Find the distance d between the plane $x + 2y + 2z = 1$ and plane $x + 2y + 2z = 4$.