This problem set is designed to provide you with some practice using both the first law and the second law to solve problems involving different kinds of thermodynamic cycles, as well as working with gas mixtures and humidity concepts. Good luck!

1. 11.105 (Otto cycle)
2. 11.115 (Stirling cycle). Also compare your answer to Carnot efficiency.
3. 11.141 (Refer to 11.29 for definition of pinch point)
4. 12.64 (Hint: Do a first law balance and find heat transferred needed. Use this heat transferred to determine if it is possible to make the second law balance.)
5. 12.79
6. 12.101
7. Explain how the following weather phenomena occur using concepts discussed in class and in the reading. Two or three sentences should be enough for each.
   a) One feels hotter on a hot day in Washington D.C. than in Phoenix, even though the temperatures may be the same. (D.C. is more humid.)
   b) A large mass of warm air meeting a large mass of colder air creates rain (or other precipitation).
   c) The accumulation of most snowfall is at temperatures just below 32°F. In fact when it's really, really cold (like, sub-zero °F) they say it's "too cold to snow".
8. Note: ****This is grungy mathematically but a good example of the engineering tradeoffs one has to make in designing real devices. If you don't do the math but set up the problem properly, you'll still get partial credit ****

Consider an internally reversible Carnot engine which is not externally reversible because heat transfer into and out of the engine occurs over a finite temperature difference. The external reservoirs of the engine are at \( T_H \), \( T_L \) respectively and the heat engine operates at highest and lowest internal temperatures of \( T_a \) and \( T_b \) respectively. The heat transfer rates into and out of the engine can be represented in the form:

\[
\dot{Q} = h_H A (T_H - T_a)
\]
\[
\dot{Q} = h_L A (T_b - T_L)
\]

Here \( h \) is a heat transfer coefficient (units of \( W/M^2K \) or \( BTU/ft^2hr \)), \( A \) is the area available for heat to be transferred over.
a) Show using the First Law of Thermodynamics that the rate of work output is:

\[ \dot{W} = \dot{Q}_H - \dot{Q}_L = \eta_{sh} \dot{Q}_H, \quad \text{where } \eta_{sh} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H}, \quad (2.1) \]

and then by expressing the second law thermal efficiency for the internally reversible heat engine in terms of the temperatures \( T_b \) and \( T_a \), show that this can be rewritten as:

\[ \dot{W} = \left(1 - \frac{T_b}{T_a}\right) \Delta h_H (T_H - T_a) \quad (2.2) \]

Explain carefully what happens to the power output when the heat engine becomes externally reversible (i.e. when both \( T_a \to T_H \) and \( T_b \to T_L \)) and also when the heat transfer rates into and out of the engine are both very high.

(b) For some intermediate choice of the working temperatures \( T_b \) and \( T_a \), the power output if the cycle must be maximized from the two extremes found above. To find these optimal values, we will consider the external reservoir temperatures as fixed, and consider varying \( T_a \) until the derivative of the power output \( \left( \frac{\partial \dot{W}}{\partial T_a}\right)_{T_H, T_L} = 0 \).

First express the temperature \( T_b \) in terms of \( T_H, T_L \) and \( T_a \) as

\[ T_b = T_L \left[ 1 - K \left( \frac{T_H}{T_a} - 1 \right) \right] \quad \text{where } K = \frac{h_H}{h_L} \quad (2.3) \]

Now differentiate the expression (2.2) with respect to \( T_a \) and set the derivative to zero to obtain the following quadratic equation:

\[ T_a^2 (1 + K)^2 - (2 K T_H (1 + K)) T_a + T_H^2 \left( K^2 - \frac{T_L}{T_H} \right) = 0 \quad (2.4) \]

Hence show that the two real roots of this equation are

\[ T_a = \frac{K}{1 + K} T_H \pm \frac{1}{1 + K} \sqrt{T_H T_L} \quad (2.5a) \]

and hence from (2.3), you can also find \( T_b \). To save you some time, you don’t need to solve for \( T_b \).
(c) It can be shown that choosing the smaller roots in the above expressions results in a minimum power output or unphysical choices of $T_a$ or $T_b$ (you should be able to see why). By using the bigger roots thus show that the thermal efficiency of the reversible cycle with maximum power output is

$$\eta_{th} = 1 - \frac{T_b}{T_a} = 1 - \frac{\sqrt{T_L}}{T_H}$$

*Is this efficiency higher or smaller than the efficiency of the Carnot cycle?*