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- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see details of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
14		10
Total:		150

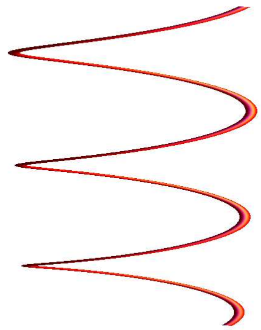
Problem 1) True/False questions (20 points). No justifications are needed.

1)	T F	The tangent plane of the surface $z = f(x,y)$ at a local maximum of f is parallel to the xy-plane
2)	T F	For any smooth functions $f(x,y), x(t), y(t)$, we have $\frac{d}{dt} f(x(t),y(t)) = f_x(x(t),y(t))x'(t) + f_y(x(t),y(t))y'(t)$.
3)	T F	At a local maximum of a function $f(x,y)$ we always have $f_{xx} \leq 0$ and $f_{yy} \leq 0$.
4)	T F	If $(0,0)$ is a critical point for a function $f(x,y)$ as well as for a function $g(x,y)$ then $(0,0)$ is a critical point of the function $f(x,y)+g(x,y)$.
5)	T F	The curves $\vec{r}(t) = \langle t, 2t \rangle$ and $\vec{s}(t) = \langle 2t, -t \rangle$ intersect at a right angle at $(0,0)$.
6)	T F	The quadric $x - y^2 + z^2 = 5$ is a hyperbolic paraboloid.
7)	T F	If $\vec{u}, \vec{v}, \vec{w}$ are unit vectors, then the length of the vector projection of $\vec{u} \times \vec{v}$ onto $\vec{w}$ is the same as the length of the vector projection of $\vec{v} \times \vec{w}$ onto $\vec{u}$.
8)	T F	The partial differential equation $u_{tt} = u_{xx}$ is called the Clairaut equation.
9)	T F	$\iint_R \sqrt{1 - x^2 - y^2}\ dx\ dy = \frac{2\pi}{3}$, where R is the region $\{(x,y) \mid x^2 + y^2 \leq 1\}$ in the xy-plane.
10)	T F	There exists a vector field $\vec{F}(x,y,z)$ in space such that $\text{curl}(\vec{F}) = \langle 5x, -11y, 7z \rangle$.
11)	T F	Let S is the upper hemisphere $x^2+y^2+z^2 = 1, z \geq 0$ with normal pointing away from the center. Then the flux integral is $\iint_S \langle 0,0,1 \rangle \cdot\ d\vec{S} = 2\pi$.
12)	T F	The points that satisfy $\theta = \pi/4$ and $\phi = \pi/4$ form a surface which is part of a cone.
13)	T F	The curvature of the curve $\vec{r}(t) = \langle t, t, t^2 \rangle$ at $t = 0$ is equal to the curvature of the curve $\vec{s}(t) = \langle t^3, t^3, t^6 \rangle$ at $t = 0$.
14)	T F	If $f(x,y,z)$ is a function and $\vec{F} = \nabla f$ then $\text{div}(\vec{F}) = 0$ everywhere (i.e. $\vec{F}$ is incompressible).
15)	T F	For any function $f(x,y,z)$ we have $\text{curl}(\text{curl}(\text{grad}(f))) = \vec{0}$.
16)	T F	For any vector field $\vec{F}$ and any curve $\vec{r}$ parametrized on $[a,b]$ we have $\int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\ dt = \vec{F}(\vec{r}(b)) - \vec{F}(\vec{r}(a))$.
17)	T F	There exist vector fields $\vec{F}$ and $\vec{G}$ in space such that $\text{curl}(\vec{F}) = \text{grad}(\vec{G})$.
18)	T F	If $\vec{F}$ is a smooth vector field in space and S is a closed oriented surface, then $\iint_S \text{curl}(\vec{F}) \cdot d\vec{S} = 0$.
19)	T F	The solid enclosed by the surfaces $z = 2 - \sqrt{x^2+y^2}$ and $z = \sqrt{x^2+y^2}$ has the volume $\int_0^{2\pi} \int_0^1 \int_r^{2-r} r\ dz dr d\theta$.
20)	T F	If $\vec{r}\,''(t) = \langle 0, 0, \sin(t) \rangle$, $\vec{r}(0) = \langle 0, 1, 0 \rangle$, $\vec{r}\,'(0) = \langle 1, 0, 0 \rangle$, then $\vec{r}(t) = \langle t, 1+t, t - \sin(t) \rangle$.

Problem 2) (6 points)

a) (4 points) Match the curves. There is an exact match.

1



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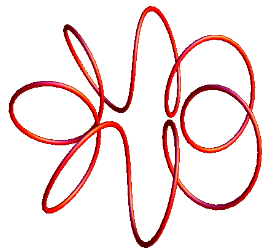

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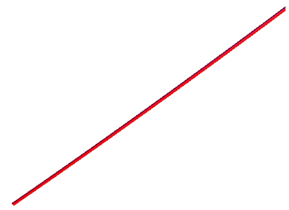

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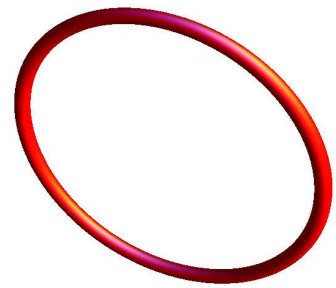
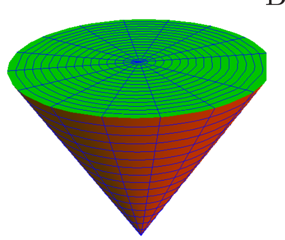


Enter 1-4	Object definition
	$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$
	$\vec{r}(t) = \langle \cos(t), 0, \sin(t) \rangle$
	$\vec{r}(t) = \langle (2+\cos(7t))\cos(t), (2+\cos(7t))\sin(t), \sin(7t) \rangle$
	$\vec{r}(t) = \langle t, t, t \rangle$

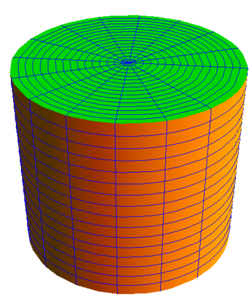
b) (4 points) Match the solids with the triple integrals. Also here, there is an exact match:

Enter A-D	3D integral computing volume
	$\int_0^{2\pi} \int_0^1 \int_r^1 r \; dz dr d\theta$
	$\int_0^{2\pi} \int_0^1 \int_{r^2}^1 r \; dz dr d\theta$
	$\int_0^{2\pi} \int_0^1 \int_{\sqrt{r}}^1 r \; dz dr d\theta$
	$\int_0^{2\pi} \int_0^1 \int_0^1 r \; dz dr d\theta$

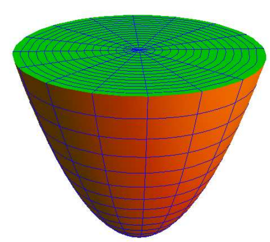
A



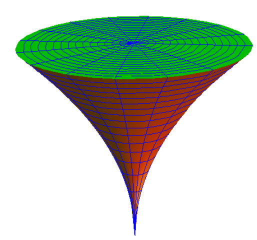
B



C



D



c) (2 points) What was the name again?

Enter one word	PDE
	$g_x = g_y$
	$g_{xx} = -g_{yy}$

Problem 3) (10 points)

a) (5 points) For the following quantities, decide whether they are vector fields or scalar fields (functions) or nonsense. Here $\vec{F} = \langle P, Q, R \rangle$ is a vector field in space, $f(x, y, z)$ is a scalar function and $\nabla = \langle \partial_x, \partial_y, \partial_z \rangle$. Recall that $\nabla \times \vec{F} = \text{curl}(\vec{F}), \nabla \cdot \vec{F} = \text{div}(\vec{F})$ and $\nabla f = \text{grad}(f)$.

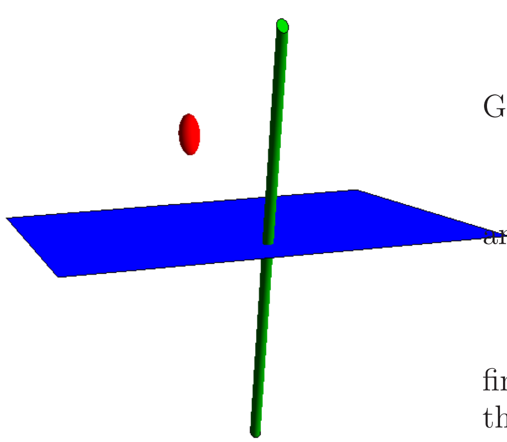
object	scalar	vector	not defined
$\nabla \vec{F}$			
$\nabla \cdot \vec{F}$			
$\nabla \times \vec{F}$			
$\nabla(\nabla \cdot \vec{F})$			
$\nabla \times (\nabla \times \vec{F})$			
$\nabla \times (\nabla \cdot \vec{F})$			
∇f			
$\nabla f \times \vec{F}$			
$\nabla f \cdot \vec{F}$			
$\nabla \times (\nabla f)$			

b) (5 points) Match the formulas for the position vector $\vec{r}(t)$ of a curve in space:

label	formula
A	$\vec{r}\,''(t)$
B	$\int_a^b \vert\vec{r}\,'(t)\vert \; dt$
C	$\vec{r}\,'(t)/\vert\vec{r}\,'(t)\vert$
D	$\vec{T}\,'(t)/\vert\vec{T}\,'(t)\vert$
E	$\vert\vec{T}\,'(t)\vert/\vert\vec{r}\,'(t)\vert$

expression	enter A-E
curvature	
unit normal vector	
unit tangent vector	
arc length	
acceleration	

Problem 4) (10 points)



Given a point $P = (4, 3, 1)$, a plane

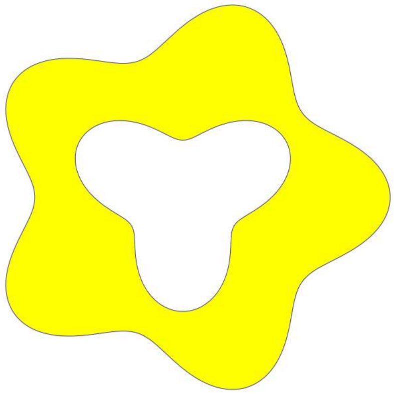
$$\Sigma : 3x + 4y - 12z = 0$$

and a line

$$L : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-1}{12} \,,$$

find the sum $d(P, L) + d(P, \Sigma)$ of the distances of P to the line and plane.

Problem 5) (10 points)



a) (5 points) Find the double integral

$$\int_0^3 \int_y^3 \frac{\sin(2x)}{x} \, dx \, dy \; .$$

b) (5 points) What is the area of the polar region

$$3 + \sin(3\theta) \leq r \leq 6 + \cos(5\theta) \; ?$$

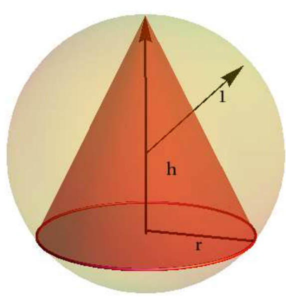
Problem 6) (10 points)

a) (8 points) Locate and classify all the local maxima, minima and saddle points of the function

$$f(x,y) = x^4 + y^4 - 8x^2 - 8y^2 \; .$$

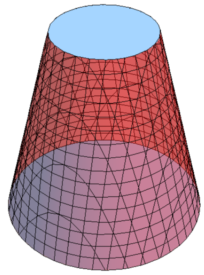
b) (2 points) Is there a global maximum or a global minimum of f? Explain.

Problem 7) (10 points)



For which base radius r and height h does a cone inscribed into the unit sphere have maximal volume $f(r,h) = \pi r^2 h/3$? The constraint is given by Pythagoras as $g(r,h) = r^2 + (h-1)^2 = 1$. Use the Lagrange method.

Problem 8) (10 points)

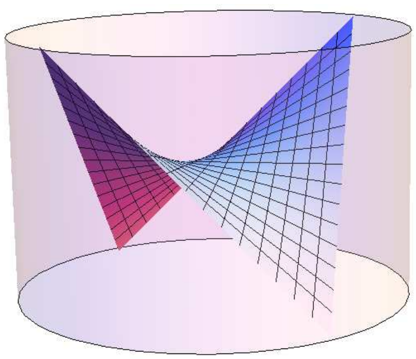


A bird's feeding cage E is part of a cone $x^2 + y^2 = 4(3 - z)^2$ with $1 < z < 2$. The cage is filled with different kind of seeds, the heavier have gone down and the density is $(3-z)$. We want to find the moment of inertia

$$\iiint_E (x^2 + y^2)(3 - z)\, dxdydz$$

so that we can know how much energy the feeding cage has if a squirrel spins it. You do not have to worry in this problem that squirrels are not birds.

Problem 9) (10 points)

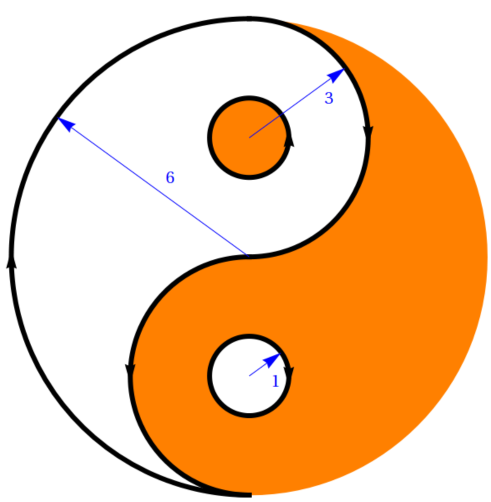


a) (4 points) Find the surface area of the surface

$$\vec{r}(s,t) = \langle s, -t, 2st \rangle$$

with $s^2 + t^2 \leq 9$.
b) (4 points) The coordinates of the surface satisfies $2xy + z = 0$. Find the tangent plane at $(1, 1, -2)$.
c) (2 points) What is the formula for the linearization of $f(x, y) = 2xy$ at the point $(1, 1)$.

Problem 10) (10 points)

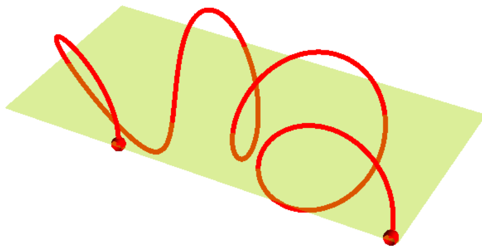


Let C be the boundary curve of the white Yang part of the Ying-Yang symbol in the disc of radius 6. You can see in the image that the curve C has three parts, and that the orientation of each part is given. Find the line integral of the vector field

$$\vec{F}(x, y) = \langle -y + \sin(e^x), x \rangle$$

around C. Notice that the Ying and the Yang have the same area.

Problem 11) (10 points)



Let C be the curve

$\vec{r}(t) = \langle (2+\cos(7t))\cos(t), (2+\cos(7t))\sin(t), \sin(7t) \rangle$

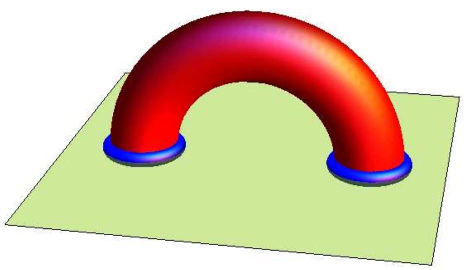
parametrized by $0 \leq t \leq \pi$ starting at $t = 0$ and ending at $t = \pi$. Calculate the line integral

$$\int_0^{\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\; dt \ ,$$

where $\vec{F}$ is the vector field

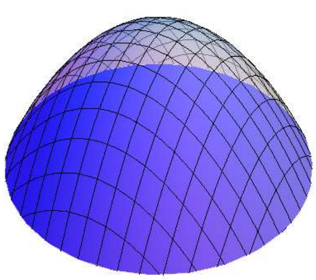
$\vec{F}(x,y,z) = \langle 4xe^{2x^2+3y^2+4z^2}, 6ye^{2x^2+3y^2+4z^2}, 8ze^{2x^2+3y^2+4z^2} \rangle$.

Problem 12) (10 points)



Find the flux of the curl of $\vec{F}(x,y,z) = \langle -y, x^2, 0 \rangle$ through a half torus surface S given by $(\sqrt{x^2+z^2}-3)^2 + y^2 = 1, z \geq 0$ which intersects the xy-plane $z = 0$ in two circles $C_1 : (x-3)^2 + y^2 = 1$ and $C_2 : (x+3)^2 + y^2 = 1$. The torus S is oriented outwards.

Problem 13) (10 points)



Find the flux of the vector field

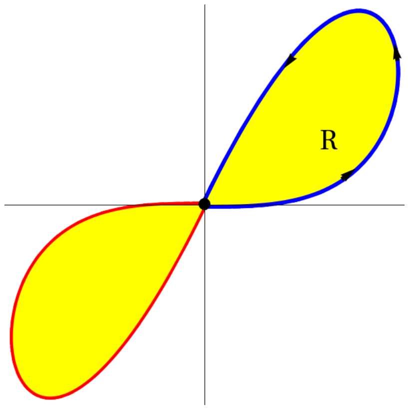
$$\vec{F}(x,y,z) = \langle x^3 z, y^3 z, 1 + e^{x^2+y^2} \rangle$$

through the paraboloid part S of the boundary of the solid

$$G : z + x^2 + y^2 \leq 1,\ z \geq 0 \ .$$

The paraboloid surface S is oriented upwards.

Problem 14) (10 points)



Find the area of the **propeller** shaped region enclosed by the figure 8 curve

$$\vec{r}(t) = \langle t - t^3, 2t^3 - 2t^5 \rangle \ ,$$

parametrized by $-1 \leq t \leq 1$. To find the total area compute the area of the region R enclosed by the right loop $0 \leq t \leq 1$ and multiply by 2.