EPS 5 Spring 2010

Problem Set 5

Due Wed, March 24th, 2010 (at the beginning of lecture)

1. The Coriolis Force

Deflection due to Coriolis force is given by $\Delta y = (\Delta x)^2 \Omega \sin(\lambda) / v$, where v is the speed, λ is the latitude, Δx is the distance traveled, and Ω is the angular velocity of the Earth (7.3×10^{-5} s^{-1}).

(a) Find the displacement of a snowball thrown 10m at 30 km/h in Cambridge (latitude 42ºN). (2 points)

$\Delta y = ((10m)^2 * 7.3 \times 10^{-5}\ s^{-1} * \sin(42^o))/(3.0 \times 10^4/(60*60)\ m\ s^{-1})$

$\Delta y = 5.9 \times 10^{-4}$ m = 0.59mm

(b) Simon Amman of Switzerland won the gold medal in ski jump in the Olympics in Vancouver (latitude 49º). He jumped 141m with a speed of around 95km/h. How much was the deflection on his jump due to the Coriolis force? (2 points)

$\Delta y = ((141m)^2 * 7.3 \times 10^{-5}\ s^{-1} * \sin(49^o))/(9.5 \times 10^4/(60*60)\ m\ s^{-1})$

Δy = 0.041 m = 4.1cm

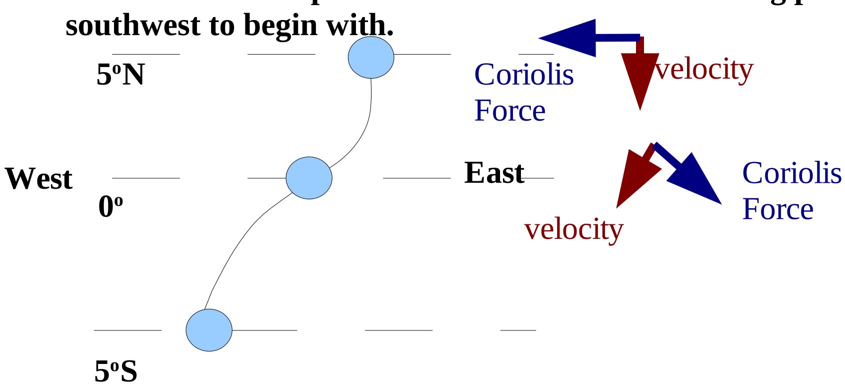
(c) What is the Coriolis force magnitude and direction of a 6 kg cannon ball fired from east to west with speed 50 km/h over 200 m at latitude 30ºS ? What is the Coriolis force magnitude and direction for the same problem with the cannon ball fired from south to north? (2 points)

$F_c = 2\ m\ \Omega\ v \sin(\lambda) = 2 * 6kg * 7.3 \times 10^{-5}\ s^{-1} * (5.0 \times 10^4/(60*60)\ m\ s^{-1}) * \sin(30^o)$

$F_c = 6.0 \times 10^{-3}$ N for both; directed to the left of motion in the Southern Hemisphere: toward south for east → west case; toward west for south → north case

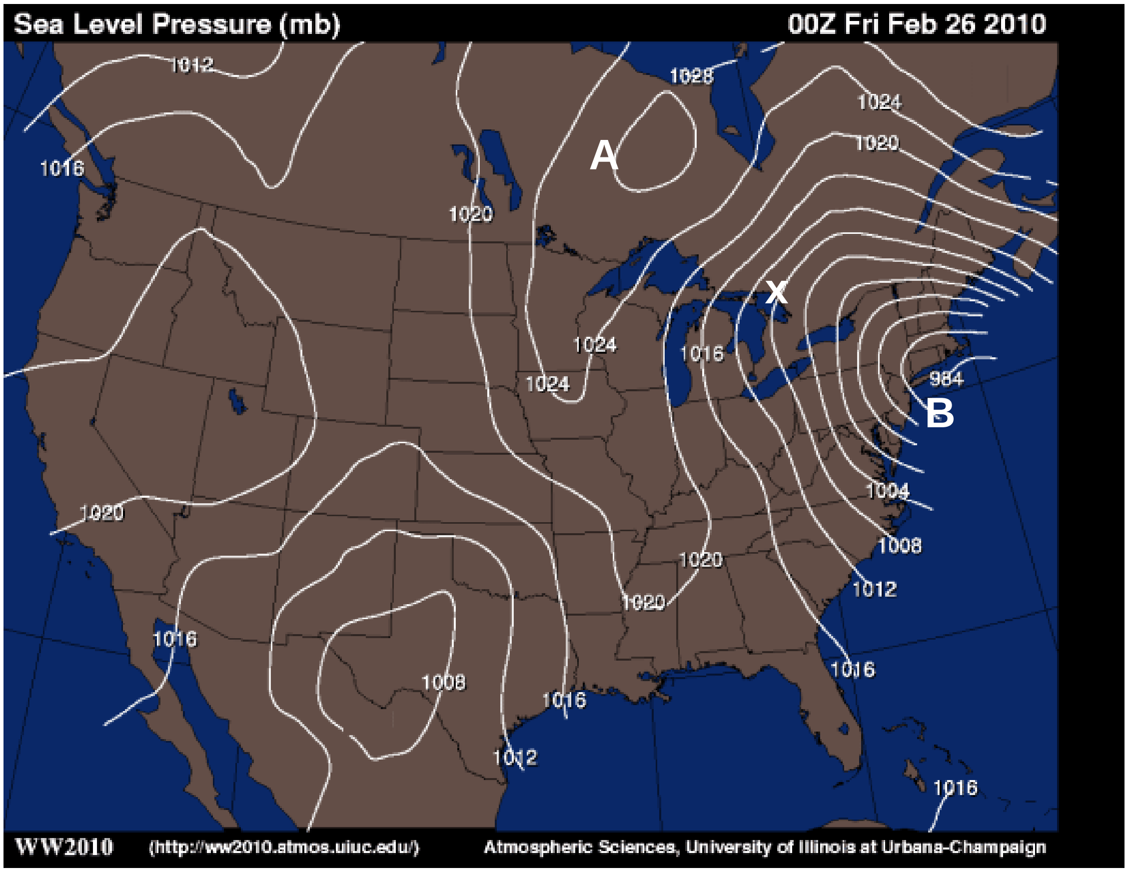
(d) Imagine a new extreme biathlon that involves sailing out into the middle of the ocean and launching missiles at a target thousands of kilometers away. Let's say you sail out to a point located at 5ºN, 150ºW. You launch the missile due south. When the missile crosses the equator, is it more to the east, more to the west, or at the same longitude as your starting position? When the missile reaches 5ºS, is the missile more to the east, more to the west, or at the same longitude as when it crossed the equator? Hint: Draw a picture. (2 points)

The missile with a south-pointing velocity is deflected to the right of its motion in the Northern Hemisphere, which is toward the west. It will be to the west of its original position when it crosses the equator. The missile is deflected to the left of its motion in the Southern Hemisphere, but will still end up more to the west than its starting position since it had an orientation to the southwest to begin with.



2. Geostrophic Balance

The figure below shows actual atmospheric surface pressure over the United States from Friday, February 26th. The white contours are isobars – lines of constant pressure – which are drawn for every change in pressure of 4mb.



(a) Consider the pressure systems at points A and B. On the map above, label the high and the low. At the midpoint X between A and B, draw and label an arrow to represent the pressure gradient force. Assuming no friction, draw another arrow to represent the Coriolis force. Finally, indicate the direction of the geostrophic wind. (4 points)

Low pressure at B. Pressure gradient from from A to B. Coriolis force in opposite direction. Wind in direction with Coriolis force 90° to the right, toward southwest.

(b) Still assuming geostrophic balance with no friction, calculate the velocity of the geostrophic wind at X, latitude 47° N. The distance between points A and B is 1800 km. The temperature at X is -2°C. (4 points)

$F_{pg} = (m/\rho)*(\Delta P/\Delta x)$

$F_c = 2\ m\ \Omega\ v\ \sin(\lambda)$

$v_g = (\Delta P/\Delta x)\ /\ (2\ \rho\ \Omega\ \sin(\lambda))$

$\Delta P=(1028mb - 984mb) = 44mb = 4.4x10^4$ Pa

$PV = nRT \rightarrow \rho = n*M_a/V \rightarrow P = \rho(R/M_a)T = \rho kT \rightarrow \rho = P/kT$

$\rho = (1.008x10^5\ Pa)/(287\ J\ kg^{-1}\ K^{-1} * 271\ K) = 1.30\ kg\ m^{-3}$

$\rightarrow$ $v_g = (4.4x10^3\ Pa/1.8x10^6\ m)\ /\ (2 * 1.30\ kg\ m^{-3} * 7.3x10^{-5}\ rad/s * \sin(47^o))$

$v_g = 17.6$ m/s (= 39mph)

(c) What do you think the weather was like in Cambridge on this day? (Sunny/rainy, windy/still, etc.) Justify your answer. (2 points)

Cambridge had rainy, windy weather. There is convergence at the low pressure system just off the coast, causing air to rise and produce clouds and rain. Air is coming in from the east, bringing moisture from over the ocean. The isobars are close together, indicating a strong pressure gradient and high winds.

3. The General Circulation of the Atmosphere

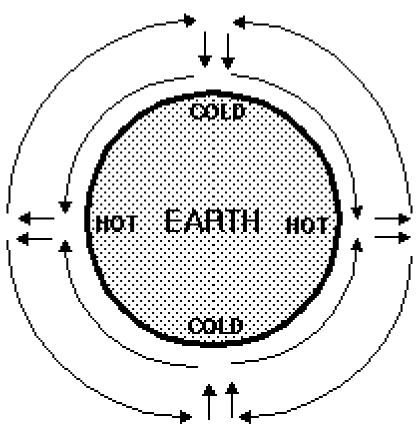
(a) Explain why trade winds blow from east to west in both the Northern Hemisphere and the Southern Hemisphere. (2 points)

The ITCZ is a region of convergence near the equator. In the Northern Hemisphere, air moves southward towards this region of convergence. The Coriolis force deflects it to the right, which is westward. In the Southern Hemisphere, air moves northward towards the ITCZ. The Coriolis deflects it to the left, which is also westward. Hence, easterly trade winds blow east to west in both hemispheres.

(b) Explain why the major deserts of the world are all located at around 30º latitude (both North and South). (2 points)

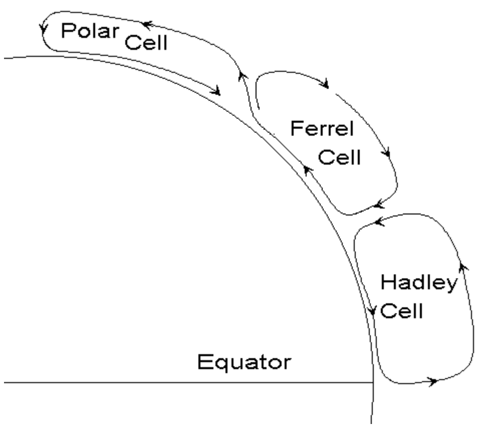
The sinking branch of the Hadley cells is typically located around 30º latitude. As air descends, its temperature increases according to the dry adiabatic lapse rate, making it impossible for water vapor to condense. The temperature of the parcel increases faster than the dew point. This also sets up a subsidence inversion, inhibiting convection. Therefore, most of the earth's deserts lie around 30º latitude .

(c) The figure below illustrates the model proposed by Hadley for the general circulation of the atmosphere. Explain Hadley's reasoning. How can this model help us to understand why the Intertropical Convergence Zone moves with the season? (2 points)



Hadley envisioned the circulation as a global sea breeze driven by the temperature contrast between the hot equator and the cold poles. This model explains the presence of the ITCZ near the equator and the seasonal variation in the location of the ITCZ (as the region of maximum heating follows the Sun from the southern tropics in January to the northern tropics in July). [*Introduction to Atmospheric Chemistry* by Daniel Jacob]

(d) Instead of having one ascending and descending cell in each hemisphere, the actual atmosphere more closely resembles the figure below. Explain the main reason why. (2 points)



A flaw is that it does not account for the Coriolis force. Air in the high-altitude branches of the Hadley circulation cells blowing from the equator to the pole is accelerated by the Coriolis force as it moves poleward, eventually breaking down into an unstable flow. Thus it is observed that the Hadley cells extend only from the equator to about 30º latitude. At 30º the air is pushed down, producing the observed subtropical high-pressure belts. The Hadley cells remain a good model for the circulation of the tropical atmosphere. The Coriolis force acting on the low-altitude branches produces the easterlies observed near the surface. [*Introduction to Atmospheric Chemistry* by Daniel Jacob]