Chapter 1N

Class 1: DC Circuits

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3Revisions: correct interchanged ground symbols (9/14); add header file (7/14); add “why?” (1/14); Some small changes (9/13);
1N.1 Overview: What happens, this first day

Today we will look at circuits made up entirely of

- DC voltage sources (things whose output voltage is constant over time; things like a battery, or a lab power supply); and . . .
- resistors.

Sounds simple, and it is. We will try to point out quick ways to handle these familiar circuit elements. We will concentrate on one circuit fragment, the voltage divider.

1N.1.1 Why?

In each day’s class notes we will sketch the sort of task that the day’s material might let us accomplish. We do this to try to head off a challenge likely to occur to any skeptical reader: OK, this is a something-or-other circuit, but what’s it for? Why do I need a something-or-other? This is an integrator—but why do I want an integrator? Here is our first try at providing such a sample application:

**Problem:** given a constant (“DC”) voltage source, design a lower voltage source, strong enough to “drive” a particular “load” resistance.

*Shorthand version of the problem:* Make a voltage divider to deliver a specified voltage. Arrange things so that increasing load current to a maximum varies $V_{\text{out}}$ by no more than a specified percentage.

1N.1.2 Preliminary: What is “the art of electronics?”

Not art that you’re likely to find in a museum, but *art* in an older sense: a craft. No doubt the title of *The Art of Electronics* (hereafter referred to as “AoE”) was chosen with an awareness of the suggestion that there’s something borderline-magical available here: perhaps a hint of “black art”?

Here’s AoE’s formulation of the subject of this course:

the laws, rules of thumb, and tricks that constitute the art of electronics as we see it.

As you may have gathered, if you have looked at the text, this course differs from an engineering electronics course in concentrating on the “rules of thumb” and the bag of “tricks.” You will learn to use rules of thumb and reliable “tricks” without apology. With their help you will be able to leave the calculator-bound novice engineer in the dust!

1N.2 What the course is *not* about

1N.2.1 Wire my basement? Fix my TV?

Alums of this course sometimes are asked for help that is beyond their capacities, and sometimes below—or beside—what they know. “So, now you can wire some outlets in my basement?” No. This course won’t help much with that task, which is easy in a sense but difficult in another, in that it requires a detailed knowledge of electrical codes (required wire gauges; types of jacketing; where ground-fault-interrupters are required).

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2But see, if you find yourself in Munich, a spectacular exception: the world’s greatest museum of science and technology, the Deutsches Museum. There you will wonderful machines demonstrating such arcana as the history of the manufacture of threaded fasteners.

And when your friend’s TV quits, you’re probably not going to want to fix it: much of the set’s circuitry will be embodied in mysterious proprietary integrated circuits; an effective repair—if it were economically worthwhile—would likely amount to ordering a replacement for a substantial module, rather than replacing a burned-out resistor or transistor, as in the good old days of big and fixable devices.

1N.2.2 Delivering Power

A subtler point is worth making as well: only now and then, in this course, do we undertake to deliver power to something (the “something” is conventionally called a “load”). Occasionally, we are interested in doing that: when we want to make a loud sound from a speaker, or want to spin a motor. But much more often, we would like to minimize the flow of power; we are concerned, instead, with the flow of information.

On the wall of the lobby of MIT’s Electrical Engineering building is a huge blowup of a photo of some MIT engineers standing among what look like large generators, each about the size of a small cow. The photo seems to date from the 1930’s.

![A Bunch of Electricals](image)

The “Electricals,” back then, were concerned mostly with those big machines: with delivering power. It was the power companies that were hiring, when one of our uncles finished at MIT, around 1936. Hoover Dam, finished in 1935, was the engineering wonder of the day. Big was beautiful. (Even now, Hoover Dam’s website boasts of the dam’s weight!—6.6 million tons, in case you were wondering.)

1N.3 What the course is about: processing information

Times have changed, as you may have noticed. Small is beautiful; nano is extra beautiful—and electronics, these days, is concerned mostly with processing information. So, we like circuits that pass and process signals while generating very little heat—using very little power. We like, for example, digital circuits made out of field-effect transistors that form switches; they offer low output impedance, gargantuan input impedance, and quiescent current of approximately zero. To a good approximation, they’re not transferring power, not

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4We guess a potentially big exception is the continuing struggle to produce an efficient and economically-viable electrically-powered car. Some glory awaits the electricals who succeed at that task.
using it, not delivering it. They’re dealing in information. That’s almost always what we’ll be doing in this course.

Obvious, perhaps? Perhaps.

We will postpone till next time—not to overload you, on the first day—discussion of a related topic: just what form the information is likely to take, in our circuits: voltage versus current. The answer may surprise you; or you may be inclined to reject the question as empty, since you know that Mr. Ohm long ago taught us that current and voltage in a device can be intimately related. Next time, we’ll try to persuade you that you ought not to reject the question; that it’s worth considering whether the signal is represented as a voltage or as a current (and see today’s supplementary note on this topic).

Now on to less abstract topics, and our first useful circuit: a voltage divider.

1N.4 Two Laws


We rely on these rules continually, in electronics. Nevertheless, we rarely will mention Kirchhoff again. We use his observations implicitly. We will see and use Ohm’s Law a lot, in contrast; no one has gotten around to doing what’s demanded by the bumper sticker one sees around MIT: Repeal Ohm’s Law!.

1N.4.1 Ohm’s Law: $E = IR$

- $E$ (old-fashioned term for voltage) is analog of water pressure or ‘head’ of water
- $R$ describes the restriction of flow
- $I$ is the rate of flow (volume/unit time)

![Figure 2: Hydraulic analogy: voltage as head of water, etc. Use it if it helps your intuition](image)

The homely hydraulic analogy works pretty well, if you don’t push it too far—and if you’re not too proud to use such an aid to intuition.

1N.4.2 What is “voltage,” and other deeper questions

For the most part, we will evade such deep questions in this course. We’re inclined to say, “Oh, a Volt is what pushes an Amp through an Ohm.” But you don’t have to be quite so glib (you don’t have to sound so much like a Harvard student!). A less circular definition of voltage is the potential energy per unit charge. Or, equivalently, it can be defined as the work done to move a unit charge against an electric field (a word that we hope doesn’t worry you; we suggest you try to get accustomed to use of the word, even if you have
reservations about its usefulness), from one electric potential (analogous to a position on a hillside) to a higher potential (higher on the hillside).

Figure 3: Voltage is work to raise a unit charge from one level to a higher level (or “potential”)

The voltage difference between two points on the hillside (or staircase, as in fig. 3) can be described as a difference in electrical potential or voltage. The so-called “electrical field” will tend to push that charge back down, just as gravity will tend to push the water down from the tank. You may or may not be interested to know that one volt is the work done as one adds one joule of potential energy to one coulomb of charge. But we’ll not again speak in these terms—which sound more like Physics than like language for the “art of electronics.”

“Ground”

Sometimes we speak of a voltage relative to some absolute reference—perhaps the planet earth (or, a bit more practically, the potential at the place where a copper spike has been driven into the ground, in the basement of the building where you are doing your electronics). In the hydraulic analogy, that absolute zero-reference might be sea-level. More often, as we will reiterate below, we are interested only in relative voltages: differences in potential, measured relative to an arbitrary reference point, not relative to planet earth.

Ohm’s is a very useful rule; but it applies only to things that behave like resistors. What are these? They are things that obey Ohm’s Law! (Sorry folks: that’s as deeply as we’ll look at this question, in this course.)

1N.4.3 Why does Ohm’s Law hold?

The restriction of current flow that we call “resistance”—which we might contrast with the very-easy flow of current in a piece of wire—occurs because the charge-carrying electrons, accelerated by an electric field, bump into obstacles (vibrations of the atomic lattice) after a short free flight, and then have to be re-accelerated in the direction of the field. Materials that are good conductors—metals—have a substantial population of electrons that are not tightly bound, and consequently are free to travel when pushed. The conductivity of a metal depends on the density of the population of charge carriers (usually, un-bound electrons), and it’s kind of reassuring to find that conductivity degrades with rising temperature: the free flights become shorter, as electrons bump into the jumpier atoms of the hotter material. This effect you will see confirmed in the first lab, if things come out right in your experiment (you’ll have to do a little reasoning to see this effect confirmed; the lab notes do not point out where this occurs). The stronger the field, the faster the drift of the electrons. Field strength goes with voltage difference between two points on the conductor; rate of drift of the electrons measures current. So, Ohm’s Law is pretty plausible.

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5 You may be inclined to wonder, as Purcell suggests, in his excellent book, “...what is as field? Is it something real, or is it merely a name for a factor in an equation which has to be multiplied by something else to give the numerical value of the force we measure in an experiment?” E. M. Purcell, Electricity and Magnetism, 2d ed. (1985), §1.7, p. 17. Purcell makes a persuasive argument that the concept of “field” is useful.

6 See Purcell, p.46, § 2.2.

7 If this remark frustrates you, see an ordinary E& M book; for example, see the good discussion of the topic in E. M. Purcell, Electricity & Magnetism, cited in the Text (2d ed., 1985), or in S. Burns & P. Bond, Principles of Electronic Circuits (1987).

8 You may prefer the contrast with a superconductor, whose resistance is not just small but is zero.
1N.4.4 What determines the value of a resistor?

A “resistor” is also, of course, a “conductor;” it may seem a bit perverse to call this thing that is inserted in a circuit to permit current flow a “resistor.” But the name comes from the assumption that the resistor is inserted where an excellent conductor—a piece of wire—might have stood, instead. To make a resistor, one can use either of two strategies: to make a “carbon composition” resistor (the sort that we’ll use in lab, because their values are relatively easy to read), one mixes up a batch of powdered insulator and powdered conductor (carbon), adjusting the proportions to give the material a particular resistivity. To make a “metal film” resistor (much the more common type, these days), one “deposits” a thin film of metal on a ceramic substrate, and then partially cuts away the thin conducting film.

1N.4.5 How generally does Ohm’s Law Apply?

We begin almost at once to meet devices that do not obey Ohm’s Law (see Lab 1: a lamp; a diode). Ohm’s Law describes one possible relation between $V$ and $I$ in a component; but there are others. As AoE says,

Crudely speaking, the name of the game is to make and use gadgets that have interesting and useful $I$ vs $V$ characteristics.

In a resistor, current and voltage are proportional in a nice, linear way: double the voltage and you get double the current. Ohm’s Law holds. Don’t expect to use it where it doesn’t fit. Even the lamp—whose filament is just a piece of metal that one might expect would behave like a resistor—doesn’t follow Ohm’s Law, as you’ll see in the lab. Why not?9

1N.4.5.1 …But we can extend the reach of Ohm’s Law: “dynamic resistance”

After today, we rarely will limit ourselves to devices that show simply resistance—and as we have said, even the resistor-like lamp that you meet in Lab 1, along with the diode, defy Ohm’s-Law treatment. But an extended version of Ohm’s Law that we’ll call dynamic resistance will allow us to apply the familiar rule in settings where otherwise it would not work. The idea is just to define a local resistance—the tangent to the slope of the device’s $V$-$I$ curve:

$$R_{\text{dynamic}} \equiv \frac{\Delta V}{\Delta I}$$

This redefinition allows us to talk about the effective resistance of a diode, a transistor, or a current source (a circuit that holds current constant). Here is a sketch of a diode’s $V$-$I$ curve—oriented so that $V$ is the vertical axis. This orientation puts the curves’ slopes into the familiar units, Ohms (rather than into $1/$Ohms,$^{10}$ as in the more standard $I$-$V$ plot).

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9Here’s a powerful clue: it would follow Ohm’s Law if you could hold the filament’s temperature constant.

10…or Siemens, the official name for inverse-Ohms.
You may like the resistor’s well-behaved straight line, because it is familiar. But the nice thing about the notion of $R_{\text{dynamic}}$ is that it is so broad-minded: it is happy to describe the $V$-$I$ curve (or “$I$-$V$ curve,” as it is more often called) for any device. It will happily fit a transistor, an exotic current source—anything. The nearly-vertical plot of the current source, implying enormous $R_{\text{dynamic}}$, will become important to your understanding of transistors.

1N.4.6 Power in a Resistor

Power is the rate of doing work, as you may recall from a course on mechanics. The concept comes up most often, in electronics, when one tries to specify a component that can safely handle the power that it’s likely to have to dissipate. High power produces high heat, and calls for a component capable of unloading or dissipating that heat. The three resistors shown below illustrate the rough relation between power rating and size—because large size usually offers large area in contact with the surroundings or “ambient.”

The indicated power ratings show the maximum that each can dissipate without damage. The tiny “surface-mount” on the left (“0805 size”) dissipates more than one might expect if one compares its size to that of the 1/4W carbon-comp (the sort we use in the lab); it does better than one might expect because it is soldered directly to a circuit board, whose copper traces help to draw off and dissipate its heat.

In the coming labs, you will sometimes run into the question whether your components can handle the power that is expected. Our usual resistors are rated at 1/4W. You can confirm that such a resistor can handle 15 volts (our usual maximum supply voltage) if the resistor’s value is at least 1k ohms. Let’s try that calculation:

$$P = I \times V.$$  

Thanks to Ohm’s Law, the formula for power can be written in any of three ways:

- $P = I \times V$ (as we just said); but since $V = IR$,
- $P = I^2R$; and since $I = V/R$,
- $P = V^2/R$

In the present case, it is the last form that is most useful. $1/4W = 15^2/R$. So, $R = 225/(1/4) = 900$. So, 1k is close to the minimum safe value, at 15V (910 would be safe, but let’s not be so fussy; call it a k).

So far, we have mentioned only power in a resistor. The notion is more general than that, and the formula, $P = V \times I$

holds for any electronic component.

A closer look at what we mean by $V$ and $I$ makes this formula seem almost obvious.

- current measures charge/time
- voltage measures work/charge

So, the product, $V \times I = \text{work/charge} \times \text{charge/time} = \text{work/time}$, and this is power.
In this course, the exceptional cases where we do worry about power are those cases in which we either use large voltage swings (for example, the 30V output swing of the “comparator” in lab 8) or want to provide unusually large currents (for example, the speaker drive of lab 6, the light-emitting diode drive of the music-transmission lab, lab 13, and the voltage regulators of lab 11).

1N.4.7 Kirchhoff’s Laws: V, I

These two ‘laws’ probably only codify what you think you know through common sense:

- Sum of voltages around loop (“circuit”) is zero.
- Sum of currents in and out of node (algebraic sum, of course) is zero.

Applications of these laws: series and parallel circuits

- **Series**: $I_{\text{total}} = I_1 = I_2$
- **Parallel**: $I_{\text{total}} = I_1 + I_2$
- **Series**: $V_{\text{total}} = V_1 + V_2$
- **Parallel**: $V_{\text{total}} = V_1 = V_2$

Query: Incidentally, where is the “loop” that Kirchhoff’s law refers to? Answer: the “loop” (or “circuit,” a near-synonym) is apparent if one draws the voltage source as a circuit element, and ties its foot to the foot of the $R$: 
Usually we don’t bother to draw the voltage source that way; we label points with voltage values, and assume that you can picture the circuit path for yourself, if you choose to.

This is kind of boring. So, let’s hurry on to less abstract circuits: to applications—and tricks. First, some labor-saving tricks.

1N.4.7.1 Parallel Resistances: calculating equivalent $R$

The conductances add: $\text{Conductance}_{\text{total}} = \text{Conductance}_1 + \text{Conductance}_2 = 1/R_1 + 1/R_2$

![Figure 9: Parallel resistors: the conductances add; unfortunately, the resistances don’t](image)

This is the easy notion to remember, but not usually convenient to apply, for one rarely speaks of conductances. The notion “resistance” is so generally used that you will sometimes want to use the formula for the effective resistance of two parallel resistors:

$$R_{\text{tot}} = (R_1 \cdot R_2)/(R_1 + R_2)$$

Believe it or not, even this formula is messier than what we like to ask you to work with in this course. So we proceed immediately to some tricks that let you do most work in your head.

Consider some easy cases:

- **two equal R’s**
- **two very unequal R’s**
- **R, 2R**

![Figure 10: Parallel R’s: Some easy cases](image)

The first two cases are especially important, because they help one to estimate the effect of a circuit one can liken to either case. Labor-saving tricks that give you an estimate are not to be scorned: if you see an easy
way to an estimate, you’re likely to make the estimate. If you have to work too hard to get the answer, you
may find yourself simply not making the estimate. A deadly trap for the student doing a lab is the thought,
“Oh, I’ll calculate this later—some time this evening, when I’m comfortable in front of a spreadsheet.” This
student won’t get to that calculation!

In this course we usually are content with answers good to 10%. So, if two parallel resistors differ by a factor
of ten or more, then we can ignore the larger of the two.

Let’s elevate this observation to a rule of thumb (our first). While we’re at it, we can state the equivalent rule
for resistors in series.

**Parallel resistances: shortcuts**

![Figure 11: Resistor calculation shortcut: parallel, series]

In a parallel circuit, a resistor much smaller than others dominates. In a series circuit, the larger resistor dominates.

**1N.5 First Application: Voltage Divider**

*Why Dividers?*

Are dividers necessary? Why not start with the right voltage? We know you aren’t troubled by this doubt;
but someone else might be. The answer, as you know, is just that a typical circuit needs several voltages, and
building a “power supply” to deliver each voltage is impracticable (meaning, mostly, expensive). You’ll soon
be designing power supplies, and certainly at that time will appreciate how much simpler a voltage divider is,
compared to a full power supply.

To illustrate the point that voltage dividers are useful, and not just an academic device used to provide an easy
introduction to circuitry, we offer here a piece of a fairly-complex device, a “function generator”—the box
that soon will be providing waveforms to the circuits that you build in lab 2. This diagram shows part of the
circuitry that converts a triangular waveform into a sinusoidal shape.

![Figure 12: Voltage dividers in a function generator: dividers are not just for beginners [Krohn-Hite 1400 function generator]]
1N.5.1 Aside: Variable Dividers or “Potentiometers”

Before we look closely at an ordinary divider, let’s note a variation that’s often useful: a voltage divider that is adjustable. This circuit, available as ready-made component, is called a “potentiometer.”

![Potentiometer Symbol](image1.png)

**Figure 13: Symbol for potentiometer, and its construction**

The name describes the circuit pretty well: the device “meters” or measures out "potential." Recalling this may help you to keep separate the two ways to use the device:

- as a potentiometer versus…
- as a variable resistor

1N.5.1.1 “Variable resistor;” just one way to use a pot

The component is called a potentiometer (a 3-terminal device), but it can be used as a variable resistor (a 2-terminal device).

![Variable Resistor](image2.png)

**Figure 14: A “pot” can be wired to operate as a variable resistor**

The pot becomes a variable resistor if one uses just one end of the fixed resistor and the slider, or if (somewhat better) one ties the slider to one end.11

1N.5.1.2 How the potentiometer is constructed

It helps to see how the thing is constructed. Here are photos of two potentiometers. In the large one on the left, in fig. 15, it is not hard to recognize how it works. A (fixed) wire-wound resistor, not insulated, follows most of the way around a circle. A sliding contact presses against this wire-wound resistor. It can be rotated to either extreme.

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11The difference between those two options is subtle. If the fixed resistance is, say 100k, the variable resistance range for either arrangement is 0 to 100k. The difference—and the reason to prefer tying slider to an end—appears if the pot becomes dirty with age. If the slider should momentarily lose contact with the fixed resistor, the tied arrangement takes the effective $R$ value to 100k. In contrast, the lazier arrangement takes $R$ to open (call it “infinite resistance,” if you prefer) when the slider loses contact.

The difference often is not important. But since it costs you nothing to use the “tied” configuration, you might as well make that your habit.
At the position shown, the contact seems to be about 70% of the way between lower and upper terminals. If the upper terminal were at 10 volts and the lower one at ground, the output at the slider terminal would be about 7 volts.

The smaller pot shown in middle and right of fig. 15 on the preceding page (and shown enlarged, relative to the left-hand device) is fundamentally the same, but constructed in a way that makes it compact. Its fixed resistor—of value 1kΩ—is made not of wound wire but of “cermet.”

1N.5.2 A Voltage Divider to Analyze

Now, here’s a simple example of the more common fixed voltage divider:

At last we have reached a circuit that does something useful. It delivers a voltage of the designer’s choice; a voltage less than the original or “source” voltage.

First, a note on labeling: we label the resistors “10k”; we omit “Ω.” It goes without saying. The “k” means kilo- or $10^3$, as you probably know.

One can calculate $V_{out}$ in several ways. We will try to push you toward the way that makes it easy to get an answer in your head.

*Three ways to analyze this circuit:*

1N.5.3 First Method: Calculate the current through the series resistance...

Calculate the current: $I = \frac{V_{IN}}{R_1 + R_2}$

Here, that’s 30v / 20kΩ = 1.5 mA

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12“Cermet” is a composite material formed of ceramic and metal.
After calculating that current, use it to calculate the voltage in the lower leg of the divider:

\[ V_{\text{out}} = I \cdot R_2 \]

Here, that product is 1.5 mA \cdot 10k = 15 V

![Figure 17: Voltage divider: first method (too hard!): calculate current explicitly](image)

*That takes too long.*

**1N.5.4 Second Method: Rely on the fact that \( I \) is the same in top and bottom...**

...but rely on this equality only *implicitly*. If you want an algebraic argument, you might say,

\[ \frac{V_2}{(V_1 + V_2)} = I \cdot \frac{R_2}{(I \cdot (R_1 + R_2))} = R_2/(R_1 + R_2) \]

or,

\[ V_{\text{out}} = V_{\text{in}} \cdot \frac{R_2}{(R_1 + R_2)} \] (1N.1)

In this case, that means

\[ V_{\text{out}} = V_{\text{in}} \cdot \frac{10k}{20k} = V_{\text{in}}/2 \]

![Figure 18: Voltage divider: second method: (a little better): current implicit](image)

That’s *much better*, and you will use formula (1N.1) fairly often. But we would like to push you not to memorize that equation, but instead to work less formally.

**1N.5.5 Third Method: Say to yourself in words how the divider works...**

...something like,

*Since the currents in top and bottom are equal, the voltage drops are proportional to the resistances* (later, *impedances*—a more general notion that covers devices other than resistors).

So, in this case, where the lower \( R \) makes up half the total resistance, it also will show half the total voltage.
For another example, if the lower leg is 10 times the upper leg, it will show about 90% of the input voltage (10/11, if you’re fussy, but 90%, to our usual tolerances).

1N.6 Loading, and “output impedance”

Now—after you’ve calculated $V_{\text{out}}$ for the divider—suppose someone comes along and puts in a third resistor:

![Voltage divider loaded](Figure 19: Voltage divider loaded)

(Query: Are you entitled to be outraged? Is this no fair?)

Again there is more than one way to make the new calculation—but one way is tidier than the other.

Two possible methods:

1N.6.1 Tedious Method:

Model the two lower R’s as one R; calculate $V_{\text{OUT}}$ for this new voltage divider:

![Voltage divider loaded: load and lower R combined in model](Figure 20: Voltage divider loaded: load and lower R combined in model)

The new divider delivers $1/3 \ V_{\text{IN}}$. That’s reasonable, but it requires you to draw a new model to describe each possible loading.
1N.6.2  Better method: Thevenin’s Model

Thevenin Model

Thevenin’s good idea: Model the actual circuit (unloaded) with a simpler circuit—the Thevenin model—which is an idealized voltage source in series with a resistor. One can then see pretty readily how that simpler circuit will behave under various loads.

Here’s how to calculate the two elements of the Thevenin model:

\[ V_{\text{Thevenin}}: \text{Just } V_{\text{opencircuit}}: \text{the voltage out when nothing is attached ("no load") } \]

\[ R_{\text{Thevenin}}: \text{often formulated as the quotient of } V_{\text{Thevenin}} / I_{\text{short-circuit}}, \text{which is the current that flows from the circuit output to ground if you simply short the output to ground. } \]

In practice, you are not likely to discover \( R_{\text{Thevenin}} \) by so brutal an experiment. Often, shorting the output to ground is a very bad idea: bad for the circuit and sometimes dangerous to you. Imagine the result, for example, if you decided to try this with a 12-volt car battery! And if you have a diagram of the circuit to look at, a much faster shortcut is available:

**Shortcut calculation of \( R_{\text{Thevenin}} \)**

Given a circuit diagram, the fastest way to calculate \( R_{\text{Thevenin}} \) is to see it as the parallel resistance of the several resistances viewed from the output.

**Figure 22:** \( R_{\text{Thevenin}} = R_1 \parallel R_2 \)

(This formulation assumes that the voltage sources are ideal, incidentally; when they are not, we need to include their output resistance. For the moment, let’s ignore this complication.)

1N.6.2.1  Justifying the Thevenin shortcut

You saw this result above—that \( R_{\text{Thevenin}} = R_{\text{parallel}} \)—but the result still may strike you as a little odd: why should \( R_1 \), going up to the positive supply, be treated as parallel to \( R_2 \)? Well, suppose the positive supply were set at 0 volts. Then surely the two resistances would be in parallel, right?

footnote 13: We don’t think you’re entitled to be outraged—but perhaps you should be mildly offended. Certainly it’s normal to see something attached to the output of your circuit (that “something” is called a “load”). You built the divider in order to provide current to something. But, as we’ll soon be saying repeatedly, you are entitled to expect loads that are not too “heavy” (that is, don’t pull too much current). By the standards of our course, this 10k load is too heavy. You’ll see why in a moment.
And try another thought experiment: redefine the positive supply as 0V. It follows then that the voltage we had been calling “ground” or “zero volts” now is -30V (to use the numbers of fig. 16 on page 12). Now it seems clear that the upper resistor, to ground, matters; the lower resistor, \( R_2 \), going to -30V now is the one that seems odd. But, of course, the circuit doesn’t know or care how we humans have chosen to define our “zero volts” or “ground.”

The point of this playing with “ground” definitions is just that the voltages at the far end of those resistors “seen” from the output do not matter. It matters only that those voltages are fixed.\(^{14}\)

Or suppose a different divider (chosen to make the numbers easy): twenty volts divided by two 1k resistors. To discover the \textit{impedance} at the output, do the usual experiment (one that we will speak of again and again):

\[ \text{ΔV} / \text{ΔI} = \frac{1\text{v}}{0.2\text{mA}} = 5\text{k} \]

And—happily—that is the parallel resistance of the two \( R \)’s. Does that argument make the result easier to accept?

You may be wondering why this model is useful. Here is one way to put the answer, though probably you will remain skeptical until you have seen the model at work in several examples:

Any non-ideal voltage source “droops” when loaded. How much it droops depends on its “output impedance”. The Thevenin equivalent model, with its \( R_{\text{Thevenin}} \), describes this property neatly in a single number.

\textit{Applying the Thevenin model}

First, let’s make sure Thevenin had it right: let’s make sure his model behaves the way the original circuit does. We found that the 10k, 10k divider from 30 volts, which put out 15v when not loaded, drooped to 10V under a 10k load. Does the model do the same?

\(^{14}\)Later we will relax this requirement, too. Thevenin’s model works as well to define the output resistance of a time-varying voltage source as it does for a DC source.
Yes, the model droops to the extent the original did: down to 10 V. What the model provides that the original circuit lacked is that single value, $R_{\text{Thev}}$, expressing how droopy/stiff the output is.

If someone changed the value of the load, the Thevenin model would help you to see what droop to expect; if, instead, you didn’t use the model and had to put the two lower resistors in parallel again and recalculate their parallel resistance, you’d take longer to get each answer, and still you might not get a feel for the circuit’s output impedance.

Let’s try using the model on a set of voltage sources that differ only in $R_{\text{Thev}}$. At the same time we can see the effect of an instrument’s input impedance.

Suppose we have a set of voltage dividers, dividing a 20V input by two. Let’s assume that we use 1% resistors (value good to ±1%).

$V_{\text{Th}}$ is obvious, and is the same in all cases. $R_{\text{Thevenin}}$ evidently varies from divider to divider.

Suppose now that we try to measure $V_{\text{out}}$ at the output of each divider. If we measured with a perfect voltmeter, the answer in all cases would be 10V. (Query: is it 10.000V? 10.0V? 15

But if we actually perform the measurement, we will see the effect of loading by the $R_{\text{IN}}$ of our imperfect lab voltmeters. Let’s try it with a VOM (“volt-ohm-meter,” the conventional name for the old-fashioned “analog” meter, which gives its answers by deflecting its needle to a degree that forms an analog to the quantity measured), and then with a DVM (“digital voltmeter,” a more recent invention, which usually can measure current and resistance as well as voltage, despite its name; both types sometimes are called simply “multimeters”).

Suppose you poke the several divider outputs, beginning from the right side, where the resistors are 1kΩ. Here’s a table showing what we found, at three of the dividers:

---

15It could be 10.0V—but it could be a bit less than 9.9V or a bit more than 10.1V, if the 20V source were good to 1% and we used 1% resistors. We should not expect perfection.
18 Class 1: DC Circuits

R values measured \( V_{\text{OUT}} \) inference
\[
\begin{array}{|c|c|c|}
\hline
\text{R values} & \text{measured} & \text{inference} \\
\hline
1k & 9.95 & \text{within R tolerance} \\
10k & 9.76 & \text{loading barely apparent} \\
100k & 8.05 & \text{loading obvious} \\
\hline
\end{array}
\]

The 8.05 v reading shows such obvious loading—and such a nice round number, if we treat it as “8 v”—that we can use this to calculate the meter’s \( R_{\text{IN}} \) without much effort:

As usual, one has a choice now, whether to pull out a formula and calculator, or whether to try, instead, to do the calculation “back-of-the-envelope” style. Let’s try the latter method.

First, we know that \( R_{\text{Thev}} \) is 100k parallel 100k: 50k. Now let’s talk our way to a solution (an approximate solution: we’ll treat the measured \( V_{\text{out}} \) as just “8 volts”):

The meter shows us 8 parts in 10; across the divider’s \( R_{\text{Thev}} \) (or call it “\( R_{\text{OUT}} \)) we must be dropping the other 2 parts in 10. The relative sizes of the two resistances are in proportion to these two voltage drops: 8 to 2, so \( R_{\text{IN-VOM}} \) must be \( 4 \cdot R_{\text{Thev}} \): 200k.

If we squint at the front of the VOM, we’ll find a little notation,

\[ \text{20,000 ohms/volt} \]

That specification means that if we used the meter on its 1V scale (that is, if we set things so that an input of 1 volt would deflect the needle fully), then the meter would show an input resistance of 20k. In fact, it’s showing us 200k. Does that make sense? It will when you’ve figured out what must be inside a VOM to allow it to change voltage ranges: a set of big series resistors. You’ll understand this fully when you have done problem 1.8 in the text; for now, take our word for it: our answer, 200k, is correct when we have the meter set to the 10 V scale, as we do for this measurement.

This is probably a good time to take a quick look at what’s inside a multimeter—VOM or DVM:

How a meter works: some meter internals types fundamentally sense current, others sense voltage. Both meter mechanisms, however, can be rigged to measure both quantities, operating as a “multimeter.”
• analog (“VOM”): senses current

Figure 27: Analog meter senses current, in its guts

• digital (“DVM”): senses voltage

Figure 28: Digital meter senses voltage, in its innards

The VOM specification, 20,000 ohms/volt, describes the sensitivity of the meter movement—the guts of the instrument. This movement puts a fairly low ceiling on the VOM’s input resistance at a given range setting.

Let’s try the same experiment with a DVM, and let’s suppose we get the following readings:

<table>
<thead>
<tr>
<th>R values</th>
<th>Measured V_{out}</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>10k</td>
<td>9.92</td>
<td>within R tolerance</td>
</tr>
<tr>
<td>1M</td>
<td>9.55</td>
<td>loading apparent</td>
</tr>
<tr>
<td>10M</td>
<td>6.69</td>
<td>loading obvious</td>
</tr>
</tbody>
</table>

Figure 29: DVM reading departs from ideal; we can infer \( R_{IN-DVM} \)

Again let’s use the case where the droop is obvious; again let’s talk our way to an answer:

This time \( R_{Th} \) is 5M; we’re dropping \( 2/3 \) of the voltage across \( R_{IN-DVM} \), \( 1/3 \) across \( R_{Th} \). So, \( R_{IN-DVM} \) must be \( 2 \times R_{Th} \), or 10M.

If we check the data sheet for this particular DVM we find that its \( R_{IN} \) is specified to be “10M, all ranges.” Again our readings make sense.
VOM vs DVM: a conclusion?

Evidently, the DVM is a better voltmeter, at least in its $R_{IN}$—as well as much easier to use. As a current meter, however, it is no better than the VOM: it drops 1/4 v full scale, as the VOM does; it measures current simply by letting it flow through a small resistor; the meter then measures the voltage across that resistor.

Digression on ground

The concept “ground” (“earth,” in Britain) sounds solid enough. It turns out to be ambiguous. Try your understanding of the term by looking at some cases:

Query: what is the resistance between points A and B? (Easy, if you don’t think about it too hard.\(^\text{16}\)) We know that the ground symbol means, in any event, that the bottom ends of the two resistors are electrically joined. Does it matter whether that point is also tied to the pretty planet we live on? It turns out that it does not.

And where is “ground” in this circuit:

1N.6.2.2 Two Senses for “Ground”

“Local ground” Local ground is what we care about: the common point in our circuit that we arbitrarily choose to call zero volts. Only rarely do we care whether or not that local reference is tied to a spike driven into the earth.

“Earth ground” But, be warned, sometimes you are confronted with lines that are tied to world ground—for example, the ground clip on a scope probe, and the “ground” of the breadboards that we use in the lab; then you must take care not to clip the scope ground to, say, +15 on the breadboard.

For a vivid illustration of “earth” grounding, see an image from Wikipedia: http://en.wikipedia.org/wiki/File:HomeEarthRodAustralia1.jpg.

\(^{16}\)Well, not easy, unless you understand the “ground” indicated in fig. 30 to be the common zero-reference in the circuit at hand. In that case, the shared “ground” connection means that the 10k and 22k resistances are in series. If you took “ground” to mean “the world, with one connection in New York, the other in Chicago,” the answer would not be so easy. Luckily, that would be a rare—and perverse—meaning to attribute to the circuit diagram.
It might be useful to use different *symbols* for the two senses of ground, and such a convention does exist:

![Figure 32: Symbols can distinguish the two senses of “ground”](image)

Unfortunately, this distinction is not widely observed, and in this book as in AoE, we will not maintain this graphical distinction. We will use the symbol that fig. 32 suggests for “world ground” to mean “ground,” meaning always “local ground,” and not “world” ground. In the rare case when we intend a connection to “world” ground, we will say so in words, not graphically.

### 1N.6.3 A Rule of Thumb for Relating $R_{OUT,A}$ to $R_{IN,B}$

The voltage dividers whose outputs we tried to measure introduced us to a problem we will see over and over again: some circuit tries to “drive” a load. To some extent, the load changes the output. We need to be able to predict and control this change. To do that, we need to understand, first, the characteristic we call $R_{IN}$ (this rarely troubles anyone) and, second, the one we have called $R_{Thevenin}$ (this one takes longer to get used to). Next time, when we meet frequency-dependent circuits, we will generalize both characteristics to “$Z_{IN}$” and “$Z_{OUT}$.”

Here we will work our way to another rule of thumb; one that will make your life as designer relatively easy. We start with a goal: *Design goal:* When circuit A drives circuit B: arrange things so that B loads A lightly enough to cause only insignificant attenuation of the signal.

And this goal leads to a rule of thumb:

![Figure 33: Circuit A drives circuit B](image)

How does this rule get us the desired result? Look at the problem as a familiar voltage divider question. If $R_{OUTA}$ is much smaller than $R_{INB}$, then the divider delivers nearly all of the original signal. If the relation is $1 : 10$, then the divider delivers $10/11$ of the signal: attenuation is just under $10\%$, and that’s good enough for our purposes.

We like this arrangement not just because we like big signals. (If that were the only concern, we could always boost the output signal, simply amplifying it.) We like this arrangement above all because it *allows us to design circuit-fragments independently:* we can design $A$, then design $B$, and so on. We need not consider $A,B$ as a large single circuit. That’s good: makes our work of design and analysis lots easier than it would be if we had to treat every large circuit as a unit.
An example, with numbers: What $R_{\text{Thev}}$ for droop of $\leq 10\%$? What $R'$s, therefore?

The effects of this rule of thumb become more interesting if you extend this chain: from A and B, on to C.

As we design C, what $R_{\text{Thev}}$ should we use for B? Is it just 10K parallel 10K? That’s the answer if we can consider B by itself, using the usual simplifying assumptions: source ideal ($R_{\text{OUT}} = 0$) and load ideal ($R_{\text{IN}}$ infinitely large).

But should we be more precise? Should we admit that the upper branch really looks like 10K + 2K: 12k? Oh dear! That’s 20% different. Is our whole scheme crumbling? Are we going to have to look all the way back to the start of the chain, in order to design the next link? Must we, in other words, consider the whole circuit at once, not just the fragment B, as we had hoped?

No. Relax. That 20% error gets diluted to half its value: $R_{\text{Thev}}$ for B is 10k parallel 12k, but that’s a shade under 5.5k. So—fumes of relief!—we need not look beyond B. We can, indeed, consider the circuit in the way we had hoped: fragment by fragment.

If this argument has not exhausted you, you might give our claim a further test by looking in the other direction: does C alter B’s input resistance appreciably ($> 10\%$)? You know the answer, but confirming it would let you check your understanding of our rule of thumb and its effects.

1N.6.3.1 Two Important Exceptions to our Rule of Thumb: Signals that are Currents, and Transmission Lines

The rule of §1N.6.3 on the preceding page for relating impedances is extremely useful and important. But we should acknowledge two important classes of circuits to which it does not apply:

- signal sources that provide \textit{current} signals rather than voltage signals. At the moment, this distinction
may puzzle you. Yet the distinction between *voltage sources*—the sort familiar to most of us—versus *current sources* is substantial and important.\(^{17}\)

- high-frequency circuits where the signal paths must be treated as “transmission lines.” (You will find a supplementary note on this topic, near the end of this book.)

Only at frequencies above what we will use in this course do such effects become apparent. These are frequencies *(or frequency components, since “steep” waveform edges include high frequency components, as Fourier teaches—see Classnotes 3)* where the period of a signal is comparable to the time required for that signal to travel to the end of its path. For example, a six-foot cable’s propagation time would be about 9ns, the period of a 110MHz sinusoid. A square wave at 22MHz would include quite a strong component at that frequency, and this component would be distorted by such a cable if we did not take care to “terminate” it properly. In fact, these calculations understate the difficulties; a path length that is between 1/10 and 1/4 of the wavelength can call for termination.

We don’t want you to worry, today, about these two points. Signals as currents are unusual, and in this course you are not likely to confront transmission-line problems. But be warned that you will meet these later, if you begin to work with signals at higher frequencies.

**1N.7 Reading in AoE**

This is the first instance in which we’ve tried to steer you toward particular sections of the text, *The Art of Electronics*. That book is, of course a sibling of the book you are reading (or is it the mother?). You should not consider the readings listed below to be *required*. But if you are proceeding by using the two books in parallel, these are the sections we consider most relevant.

<table>
<thead>
<tr>
<th>Reading:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 1, §§ 1.1 - 1.32.</td>
</tr>
<tr>
<td>Appendix on “Resistor Types,” <em>Re</em>: resistor color code and precision resistors</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems in text.</td>
</tr>
<tr>
<td>Additional Exercises 1.2.</td>
</tr>
</tbody>
</table>

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\(^{17}\)You may be inclined to protest that you can convert a current into a voltage and vise versa. You will build your first circuit with an output that is a determined *current* in Lab 4, and you will meet your first current-source transducer (a photodiode) in Lab 6.