frame is important

- in graphics, we often keep track of a number of frames
  - each object, the camera, the world ...
  - so we need to be careful how we use matrices.
- given point and matrix is not enough to specify mapping
- for example point \( \tilde{p} \) and the matrix

\[
S = \begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

- the matrix is non-uniform scaling
- fix a frame \( \tilde{f} \)
- in this frame \( \tilde{p} = \tilde{f}c \)
- transform with matrix \( \tilde{f}c \rightarrow \tilde{f}Sc = \tilde{f}(Sc) =: \tilde{f}c' \)
  - the stretches by factor of two in first axis of \( \tilde{f} \)
- see fig

other frame

- pick some other frame \( \tilde{a}f \).
- relationship between bases \( \tilde{a}f = \tilde{f}A \).
- express same point as \( \tilde{p} = \tilde{f}c = \tilde{a}f(A^{-1}c) =: \tilde{a}f d \),
- now use matrix \( S \) we get \( \tilde{a}f d \rightarrow \tilde{a}f(Sc) =: \tilde{a}f' d' \).
- the same point \( \tilde{p} \) is stretched about first axis of \( \tilde{a}f \)
- see fig
- also rot fig

left-of rule

- point is transformed with respect to the the frame that appears immediately to the left of the transformation matrix in the expression.
- We read

\[
\tilde{p} = \tilde{f}c \rightarrow \tilde{f}Sc
\]

- as “\( \tilde{p} \) is transformed by \( S \) with respect to \( \tilde{f} \)”.
- We read

\[
\tilde{p} = \tilde{a}f A^{-1} c \rightarrow \tilde{a}f SA^{-1} c
\]

- as “\( \tilde{p} \) is transformed by \( S \) with respect to \( \tilde{a}f \)”.

more generally

- We read

\[
\tilde{p} = \tilde{f}ABc \rightarrow \tilde{f}ASBc
\]

- as “\( \tilde{p} \) is transformed by \( S \) with respect to \( \tilde{f}A \)”.

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for frames

• same for transformations of frames
  
  \[ \vec{f} \rightarrow \vec{f}S \]
  
  – “\( \vec{f} \) is transformed by \( S \) with respect to \( \vec{f} \).”

• We read
  
  \[ \vec{f} = \vec{a}A^{-1} \rightarrow \vec{a}SA^{-1} \]
  
  – as “\( \vec{f} \) is transformed by \( S \) with respect to \( \vec{a} \).”

more generally

• We read
  
  \[ \vec{g} = \vec{f}AB \rightarrow \vec{f}ASB \]
  
  – as “\( \vec{g} \) is transformed by \( S \) with respect to \( \vec{f}A \).”

auxiliary frame

• we may wish to transform a frame \( \vec{f} \) in some specific way represented by a matrix \( M \), with respect to some auxiliary frame \( \vec{a} \).
  
  – For example, we may be using some frame to model the planet Earth, and we now wish the Earth to rotate around the Sun’s frame.

• let \( \vec{a} = \vec{f}A \)

• then The transformed frame can then be expressed as

\[
\begin{align*}
\vec{f} & = \vec{a}A^{-1} \\
& \rightarrow \vec{a}MA^{-1} \\
& = \vec{f}AMA^{-1}
\end{align*}
\]

multiple transformations

• using the “left of” rule

• example:
  
  – a rotation matrix \( R \) rotating a point by \( \theta \) degrees about origin
  
  – translation matrix \( T \), translating the point by one unit in the direction of the first frame axis.

interp 1

• given tform

\[ \vec{f} \rightarrow \vec{f}TR \]

• break into 2 steps

• In the first step

\[ \vec{f} \rightarrow \vec{f}T = \vec{f'} \]

  – \( \vec{f} \) is transformed by \( T \) with respect to \( \vec{f} \) and we call the resulting frame \( \vec{f'} \).
• In the second step,

\[ \tilde{\mathbf{f}}^T \rightarrow \tilde{\mathbf{f}}^T R \]
\[ \tilde{\mathbf{f}}^t \rightarrow \tilde{\mathbf{f}}^t R \]

- This is interpreted as: \( \tilde{\mathbf{f}}^t \) is transformed by \( R \) with respect to \( \tilde{\mathbf{f}}^t \).

other way

• In the first step

\[ \tilde{\mathbf{f}}^t \rightarrow \tilde{\mathbf{f}}^t R = \tilde{\mathbf{f}}^{ot} \]

\( \tilde{\mathbf{f}}^{ot} \) is transformed by \( R \) with respect to \( \tilde{\mathbf{f}}^{ot} \) and we call the resulting frame \( \tilde{\mathbf{f}}^{ot} \).

• In the second step,

\[ \tilde{\mathbf{f}}^{ot} R \rightarrow \tilde{\mathbf{f}}^{ot} TR \]

\( \tilde{\mathbf{f}}^{ot} \) is transformed by \( T \) with respect to \( \tilde{\mathbf{f}}^{ot} \).

summary

• both interp can be useful
• left to right, wrt latest (local)
  - right to left, wrt original frame (global)