Name:

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TTH 10 Jun-Hou Fung
TTH 10 Peter Smillie
TTH 11:30 Aukosh Jagannath
TTH 11:30 Sebastian Vasey

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False (TF) questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.1) T F The

2) T F Any three distinct points A, B, C in space determine a unique plane which passes through these points.

3) T F For any two non-intersecting lines L, K, there are two parallel planes Σ, Δ whose distance $d(\Sigma, \Delta)$ is equal to the distance $d(L, K)$ such that L is in Σ and K is in Δ.

4) T F If $z - f(x, y) = g(x, y, z)$ then the graph of $f(x, y)$ is a level surface $g(x, y, z) = c$ of $g(x, y, z)$.

5) T F The graph of the function $f(x, y) = x^2 + y$ is called an elliptic paraboloid.

6) T F The equation $\rho \sin(\phi) \sin(\theta) = 1$ in spherical coordinates defines a plane.

7) T F The vector $\langle 1, 2, 3 \rangle$ is parallel to the plane $2x + 4y + 6z = 4$.

8) T F The cross product between $\langle 2, 3, 1 \rangle$ and $\langle 1, 1, 1 \rangle$ is 6.

9) T F The curve $\vec{r}(t) = \langle \cos(t), t^2, \sin(t) \rangle, 1 \leq t \leq 9$ and the curve $\vec{r}(t) = \langle \cos(t^2), t^4, \sin(t^2) \rangle, 1 \leq t \leq 3$ have the same length.

10) T F The point $(1, -1, 1)$ has the spherical coordinates of the form $(\rho, \theta, \phi) = (\sqrt{3}, \pi/4, \pi/4)$.

11) T F The distance between two parallel lines in space is the distance of any point on one line to the other line.

12) T F For two nonzero vectors $\vec{v}$ and $\vec{w}$, the identity $\text{Proj}_{\vec{w}}(\vec{v} \times \vec{w}) = \vec{0}$ holds.

13) T F The vector projection of $\langle 2, 3, 4 \rangle$ onto $\langle 1, 0, 0 \rangle$ is $\langle 2, 0, 0 \rangle$.

14) T F The triple scalar product $\vec{u} \cdot (\vec{v} \times \vec{w})$ between three vectors $\vec{u}, \vec{v}, \vec{w}$ is zero if and only if two or more of the 3 vectors are parallel.

15) T F There are two vectors $\vec{v}$ and $\vec{w}$ so that the dot product $\vec{v} \cdot \vec{w}$ is equal to the length of the cross product $|\vec{v} \times \vec{w}|$.

16) T F The distance between two spheres of radius 2 whose centers have distance 8 is 4.

17) T F If two vectors $\vec{v}$ and $\vec{w}$ are both parallel and perpendicular, then at least one of the vectors must be the zero vector.

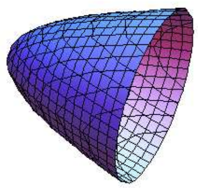
18) T F The curvature $\kappa(\vec{r}(t))$ is always smaller than or equal to the length $|\vec{r}\,''(t)|$ of the acceleration vector $\vec{r}\,''(t)$.

19) T F The curve $\vec{r}(t) = \langle \cos(t^2) \sin(t^2), \sin(t^2) \sin(t^2), \cos(t^2) \rangle$ is located on a sphere.

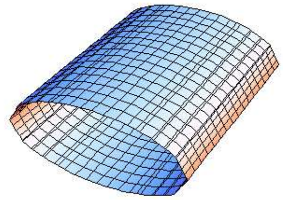
20) T F The surface $x^2 + y^2 + z^2 = 2z$ is a sphere.

Problem 2) (10 points)

a) (6 points) Match the surfaces the equations $g(x,y,z)=0$.



I



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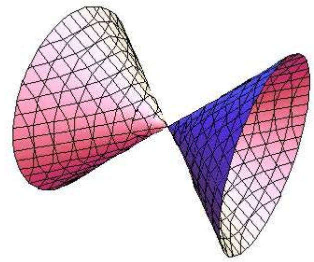
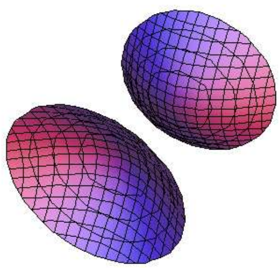
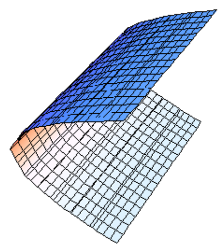


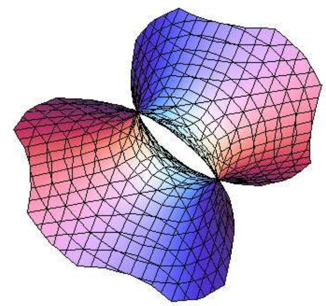
III



IV



V



VI

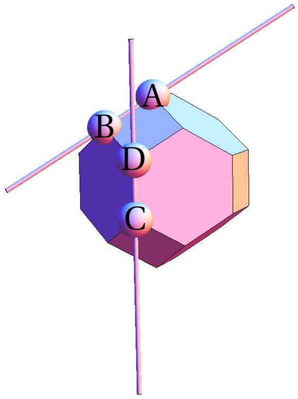
Function $g(x,y,z)=0$	Enter I,II,III,IV,V,VI	Function $g(x,y,z)=$	Enter I,II,III,IV,V,VI
y^2+z^2-x		$x^2-y^2-z^2+1$	
$y^2/4+z^2/4-1$		$x-z^2$	
$x^2-y^2-z^2-1$		$y^2-z^2+x^2$	

b) (4 points) Match the surfaces given in cylindrical and spherical coordinates with the surfaces given in Cartesian coordinates:

	surface
A	$r=1$
B	$\sin(\theta)=0$
C	$\cos(2\phi)=0$
D	$\rho=1$

Enter A-D	surface
	$x^2+y^2=1$
	$x^2+y^2=z^2$
	$x^2+y^2+z^2=1$
	$y=0$

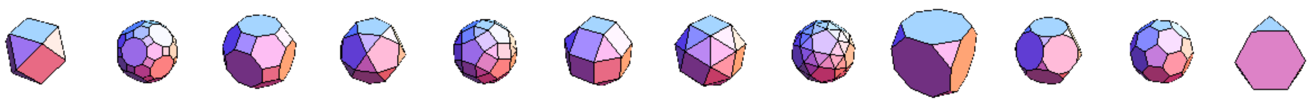
Problem 3) (10 points)



A **truncated octahedron** has an edge connecting the vertices $A = (-1, 3, 0), B = (-1, 1, -1)$ and an edge connecting the vertices $C = (-3, -1, 0), D = (-3, 1, 0)$.

a) (5 points) Find the distance of C to the line through A, B.

b) (5 points) Find the distance between the line L through A, B and the line K through C, D.



Here are the remaining 12 **Archimedean solids**. These are polyhedra bound by different types of regular polygons but for which each vertex of the polyhedron looks the same. There are 13 such semiregular polyhedra. Archimedes studied them first in 287BC. Kepler was the first to describe the complete set of 13 in his work **"Harmonices Mundi"** of 1619.

Problem 4) (10 points)

a) (3 points) Give a parametrization $\vec{r}(\theta, z) = \langle x(\theta, z), y(\theta, z), z(\theta, z) \rangle$ of the surface which is in cylindrical coordinates given by

$$r = z^4 .$$

b) (2 points) Find a parametrization $\vec{r}(u, v)$ of the graph $z = \sin(xy)$.

c) (2 points) Find a parametrization $\vec{r}(u, v)$ of the yz-plane $x = 0$.

d) (3 points) Give a parametrization $\vec{r}(\phi, \theta)$ of the surface which is in spherical coordinates given by

$$\rho = 2 + \cos(13\phi) .$$

Problem 5) (10 points)

a) (7 points) Find the arc length of the curve

$$\vec{r}(t) = \langle \cos(t^2/2), \sin(t^2/2), (1/3)(1 - t^2)^{3/2} \rangle$$

from $0 \leq t \leq 1$.

b) (3 points) Decide whether the function

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is continuous.

Problem 6) (10 points)

Wall-e explores a planet, where a strong solar wind produces a time-dependent magnetic field and where the combined force of gravity and magnetic lift produces a time dependent vertical acceleration

$$\vec{r}\,''(t) = \langle 0, 0, 10\cos(t) \rangle \ .$$

a) (6 points) Wall-e knows that he is at time $t = 0$ at $\vec{r}(0) = \langle 1, 2, 3 \rangle$ with velocity $\langle 0, 1, 2 \rangle$. Where is he at time $t = \pi$?

b) (4 points) What speed does he have at time $t = \pi$?

Problem 7) (10 points)

Potter plays Quidditch. At time $t = 0$ he is at $P = (1, 3, 5)$. At time $t = 1$ he is at $Q = (0, 1, 3)$. Harry is spell-bound and can not change direction, nor change speed and crashes into a tilted side wall of the stadium crushing his knee (*).

a) (3 points) If Potter flies on a straight line through PQ, find a parametrization for that line.

b) (4 points) Where and when does he hit the tilted side wall $x + y + z = 1$ of the stadium?

c) (3 points) What is the angle between Harry's velocity vector and the upwards pointing normal vector of the side wall?

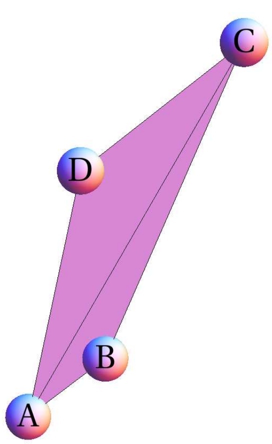
(*) Don't worry, Madam Pomfrey will fix it.

Problem 8) (10 points)

No justifications are needed in this problem. All vectors $\vec{v}, \vec{w}, \vec{r}\,', \vec{r}\,'', \vec{T}, \vec{T}\,'$ can be assumed to be nonzero. The vector $\vec{N} = \vec{T}'/|\vec{T}'|$ is the normal vector and $\vec{B}$ is the binormal vector. Recall that two vectors are perpendicular, if and only if their dot product is zero and that two vectors are parallel if and only if their cross product is the zero vector.

first vector	second vector	always parallel	always perpendicular	neither
$\vec{T}'(0)$	$\vec{r}''(0)$			
$\vec{w}$	$(\vec{v} \times \vec{w}) \times \vec{v}$			
$\vec{v}$	$\vec{v}/\lvert\vec{v}\rvert$			
$\vec{v} = \langle a, b, c \rangle$	normal to $ax + by + cz = 4$			
$\vec{T}$	$\vec{r}\,'$			
$\vec{T}$	$\vec{T}\,'$			
$\text{Proj}_{\vec{v}}(\vec{w})$	$\vec{v}$			
$\text{Proj}_{\vec{v}}(\vec{w})$	$\vec{w}$			
$(\vec{v} + \vec{w}) \times \vec{w}$	$\vec{v} \times \vec{w}$			
$\vec{B}$	$\vec{N}$			

Problem 9) (10 points)



The four points $A = (0, 0, 5), B = (1, 1, 6), C = (2, 4, 11), D = (0, 2, 9)$ are in a plane.

a) (5 points) Find the equation $ax + by + cz = d$ for this plane.

b) (5 points) The quadrilateral $ABCD$ is the union of two triangles ABC and ACD. Find the area of the quadrilateral.

Problem 10) (10 points)

In this problem we find some parametrizations of surfaces which is of the form

$$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle \,.$$

a) (2 points) Parametrize the **paraboloid** $z = x^2 - y^2$.

b) (3 points) Parametrize (the entire!) **ellipsoid** $(x-1)^2 + \frac{(y-2)^2}{4} + z^2 = 1$.

c) (2 points) Parametrize the **plane** $x + y + z = 3$.

d) (3 points) Parametrize the **cylinder** $x^2 + z^2 = 1$.