1.1
a) Series: $R = R_1 + R_2 = 15 \, k$

b) Parallel: $R = \frac{R_1 \cdot R_2}{R_1 + R_2} = 5.33 \, k$
   
   (or, using shortcut $a_2$, $R = 10 \, k / 3$)

1.2
- $I = 12 \, A$
- $P = VI = 144 \, W$ (or $P = \frac{V^2}{R}$)

1.4
- as above for 1.3

1.5
- $P = I \cdot V = \frac{V^2}{R}$
- $V_{\text{max}} = 15 \, V \Rightarrow P_{\text{max}} = \frac{15^2}{10} = 0.225 \, W$
- This is $\frac{1}{4}$ W.
- (What if $R$ is 10% low? $P = P_{\text{max}} \cdot 0.9$
  
  $= 0.225 \, W \times 0.9 = 0.2025 \, W$)

1.6
- $10^{10} \, W, 110 \, V \Rightarrow I = 9 \times 10^7 \, \text{amps, approximately}$

a) $I^2 R = (9 \times 10^7)^2 \times 5 \times 10^8 \, \text{watts/ft}$
  
  $= 4.13 \times 10^{10} \, \text{watts/ft}$

b) $\Rightarrow$ lose all $10^8 \, \text{watts}$ in $24.2 \, \text{ft}$

c) $P = \lambda \sigma T^4$

- $A = 2 \pi R L = 2 \pi 20 \, \text{cm}^2 / \text{ft}$
- $\sigma = 6 \times 10^{-8} \, \text{W/} \Omega \text{K}^2 \text{cm}^{-2}$

- Using $P$ from above,
  
  $T_4 \sigma = \frac{P}{2 \pi R L} = \frac{4.13 \times 10^8}{6 \times 10^{-8} \times 2.9 \times 10^3} = 2.36 \times 10^{16} \, \text{K}^4$
  
  $T = 12,400 \, \text{K}$

This is hotter than sun's surface ($\sim 5000 \, \text{K}$)

Copper would melt (unpreventable)?
20000 x 1V meters (8½A movement) \(\rightarrow\) 20k on 1V (f.s.e.) range

\[ \frac{20k}{20k + 10k} \times 1V = \frac{2}{3} \text{ volt} \]

\(\Rightarrow\) 1V source, int. res. = 10k

It's a voltage divider: \( V_{\text{mkt}} = \frac{20k}{20k + 10k} \times 1V = \frac{2}{3} \text{ volt} \)

\(\text{Same thing: } V_{\text{meas}} = \frac{20/3}{20/3 + 10} \times 1V = 0.4 \text{ volt} \)

(alternate method: \(\frac{10k}{1k + 10k} = 0.5V \))

(load with 20k: \( V_{\text{meas}} = \frac{20k}{20k + 10k} \times 1V = 0.4 \text{ volt} \))
50 µA, 5 kΩ full-scale for 0.25 volt applied

\[ R_{\text{shunt}} \]
\[ 5k \quad \text{50 µA} \quad \text{perfect} \]
\( (R=0) \)

We want 0.25 volts across \( R_{\text{shunt}} \) for \( I = 1A \)

Ignoring the 5k, \( R_{\text{shunt}} \times 0.25A \)

(to be exact, \( 5k/V_{\text{shunt}} = 0.25A, \quad \Rightarrow R_{\text{shunt}} = 0.2500\text{kΩ} \))

To make it 0-10 volts, want \( 10/\left( R_{\text{series}} + 5k \right) \) = 50 µA

\[ R_{\text{series}} \quad 5k \quad 0-50\text{µA} \quad (R=0) \]

\[ \Rightarrow R_{\text{series}} + 5k = 200k \]
\[ R_{\text{series}} = 155k \]

1.9

a) No load, 15V - (divider)

b) With load, 30V

\[ V_{\text{out}} = 10V \]

\[ \frac{5k}{10k} \]

\[ V_{\text{out}} = 15V \]

\[ \frac{5k}{10k} \]

(c) \[ 30V \]

\[ \frac{5k}{10k} \]

\[ V_{\text{out}} = 15V \]

\[ \frac{5k}{10k} \]

(d) \[ 15V \]

\[ \frac{5k}{10k} \]

\[ V_{\text{out}} = 15 \times 15 = 10V \]

(e) \[ P_1 = V^2/R = 20/10^4 = 0.002 \text{W} \]
\[ P_2, P_3 : \quad P = V^2/R = 10^2/10^4 = 10\text{mW} \text{ each} \]

\[ (1/2) \]
\[ V_{\text{load}} = \frac{R_1}{R_1 + R_3} V \]

\[ P_{\text{load}} = \frac{V_{\text{load}}}{R_{\text{load}}} = V^2 \frac{R_1}{(R_1 + R_3)^2} \]

Only \( R_2 \) is adjustable.

\[ \frac{\partial P_{\text{load}}}{\partial R_{\text{load}}} = V^2 \left[ \frac{1}{(R_1 + R_3)^2} + \frac{R_1}{(R_1 + R_3)^3} \right] = 0 \]

\[ \Rightarrow 1 = \frac{2R_4}{R_1 + R_3} \]

\[ R_4 = R_3 \]

\[ dB = 20 \log_{10} \frac{V_2}{V_1} \]

\[ \frac{V_2}{V_1} = 10 \frac{dB}{20} : \quad \begin{array}{ccc} 3\text{dB} & \frac{V_2}{V_1} & 2 \\ 6\text{dB} & 2 & 4 \\ 10\text{dB} & \frac{V_2}{10} & 10 \\ 20\text{dB} & 10 & 100 \end{array} \]
1.12

\[ V_1 = \frac{Q}{C_1} \quad V_2 = \frac{Q}{C_2} \quad \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{V} \]

\[ V_{\text{total}} = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_1 + C_2} \]

So, \( C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} \) \( \text{(same as resistors in parallel)} \)

1.16

a) \[ Z_1 = -\frac{j}{\omega C_1} \quad Z_2 = -\frac{j}{\omega C_2} \]

\[ Z_{\text{total}} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = -\frac{j}{\omega (C_1 + C_2)} = -\frac{j}{\omega C_{\text{total}}} \]

\[ \Rightarrow C_{\text{total}} = C_1 + C_2 \]

b) \[ C_1 \quad C_2 \]

\[ Z_{\text{total}} = Z_1 + Z_2 = \frac{1}{\omega C_1} - \frac{1}{\omega C_2} = -\frac{j}{\omega C_1} \frac{1}{C_1} \frac{1}{C_2} = -\frac{j}{\omega C_{\text{total}}} \]

\[ \Rightarrow C_{\text{total}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} \]

1.17

\[ A = BC \quad A = Ae^{i\theta_2} \quad \frac{B}{C} = Be^{i\theta_1} \]

So \( Ae^{i\theta_2} = BCe^{i(\theta_1 + \theta_2)} \)

\[ \text{So, } A = BC \]
1.18 Choose t = 0 when one quantity (voltage, current) is zero. Then,
\[
\mathcal{P} = \frac{1}{2\pi} \int_{0}^{2\pi} \sin x \cos x \, dx = \frac{1}{2\pi} \int_{0}^{2\pi} \sin x \, d(sin x)
\]
\[
= \frac{1}{4\pi} \sin^2 x \bigg|_{0}^{2\pi} = 0
\]

1.19

\[
V_0 \cos wt \quad \rightarrow \quad \text{magnitude of } V_k = \frac{R}{\sqrt{R^2 + \omega^2 C^2}} V_0 \quad \text{(that's } \frac{V_0}{\frac{1}{|Z_k|}} \text{)}
\]

so \[
\frac{V_k^2}{P} = P(e) = \frac{V_0^2 R}{R^2 + \omega^2 C^2}
\]

in agreement with the calculation using \( P = \text{Re}(V \times I^*) \)

1.20

\[
\bar{z}_{\text{series}} = R + j\omega L - \frac{j}{\omega C} = R + j\omega L - \frac{1}{\omega C} = R
\]

impedance is purely resistive; \( \text{PF} = 1.0 \)

\[
\bar{z}_{\text{parallel}} = \frac{1}{R} \frac{1}{\frac{1}{L} + \frac{1}{j\omega C}} = \frac{1}{R + \frac{1}{j\omega L} - \frac{1}{j\omega C}} = R
\]

same conclusion as above
\[
I = \frac{V_{in}}{Z_{in}} = \frac{V_{in}}{R - j\omega C} = \frac{V_{in}}{R^2 + (\omega C)^2}
\]
\[
V_{out} = I Z_{out} = V_{in} \frac{R + j\omega C}{R^2 + (\omega C)^2} \cdot V_{in} \frac{1}{R^2 + (\omega C)^2}
\]

Guided way: \( V_{out} = \frac{12V}{11} \cdot V_{in} \) →

---

Diagram:

The hypotenuse represents the input, and the real leg \( R \) represents the output voltage. So, \( V_{out} = \frac{R}{(R^2 + \omega^2 C^2)} \cdot V_{in} \) (magnitudes).
EXERCISE 1.23
At what frequency does an RC low-pass filter attenuate by 6 dB (output voltage equal to half the input voltage)? What is the phase shift at that frequency?

\[
\frac{V_{out}}{V_{in}} = \frac{\frac{1}{C}}{\frac{1}{R} + \frac{1}{C} + j\omega C} = \frac{1}{1 + j\omega RC}
\]

To get magnitude of this expression, multiply by complex conjugate, then take square root (see p. 34):

\[
\left|\frac{V_{out}}{V_{in}}\right| = \sqrt{\frac{1}{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}
\]

At what frequency is this \(\frac{1}{2}\)?

\[
1 + (\omega RC)^2 = 2^2 = 4 \Rightarrow \omega^2 = \frac{3}{RC}^2 \Rightarrow \omega = \frac{\sqrt{3}}{RC}
\]

Or \( f = \frac{\sqrt{3}}{2\pi RC} \)

What's phase shift? Here, see phasor diagram just below: \( \theta = \arccos(\frac{1}{2}) = 60^\circ \) (as said below)

\[ V_0 \]
\[ V_{in} \]
\[ \frac{V_0}{V_{in}} = \frac{\frac{1}{C}}{\frac{1}{R} + \frac{1}{C} + j\omega C} \text{ (magnitudes)} \]

\[ \frac{1}{\sqrt{1 + (\omega RC)^2}} \]

phase shift: \( \frac{\sqrt{3}}{2} \)

@ \( \omega_{1/2B} \): \( \theta = \arccos(\frac{1}{2}) = 60^\circ \)

(\( @ \omega_{1/2B}, \theta = 45^\circ \))
For amplitude response, we use the (by now) usual trick:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{\left| \frac{z_{\text{out}}}{z_{\text{in}}} \right|}{1 + \frac{R^2}{(\omega L - \omega C)^2}} = \frac{\omega L - \omega C}{\left[ R^2 + (\omega L - \omega C)^2 \right]^{1/2}}
\]

\[
= \frac{1}{\left[ 1 + \frac{R^2}{(\omega L - \omega C)^2} \right]}
\]

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \begin{cases} 
1 @ \omega = 0 \\
0 @ \omega = \frac{1}{\sqrt{LC}} \\
1 @ \omega = \infty
\end{cases}
\]

**Exercise 1.26**

Design a full-wave bridge rectifier circuit to deliver 10 volts dc with less than 0.1 volt (pp) ripple into a load drawing up to 10mA. Choose the appropriate ac input voltage, assuming 0.6 volt diode drops. Be sure to use the correct ripple frequency in your calculation.

\[\text{Need 10V plus 2 diode drops caused by bridge: } \Rightarrow 11.2V\]

\[\text{Ripple: } < 0.1V\]

\[\text{Output waveform:}\]

\[\text{C: } 800 \mu F\]

\[\text{Load: } 10mA\]

\[\Delta t = 8ms\]

\[\text{Time between peaks of rectified waveform, } T = \frac{1}{2 \times 60Hz} \times 8ms\]

\[\text{16ms full-wave rectified, } 8ms\]
Average value of $I = \frac{1}{2} \times 2 = 1 \text{ Amp}$.
Average value of $I^2 = \frac{1}{2} \times (2^2) = 2\text{ watt}$.
Requires fuse of rating $\geq \sqrt{2}\text{ A}$ or $1.4\text{ A}$.

Add resistor as shown.

OR:

- $1k$ ohm resistor
diodes
AE1.1

\[ 5V \text{ - open circuit } V \rightarrow I_{n} R_{n} \]

\[ 1mA \text{ - short circuit } I \rightarrow I_{n} \]

\[ I_{n} = 1mA \]

\[ R_{n} = \frac{V_{oc}}{I_{sc}} = \frac{5V}{1mA} = 5k \Omega = 1mA \]

Load with 5kΩ

\[ 10V \text{ - } 10k \Omega \]

\[ 1mA \text{ - } 2.5k \Omega \]

\[ V_{out} = \frac{10/2}{10/2 + 10} = 2.5V \]

\[ V_{out} = 2.5V \]

\[ 5V \text{ - } 5k \Omega \]

\[ V_{out} = \frac{1}{5} \times 5V = 2.5V \]

AE1.2

\[ 0.5mA \uparrow \]

\[ V_{oc} = 0.5mA \times 10k = 5V \]

\[ I_{sc} = 0.5mA \]

\[ \Rightarrow 5V \]

same open circuit voltage, and same resistor network, but different. Then we're equivalent? More on this later.
ADDITIONAL EXERCISES

(1) Find the Norton equivalent circuit (a current source in parallel with a resistor) for the voltage divider in Figure 1.101. Show that the Norton equivalent gives the same output voltage as the actual circuit when loaded by a 5k resistor.

Figure 1.101

Thevenin equivalent:

Norton equiv:

I get it from the Thevenin model, since I'm used to That one: we rarely use the Norton model

Try applying load:

(a) To original:

\[
\begin{align*}
10V & \quad \frac{10k}{10k} \quad \frac{5k}{10k} \\
\end{align*}
\]

\[
R' = \frac{10k \cdot 5k}{10k + 5k} = 3.3k
\]

\[
\Rightarrow \quad \frac{10V}{10V} = \frac{3.3}{15.3} \approx \frac{1}{4} \Rightarrow Uo = 2.5V
\]

(b) To Norton model:

\[
\begin{align*}
1mA & \quad \frac{1mA}{1k} \quad \frac{5k}{1k} \\
\end{align*}
\]

\[
\Rightarrow \quad Uo = 2.5V
\]
Find the Thévenin equivalent for the circuit shown in Figure 1.102. Is it the same as the Thévenin equivalent for exercise 1?

\[ V_{TH} = \text{Vout-open-circuit} = I \cdot R_1 = 0.5 \text{mA} \cdot 10k = 5 \text{V} \]

\[ R_{TH} = \frac{V_{TH}}{I_{SHORT-CIRCUIT}} = \frac{5 \text{V}}{0.5 \text{mA}} = 10k \]

Our usual shortcut yields the same result:

\[ R_m = \text{all paths in parallel} = 10k \parallel (10k + R_{E-source}) = 10k \quad \text{(Infinite, since \( AV \), \( AE \), zero!)} \]

No, not same as for ex. 1:

\[ R_{TH} \text{ differs.} \]
(3) Design a "rumble filter" for audio. It should pass frequencies greater than 20Hz (at the -3dB point at 10Hz). Assume zero source impedance (perfect voltage source) and 10k (minimum) load impedance (that's important so that you can choose R and C such that the load doesn't affect the filter operation significantly).

$$T_{3dB} = 10 \times 10^3, \quad R_{load} = 10k$$

Rumble Filter = high-pass

\[
\begin{align*}
C &= \frac{1}{2 \pi f \cdot R} \approx \frac{1}{6 \times 10^{-3}} \cdot \frac{1}{0.016 \times 10^{-3}} \\
&= 16 \mu F
\end{align*}
\]

Use \( R \) as \( Z_{out} \) and \( C \) as \( Z_{out} \) \( \Rightarrow R = 1k \)

\( R_c = \frac{R}{2} \) and \( C \)

As above,  
1. choose \( R = R_{load} \) \( \Rightarrow R = 1k \)  
2. choose \( R \cdot C = \frac{1}{2 \pi f} \) \( \Rightarrow C = 0.016 \mu F \)

choose \( R \cdot C = \frac{1}{2 \pi f} \)
(6) Design a bandpass RC filter (Fig. 1.104); $f_1$ and $f_2$ are the 3dB points. Choose impedances so that the first stage isn't much affected by the loading of the second stage.

![](https://via.placeholder.com/150)

A high-pass...

...a low-pass.

The signal must make it past both:

$\Rightarrow$ the two filters are in series

(Signal) \[ \overset{\text{(Signal)}}{W} \rightarrow \overset{\text{FILTER1}}{\rightarrow} \overset{\text{FILTER2}}{\rightarrow} \]

(The order doesn't matter.)

\[ \frac{1}{R} - \frac{1}{C_{CH1}} \]

\[ \frac{1}{10R} \]

"first stage" \hspace{1cm} "second stage"

\[ w_1 \text{ in } w_{3\text{dB}} \text{ for the high-pass} \]

\[ w_2 \text{ in } w_{3\text{dB}} \text{ for the low-pass} \]

\[ \Rightarrow C_{CL0} = \frac{1}{w_2 R} \]

\[ C_{CH1} = \frac{1}{w_1 (10R)} \]

(Not to load 1st stage, we follow our "10x" rule-of-thumb)

(This all seems very abstract, perhaps, numbers would help.)
With the values shown, you have a box attenuator for all input frequencies. The resulting input impedance is equivalent to $10\, \text{MO}\| \, 12\, \text{pF}$ (i.e., ten times the input impedance you began with.)

\[ (\text{end CL1 SOLUS}) \]