Homework 1: Linear Equations

This homework is due on Monday, January 29, respectively Tuesday January 30, 2018. Homework is due at the beginning of each class in the classroom.

1. Find all solutions of the linear system

\[
\begin{align*}
x + y + z + u &= 1 \\
x + y - u + v &= 2 \\
x + z &= 3 \\
x + y + u &= 4 \\
y + v &= 5
\end{align*}
\]

2. On the iWatch one can play a $4 \times 4$ Suduku. The rules are that in each of the four $2 \times 2$ sub-squares, in each of the four rows and each of the four columns, the entries 1 to 4 have to appear and so add up to 10. The game initially gives 4 numbers. We have already started to enter 4 more numbers. There are still 8 missing. Introduce 8 variables and write down a system of 12 linear equations for these 8 variables, then solve the system.

3. A 10 km trip from the Swiss waterfall “Rheinfall” to the village “Rheinau” takes 30 minutes. The return trip takes an hour. How fast is the speed $v$ (in km/h) of the boat traveling relative to the water, and how fast is the speed $s$ (in km/h) of the river?
On a heating mesh, the temperature at exterior mesh points is 0, 400 or 800 F as given in the picture. In thermal equilibrium, each interior mesh point has the average of the temperatures at the 4 adjacent points. For example $T_2 = (T_3 + T_1 + 400 + 0)/4$. Find the temperatures $T_1, T_2, T_3$.

A polyhedron has $v$ vertices, $e$ edges and $f$ triangular faces. Euler proved his famous formula $v - e + f = 2$ for such ”discrete spheres”. There is an other relation, $3f = 2e$ called a Dehn-Sommerville relation which always holds. Lets call this number $f$ the ”area”. You get a polyhedron with area $f=288$. Write down a system of equations in matrix form $Ax = b$. Then determine the number of vertices and edges.

Main definitions

A **linear equation** for variables $x_1, x_2, \ldots, x_n$ is an equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$. Given $m$ equations of this type, we get a **system of linear equations**. It can be written in matrix form $A\vec{x} = \vec{b}$, where $\vec{x}$ is a column vector containing the $n$ variables and the $m \times n$ matrix $A$ lists the $m \cdot n$ coefficients and $\vec{b}$ is the column vector. The system $x + 2y + z = 8, 3x - y - 7z = 4$ for example can be written as

$$
\begin{bmatrix}
1 & 2 & 1 \\
3 & -1 & -7
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
8 \\
4
\end{bmatrix}.
$$