frame is important

- in graphics, we often keep track of a number of frames
  - each object, the camera, the world ...
  - so we need to be careful how we use matrices.
- given point and matrix is not enough to specify mapping
- for example point $\tilde{p}$ and the matrix
  $S = \begin{bmatrix}
  2 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 
\end{bmatrix}$
  - the matrix is non-uniform scaling
- fix a frame $\vec{f}^t$
  - in this frame $\tilde{p} = \vec{f}^t c$
  - transform with matrix $\vec{f}^t c \Rightarrow \vec{f}^t Sc = \vec{f}^t (Sc) =: \vec{f}^t c'$
    - the stretches by factor of two in first axis of $\vec{f}^t$
- see fig

other frame

- pick some other frame $\vec{a}^r$.
- relationship between bases $\vec{a}^r = \vec{f}^t A$.
- express same point as $\tilde{p} = \vec{f}^t c = \vec{a}^r (A^{-1} c) =: \vec{a}^r d$,
- now use matrix $S$ we get $\vec{a}^r d \Rightarrow \vec{a}^r (Sd) =: \vec{a}^r d'$.
  - the same point $\tilde{p}$ is stretched about first axis of $\vec{a}^r$
- see fig
- also rot fig

left-of rule

- point is transformed with respect to the the frame that appears immediately to the left of the transformation matrix in the expression.
  - We read $\tilde{p} = \vec{f}^t c \Rightarrow \vec{f}^t Sc$ as “$\tilde{p}$ is transformed by $S$ with respect to $\vec{f}^t$”.
  - We read $\tilde{p} = \vec{a}^r A^{-1} c \Rightarrow \vec{a}^r SA^{-1} c$ as “$\tilde{p}$ is transformed by $S$ with respect to $\vec{a}^r$”.

more generally

- We read $\tilde{p} = \vec{f}^t ABc \Rightarrow \vec{f}^t ASBc$ as “$\tilde{p}$ is transformed by $S$ with respect to $\vec{f}^t A$”.

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for frames

- same for transformations of frames
  - We read
    \[ \vec{f} \Rightarrow \vec{f} S \]
    - “\( \vec{f} \) is transformed by \( S \) with respect to \( \vec{f} \).”
  - We read
    \[ \vec{f} = a^t A^{-1} \Rightarrow a^t S A^{-1} \]
    - as “\( \vec{f} \) is transformed by \( S \) with respect to \( a^t \).”

more generally

- We read
  \[ g^t = \vec{f} AB \Rightarrow \vec{f} ASB \]
  - as “\( g^t \) is transformed by \( S \) with respect to \( \vec{f} A \).”

auxiliary frame

- we may wish to transform a frame \( \vec{f} \) in some specific way represented by a matrix \( M \), with respect to some auxiliary frame \( a^t \).
  - For example, we may be using some frame to model the planet Earth, and we now wish the Earth to rotate around the Sun’s frame.
- let \( a^t = \vec{f} A \)
- then The transformed frame can then be expressed as
  \[
  \begin{align*}
  \vec{f} & \quad (1) \\
  = & \quad a^t A^{-1} \quad (2) \\
  \Rightarrow & \quad a^t MA^{-1} \quad (3) \\
  \Rightarrow & \quad \vec{f} AMA^{-1} \quad (4)
  \end{align*}
  \]

multiple transformations

- using the “left of” rule
- example:
  - a rotation matrix \( R \) rotating a point by \( \theta \) degrees about origin
  - translation matrix \( T \), translating the point by one unit in the direction of the first frame axis.

interp 1

- given tform
  \[ \vec{f} \Rightarrow \vec{f} TR \]
- break into 2 steps
- In the first step
  \[ \vec{f} \Rightarrow \vec{f} T = \vec{f}^t \]
  - \( \vec{f} \) is transformed by \( T \) with respect to \( \vec{f} \) and we call the resulting frame \( \vec{f}^t \).
• In the second step,

\[ \vec{f}^T \Rightarrow \vec{f}^T R \]
\[ \vec{f}^t \Rightarrow \vec{f}^t R \]

- This is interpreted as: \( \vec{f}^t \) is transformed by \( R \) with respect to \( \vec{f}^t \).

other way

• In the first step

\[ \vec{f}^t \Rightarrow \vec{f}^t R = \vec{f}^{ot} \]

\( \vec{f}^t \) is transformed by \( R \) with respect to \( \vec{f}^t \) and we call the resulting frame \( \vec{f}^{ot} \).

• In the second step,

\[ \vec{f}^t R \Rightarrow \vec{f}^t TR \]

\( \vec{f}^{ot} \) is transformed by \( T \) with respect to \( \vec{f}^t \).

summary

• both interps can be useful
• left to right, wrt latest (local)
  - right to left, wrt original frame (global)