# Computer Science S-111
## Intensive Introduction to Computer Science

Harvard Summer School 2020  
David G. Sullivan, Ph.D.

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Welcome to CS S-111!

*Computer science is not so much the science of computers as it is the science of solving problems using computers.*

Eric Roberts

- This course covers:
  - the process of developing *algorithms* to solve problems
  - the process of developing computer programs to express those algorithms
  - fundamental *data structures* for imposing order on a collection of information
  - the process of *comparing* data structures & algorithms for a given problem
Computer Science and Programming

• There are many different fields within CS, including:
  • software systems
  • computer architecture
  • networking
  • programming languages, compilers, etc.
  • theory
  • AI

• Experts in many of these fields don’t do much programming!

• However, learning to program will help you to develop ways of thinking and solving problems used in all fields of CS.

A Rigorous Introduction

• Intended for:
  • future concentrators who plan to take more advanced courses
  • others who want a rigorous introduction
  • no programming background required, but can also benefit people with prior background

• Allow for **20-30 hours** of work per week
  • start work early!
  • come for help!
  • don’t fall behind!
CS 111 Requirements

• Lectures and sections
  • attendance at both is required

• Ten problem sets (40%)
  • part I = "written" problems
  • part II = "programming" problems
  • grad-credit students will have extra work on most assts.

• Four unit tests (25%)
  • given at the end of lecture (see the schedule)

• Final exam (35%)
  • Friday, August 7, 8:30-11:30 a.m.

Textbooks

• Required: The CSCI S-111 Coursepack
  • contains all of the lecture notes
  • print it and mark it up during lecture

• Optional resource for the first half:
  Building Java Programs by Stuart Reges and Marty Stepp (Addison Wesley).

• Optional resource for the second half:
  Data Structures & Algorithms in Java, 2nd edition
  by Robert Lafore (SAMS Publishing).
Other Course Staff

• Teaching Assistants (TAs):
  Ashby Hobart
  Libby James

• See the course website for contact info.

• **Piazza is your best bet for questions.**

• For purely administrative questions: libs111@fas.harvard.edu
  • will forward your email to the full course staff

Other Details of the Syllabus

• Schedule:
  • note the due dates and test dates
  • no lectures or sections on most Wednesdays
    • **exceptions:** July 1 (July 3 is off), July 8 (July 10 is off),
      August 5 (August 6 is off)

• Policies:
  • 10% penalty for submissions that are one day late
  • please don't request an extension unless it's an emergency!
  • grading

• Please read the syllabus carefully and make sure that you
  understand the policies and follow them carefully.

• Let us know if you have any questions.
Algorithms

• In order to solve a problem using a computer, you need to come up with one or more *algorithms*.

• An algorithm is a step-by-step description of how to accomplish a task.

• An algorithm must be:
  • *precise*: specified in a clear and unambiguous way
  • *effective*: capable of being carried out

Programming

• Programming involves expressing an algorithm in a form that a computer can interpret.

• We will primarily be using the Java programming language.
  • one of many possible languages

• The key concepts of the course transcend this language.
What Does a Program Look Like?

• Here’s a Java program that displays a simple message:

```java
public class HelloWorld {
    public static void main(String[] args) {
        System.out.println("hello, world");
    }
}
```

• Like all programming languages, Java has a precise set of rules that you must follow.
  • the syntax of the language

• To quickly introduce you to a number of key concepts, we will begin with a simpler language.

---

Scratch

• A simple but powerful graphical programming language
  • developed at the MIT Media Lab
  • makes it easy to create animations, games, etc.
Scratch Basics

- Scratch programs (scripts) control characters called sprites.
- Sprites perform actions and interact with each other on the stage.

Program Building Blocks

- Grouped into color-coded categories:

  - The shape of a building block indicates where it can go.
Program Building Blocks: Statements

- Statement = a command or action
  - move 10 steps
  - play sound meow
  - turn 15 degrees
  - hide
  - wait 1 secs

- Statements have bumps and/or notches that allow you to stack them.
  - each stack is a single script

- A statement may have:
  - an input area that takes a value (10, 1, etc.)
  - a pull-down menu with choices (meow)

Program Building Blocks: Statements (cont.)

- Clicking on any statement in a script executes the script.

- When rearranging blocks, dragging a statement drags it and any other statements below it in the stack.
  - example: dragging the wait command below
Flow of Control

- Flow of control = the order in which statements are executed
- By default, statements in a script are executed sequentially from top to bottom when the script is clicked.

- **Control blocks** (gold in color) allow you to affect the flow of control.
  - simple example: the *wait* statement above pauses the flow of control

Flow of Control: Repetition

- Many control statements are C-shaped, which allows them to control other statements.
- Example: statements that repeat other statements.

- Drag statements inside the opening to create a repeating stack.

- In programming, a group of statements that repeats is known as a *loop*. 
Flow of Control: Responding to an Event

- *Hat blocks* (ones with rounded tops) can be put on top of a script.
  - They wait for an event to happen.
  - When it does, the script is executed.

What Does a Program Look Like?

- Recall our earlier Java program:

  ```java
  public class HelloWorld {
      public static void main(String[] args) {
          System.out.println("hello, world");
      }
  }
  ```

  - Here’s the Scratch version
  - And here’s the result:
Stage Coordinates

- Dimensions: 480 units wide by 360 units tall
- Center has coordinates of 0, 0

What does this program draw?
How many changes would be needed to draw this figure instead? (What are they?)

How could we draw this figure?
Flow of Control: Repeating a Repetition!

• One loop inside another loop!
  • known as a *nested loop*

• How many times is the *move* statement executed above?

Making Our Program Easier to Change

• It would be nice to avoid having to manually change *all* of the numbers.

• Take advantage of relationships between the numbers.
  • what are they?
Program Building Blocks: Variables

- A variable is a named location in the computer's memory that is used to store a value.

- Can picture it as a named box:

- To create a variable:

Using Variables in Your Program

- note: you must drag a variable into place, not type its name
Program Building Blocks: Operators

- Operators create a new value from existing values/variables.

Our Program with Variables and Operators
Getting User Input

• Use the ask command from the sensing category.

```
ask [text] and wait
```

• The value entered by the user is stored in the special variable `answer`, which is also located in the sensing category.

• Allowing the user to enter `numSides` and `numCopies`:

```
ask number of sides for polygon? and wait
set numSides to answer
ask number of copies? and wait
set numCopies to answer
```

Program Building Blocks: Boolean Expressions

• Blocks with pointed edges produce boolean values:
  • `true` or `false`

• Boolean operators:

- `<` Reports true if first value is less than second.
- `=` Reports true if two values are equal.
- `>` Reports true if first value is greater than second.
- `and` Reports true if both conditions are true.
- `or` Reports true if either condition is true.
- `not` Reports true if condition is false; reports false if condition is true.
Flow of Control: Conditional Execution

• conditional execution = deciding whether to execute one or more statements on the basis of some condition

• There are C-shaped control blocks for this:

• They have an input area with pointed edges for the condition.

Flow of Control: Conditional Execution (cont.)

• If the condition is true:
  • the statements under the *if* are executed
  • the statements under the *else* are *not* executed

• If the condition is false:
  • the statements under the *if* are *not* executed
  • the statements under the *else* are executed
How can we deal with invalid user inputs?

More Info on Scratch

- We're using the latest version:
  https://scratch.mit.edu/projects/editor

- Creating a Scratch account is not required for this course.
Final version

- We use two if-else statements to check for invalid inputs:
  - one checks for numSides < 3
  - one checks for numCopies < 1

- If an invalid input is found, we:
  - show the sprite
  - have the sprite say an error message
  - end the program

- Otherwise, we continue with the rest of the program.
Programming in Java

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Harvard University
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Programs and Classes

• In Java, all programs consist of one of more *classes*.

• For now:
  • we'll limit ourselves to writing a single class
  • you can just think of a class as a container for your program

• Example: our earlier program:

```java
public class HelloWorld {
    public static void main(String[] args) {
        System.out.println("hello, world");
    }
}
```

• A class must be defined in a file with a name of the form `<classname>.java`
  • for the class above, the name would be HelloWorld.java
Using an IDE

• An integrated development environment (IDE) is an application that helps you develop programs.

• We’ll use the Dr. Java IDE.
  • PS 0 told you how to obtain and install it.

• With an IDE, you do the following:
  • use its built-in text editor to write your code
  • instruct the IDE to compile the code
    • turns it into lower-level instructions that can be run
    • checks for violations of the syntax of the language
  • instruct the IDE to run the program
  • debug as needed, using the IDE’s debugging tools

Using Dr. Java
**Format of a Java Class**

- General syntax:
  ```
  public class <name> {
  code goes here...
  }
  ```
  where `<name>` is replaced by the name of the class.

- Notes:
  - the class begins with a *header*:
    ```
    public class <name>
    ```
  - the code inside the class is enclosed in curly braces (`{` and `}`)

**Methods**

- A method is a collection of instructions that perform some action or computation.

- Every Java program must include a method called `main`.
  - contains the instructions that will be executed first when the program is run

- Our example program includes a `main` method with a single instruction:
  ```
  public class HelloWorld {
  public static void main(String[] args) {
    System.out.println("hello, world");
  }
  }
  ```
Methods (cont.)

• General syntax for the main method:

```java
public static void main(String[] args) {
    <statement>;
    <statement>;
    ...
    <statement>;
}
```

where each `<statement>` is replaced by a single instruction.

• Notes:
  • the main method always begins with the same header:
    ```java
    public static void main(String[] args)
    ```
  • the code inside the method is enclosed in curly braces
  • each statement typically ends with a semi-colon
  • the statements are executed sequentially

Identifiers

• Used to name the components of a Java program like classes and methods.

• Rules:
  • must begin with a letter (a-z, A-Z), $, or _
  • can be followed by any number of letters, numbers, $, or _
  • spaces are not allowed
  • cannot be the same as a keyword – a word like `class` that is part of the language itself (see the book)

• Which of these are not valid identifiers?

  n1  num_values  2n
  avgSalary  course  name

• Java is case-sensitive (for both identifiers and keywords).
  • example: HelloWorld is not the same as helloworld
Conventions for Identifiers

- Capitalize class names.
  - example: HelloWorld

- Do not capitalize method names.
  - example: main

- Capitalize internal words within the name.
  - example: HelloWorld

Printing Text

```java
public class HelloWorld {
    public static void main(String[] args) {
        System.out.println("hello, world");
    }
}
```

- Our program contains a single statement that prints some text.
- The printed text appears in a window known as the console.
Printing Text (cont.)

- The general format of such statements is:
  ```java
  System.out.println("<text>");
  ```
  where `<text>` is replaced by the text you want to print.

- A piece of text like "Hello, world" is referred to as a **string literal**.
  - string: a collection of characters
  - literal: specified explicitly in the program ("hard-coded")

- A string literal must be enclosed in double quotes.

- You can print a blank line by omitting the string literal:
  ```java
  System.out.println();
  ```

Printing Text (cont.)

- A string literal cannot span multiple lines.
  - example: this is not allowed:
    ```java
    System.out.println("I want to print a string on two lines.");
    ```

- Instead, we can use two different statements:
  ```java
  System.out.println("I want to print a string");
  System.out.println("on two lines.");
  ```
println vs. print

• After printing a value, `System.out.println` "moves down" to the next line on the screen.

• If we don’t want to do this, we can use `System.out.print` instead:

```
System.out.print("<text>");
```

The next text to be printed will begin *just after* this text – on the same line.

• For example:

```
System.out.print("I ");
System.out.print("program ");
System.out.println("with class!");
```

is equivalent to

```
System.out.println("I program with class!");
```

---

Escape Sequences

• Problem: what if we want to print a string that includes double quotes?

  • example: `System.out.println("Jim said, "hi!" ");`
  • this won’t compile. why?

• Solution: precede the double quote character by a `\`

```
System.out.println("Jim said, \"hi!\" ");
```

• `\"` is an example of an *escape sequence*.

• The `\` tells the compiler to interpret the following character differently than it ordinarily would.

• Other examples:
  • `\n` a newline character (goes to the next line)
  • `\t` a tab
  • `\\` a backslash
Procedural Decomposition
(How to Use Methods to Write Better Programs)

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Example Program: Writing Block Letters

- Here’s a program that writes the name "DEE" in block letters:

```java
class BlockLetters {
    public static void main(String[] args) {
        System.out.println("    -----");
        System.out.println("     |   \\");
        System.out.println("     |    |");
        System.out.println("     |   /");
        System.out.println("    -----");
        System.out.println();
        System.out.println("    +-----");
        System.out.println("    |      ");
        System.out.println("    +----");
        System.out.println("    |      ");
        System.out.println("    +-----");
        System.out.println();
        System.out.println("    +-----");
        System.out.println("    |      ");
        System.out.println("    +----");
        System.out.println("    |      ");
        System.out.println("    +-----");
    }
}
```
Example Program: Writing Block Letters

• The output looks like this:

```
-----
|   \
|    |
|   /
-----

+-----
|      
+-----
|      
+-----
|      
+-----

+-----
|      
+-----
|      
+-----
```

Code Duplication

```java
public class BlockLetters {
    public static void main(String[] args) {
        System.out.println("-----");
        System.out.println(" | \\");
        System.out.println(" | |");
        System.out.println(" | /");
        System.out.println(" -----");

        System.out.println("+-----");
        System.out.println("| ");
        System.out.println("| ----");
        System.out.println("| ");
        System.out.println("+-----");

        System.out.println("+-----");
        System.out.println("| ");
        System.out.println("| ----");
        System.out.println("| ");
        System.out.println("+-----");
    }
}
```

• The code that writes an E appears twice – it is duplicated.
Code Duplication (cont.)

• Code duplication is undesirable. Why?

• Also, what if we wanted to create another word containing the letters D or E? What would we need to do?

• A better approach: create a command for writing each letter, and execute that command as needed.

• To create our own command in Java, we define a method.

Defining a Simple Static Method

• We've already seen how to define a main method:

```java
public static void main(String[] args) {
    <statement>;
    <statement>;
    ...
    <statement>;
}
```

• The simple methods that we'll define have a similar syntax:

```java
public static void <name>() {
    <statement>;
    <statement>;
    ...
    <statement>;
}
```

• This type of method is known as static method.
Defining a Simple Static Method (cont.)

• Here's a static method for writing a block letter E:

```java
public static void writeE() {
    System.out.println("  +-----");
    System.out.println("  | ");
    System.out.println("  +----");
    System.out.println("  | ");
    System.out.println("  +-----");
}
```

• It contains the same statements that we used to write an E in our earlier program.

• This method gives us a command for writing an E.

• To use it, we simply include the following statement:
  ```java
  writeE();
  ```

Calling a Method

• The statement
  ```java
  writeE();
  ```
  is known as a *method call*.

• General syntax for a static method call:
  ```java
  <method-name>()
  ```

• Calling a method causes the statements inside the method to be executed.
Using Methods to Eliminate Duplication

• Here’s a revised version of our program:

```java
public class BlockLetters2 {
    public static void writeE() {
        System.out.println("    +-----");
        System.out.println("    |"rieb);
        System.out.println("    +----");
        System.out.println("    |"rieb);
        System.out.println("    +-----");
    }

    public static void main(String[] args) {
        System.out.println("    -----");
        System.out.println("     |   ");
        System.out.println("     |    |");
        System.out.println("     |   /");
        System.out.println("    -----");
        System.out.println();
        writeE();
        System.out.println();
        writeE();
    }
}
```

Methods Can Be Defined In Any Order

• Here’s a version in which we put the `main` method first:

```java
public class BlockLetters2 {
    public static void main(String[] args) {
        System.out.println("    -----");
        System.out.println("     |   ");
        System.out.println("     |    |");
        System.out.println("     |   /");
        System.out.println("    -----");
        System.out.println();
        writeE();
        System.out.println();
        writeE();
    }

    public static void writeE() {
        System.out.println("    +-----");
        System.out.println("    |"rieb);
        System.out.println("    +----");
        System.out.println("    |"rieb);
        System.out.println("    +-----");
    }
}
```

• By convention, the `main` method should appear first or last.
Flow of Control

- A program's flow of control is the order in which its statements are executed.

- By default, the flow of control:
  - is sequential
  - begins with the first statement in the main method

Flow of Control (cont.)

- Example: consider the following program:
  ```java
  public class HelloWorldAgain {
      public static void main(String[] args) {
          System.out.println("hello");
          System.out.println("world");
          System.out.println();
      }
  }
  ```

- We can represent the flow of control using a flow chart:
Method Calls and Flow of Control

- When we call a method, the flow of control jumps to the method.
- After the method completes, the flow of control jumps back to the point where the method call was made.

```java
public class BlockLetters2 {
    public static void writeE() {
        System.out.println("    +-----");
        System.out.println("    |  ");
        System.out.println("    \-----");
        System.out.println("    |  ");
        System.out.println("    +-----");
    }
    public static void main(String[] args) {
        System.out.println("    -----"하였습니다);
        System.out.println("      |   \"주의함");
        System.out.println("      |    |  ");
        System.out.println("      |   /  ");
        System.out.println("    -----");  
        System.out.println();
        writeE();
        System.out.println();
        ...  

```

Method Calls and Flow of Control (cont.)

- Here's a portion of the flowchart for our program:

```
main method:

... System.out.println();
writeE(); System.out.println("   +-----");
System.out.println("   |  ");
System.out.println("   +-----");
... System.out.println();
writeE(); System.out.println("   +-----");
System.out.println("   |  ");
System.out.println("   +-----");
System.out.println();
writeE(); System.out.println();
...  

writeE method:

... System.out.println("   +-----");
System.out.println("   |  ");
System.out.println("   +-----");
... System.out.println();
writeE(); System.out.println("   +-----");
System.out.println("   |  ");
System.out.println("   +-----");
System.out.println();
writeE(); System.out.println();
...  
```
Another Use of a Static Method

```java
public class BlockLetters3 {
    public static void writeD() {
        System.out.println("    -----");
        System.out.println("     |   \\");
        System.out.println("     |   |");
        System.out.println("     |   /");
        System.out.println("    -----");
    }

    public static void writeE() {
        System.out.println("    +-----");
        System.out.println("    |");
        System.out.println("    +----");
        System.out.println("    |");
        System.out.println("    +-----");
    }

    public static void main(String[] args) {
        writeD();
        System.out.println();
        writeE();
        System.out.println();
        writeE();
    }
}
```

Another Use of a Static Method (cont.)

- The code in the `writeD` method is only used once, so it doesn't eliminate code duplication.

- However, using a separate static method still makes the overall program more readable.

- It helps to reveal the structure of the program.
Procedural Decomposition

- In general, methods allow us to *decompose* a problem into smaller subproblems that are easier to solve.
  - the resulting code is also easier to understand and maintain

- In our program, we've decomposed the task "write DEE" into two subtasks:
  - write D
  - write E (which we perform twice).

- We can use a *structure diagram* to show the decomposition:

```
write DEE
write D               write E
```

Procedural Decomposition (cont.)

- How could we use procedural decomposition in printing the following lyrics?

Dashing through the snow in a one-horse open sleigh,
O'er the fields we go, laughing all the way.
Bells on bobtail ring, making spirits bright.
What fun it is to ride and sing a sleighing song tonight!

Jingle bells, jingle bells, jingle all the way!
O what fun it is to ride in a one-horse open sleigh!
Jingle bells, jingle bells, jingle all the way!
O what fun it is to ride in a one-horse open sleigh!

A day or two ago, I thought I'd take a ride,
And soon Miss Fanny Bright was seated by my side.
The horse was lean and lank; misfortune seemed his lot;
We got into a drifted bank and then we got upsot.

Jingle bells, jingle bells, jingle all the way!
O what fun it is to ride in a one-horse open sleigh!
Jingle bells, jingle bells, jingle all the way!
O what fun it is to ride in a one-horse open sleigh!
Procedural Decomposition (cont.)

Dashing through the snow in a one-horse open sleigh,
O'er the fields we go, laughing all the way.
Bells on bobtail ring, making spirits bright.
What fun it is to ride and sing a sleighing song tonight!

Jingle bells, jingle bells, jingle all the way!
O what fun it is to ride in a one-horse open sleigh!
Jingle bells, jingle bells, jingle all the way!
O what fun it is to ride in a one-horse open sleigh!

A day or two ago, I thought I'd take a ride,
And soon Miss Fanny Bright was seated by my side.
The horse was lean and lank; misfortune seemed his lot;
We got into a drifted bank and then we got upset.

Code Reuse

• Once we have a set of methods, we can use them to solve other problems.

• Here's a program that writes the name "ED":

```java
public class BlockLetters4 {
    // these methods are the same as before
    public static void writeD() {
        ...
    }

    public static void writeE() {
        ...
    }

    public static void main(String[] args) {
        writeE();
        System.out.println();
        writeD();
    }
}
```
Tracing the Flow of Control

• What is the output of the following program?

```java
public class FlowControlTest {
    public static void methodA() {
        System.out.println("starting method A");
    }
    public static void methodB() {
        System.out.println("starting method B");
    }
    public static void methodC() {
        System.out.println("starting method C");
    }
    public static void main(String[] args) {
        methodC();
        methodA();
    }
}
```

Methods Calling Methods

• The definition of one method can include calls to other methods.

• We've seen this already in the main method:

```java
public static void main(String[] args) {
    writeE();
    System.out.println();
    writeD();
}
```

• We can also do this in other methods:

```java
public static void foo() {
    System.out.println("This is method foo.");
    bar();
}

public static void bar() {
    System.out.println("This is method bar.");
}
```
Methods Calling Methods (cont.)

- What is the output of the following program?

```java
public class FlowControlTest2 {
    public static void methodOne() {
        System.out.println("boo");
        methodThree();
    }
    public static void methodTwo() {
        System.out.println("hoo");
        methodOne();
    }
    public static void methodThree() {
        System.out.println("foo");
    }
    public static void main(String[] args) {
        methodOne();
        methodThree();
        methodTwo();
    }
}
```

Comments

- Comments are text that is ignored by the compiler.
- Used to make programs more readable
- Two types:
  1. line comments: begin with `//`
     - compiler ignores from `//` to the end of the line
     - examples:
       `// this is a comment`
       `System.out.println(); // so is this`
  2. block comments: begin with `/*` and end with `*/`
     - compiler ignores everything in between
     - typically used at the top of each source file
Comments (cont.)
/*
* DrawTriangle.java
* Dave Sullivan (dgs@cs.bu.edu)
* This program draws a triangle.
*/

block comments

public class DrawTriangle {
public static void main(String[] args) {
System.out.println("Here's my drawing:");

line comments
// Draw the triangle using characters.
System.out.println("
^");
System.out.println("
/ \\");
System.out.println(" /
\\");
System.out.println(" /
\\");
System.out.println(" -------");
}
}

Comments (cont.)
• Put comments:
• at the top of each file, naming the author and explaining
what the program does
• at the start of every method other than main,
describing its behavior
• inside methods, to explain complex pieces of code
(this will be more useful later in the course)
• We will deduct points for failing to include the correct comments
and other stylistic problems.

CSCI S-111

David G. Sullivan, Ph.D.

39


Primitive Data, Variables, and Expressions;
Simple Conditional Execution

Computer Science S-111
Harvard University
David G. Sullivan, Ph.D.

Overview of the Programming Process

Analysis/Specification
Design
Implementation
Testing/Debugging
Example Problem: Adding Up Your Change

- Let's say that we have a bunch of coins of various types, and we want to figure out how much money we have.
- Let's begin the process of developing a program that does this.

Step 1: Analysis and Specification

- *Analyze* the problem (making sure that you understand it), and *specify* the problem requirements clearly and unambiguously.
- Describe exactly *what* the program will do, without worrying about *how* it will do it.
Step 2: Design

• Determine the necessary algorithms (and possibly other aspects of the program) and sketch out a design for them.

• This is where we figure out how the program will solve the problem.

• Algorithms are often designed using pseudocode.
  • more informal than an actual programming language
  • allows us to avoid worrying about the syntax of the language
  • example for our change-adder problem:
    get the number of quarters
    get the number of dimes
    get the number of nickels
    get the number of pennies
    compute the total value of the coins
    output the total value

Step 3: Implementation

• Translate your design into the programming language.
  pseudocode \(\rightarrow\) code

• We need to learn more Java before we can do this!

• Here’s a portion or fragment of a Java program for computing the value of a particular collection of coins:

  quarters = 10;
dimes = 3;
nickels = 7;
pennies = 6;
cents = 25*quarters + 10*dimes + 5*nickels + pennies;
System.out.println("Your total in cents is:");
System.out.println(cents);

• In a moment, we’ll use this fragment to examine some of the fundamental building blocks of a Java program.
Step 4: Testing and Debugging

- A bug is an error in your program.

- Debugging involves finding and fixing the bugs.

  The first program bug! Found by Grace Murray Hopper at Harvard. (http://www.hopper.navy.mil/grace/grace.htm)

- Testing – trying the programs on a variety of inputs – helps us to find the bugs.

Overview of the Programming Process

Analysis/Specification

Design

Implementation

Testing/Debugging
Program Building Blocks: Literals

```java
quarters = 10;
dimes = 3;
nickels = 7;
pennies = 6;
cents = 25*quarters + 10*dimes + 5*nickels + pennies;
System.out.println("Your total in cents is: ");
System.out.println(cents);
```

- **Literals** specify a particular value.

- They include:
  - string literals: "Your total in cents is:"
    - are surrounded by double quotes
  - numeric literals: 25 3.1416
    - commas are not allowed!

Program Building Blocks: Variables

```java
quarters = 10;
dimes = 3;
nickels = 7;
pennies = 6;
cents = 25*quarters + 10*dimes + 5*nickels + pennies;
System.out.println("Your total in cents is: ");
System.out.println(cents);
```

- We’ve already seen that variables are named memory locations that are used to store a value:

  quarters 10

- Variable names must follow the rules for identifiers (see previous notes).
Program Building Blocks: Statements

```java
quarters = 10;
dimes = 3;
nickels = 7;
pennies = 6;

cents = 25*quarters + 10*dimes + 5*nickels + pennies;
System.out.println("Your total in cents is: ");
System.out.println(cents);
```

- In Java, a single-line statement typically ends with a semi-colon.
- Later, we will see examples of control statements that contain other statements, just as we did in Scratch.

Program Building Blocks: Expressions

```java
quarters = 10;
dimes = 3;
nickels = 7;
pennies = 6;

cents = 25*quarters + 10*dimes + 5*nickels + pennies;
System.out.println("Your total in cents is: ");
System.out.println(cents);
```

- *Expressions* are pieces of code that evaluate to a value.
- They include:
  - literals, which evaluate to themselves
  - variables, which evaluate to the value that they represent
  - combinations of literals, variables, and *operators*:
    \[ 25\times\text{quarters} + 10\times\text{dimes} + 5\times\text{nickels} + \text{pennies} \]
Program Building Blocks: Expressions (cont.)

- Numerical operators include:
  - `+` addition
  - `-` subtraction
  - `*` multiplication
  - `/` division
  - `%` modulus or mod: gives the remainder of a division
    example: `11 % 3` evaluates to `2`

- Operators are applied to operands:
  
  \[
  25 \times \text{quarters} + (2 \times \text{length}) + (2 \times \text{width})
  \]

  operands of the `*` operator
  operands of the `+` operator

Evaluating Expressions

- With expressions that involve more than one mathematical operator, the usual order of operations applies.
  - example:
    \[
    3 + 4 \times 3 / 2 - 7
    \]
    =
    =
    =
    =

- Use parentheses to:
  - force a different order of evaluation
    - example:
      \[
      \text{radius} = \text{circumference} / (2 \times \pi);
      \]
  - make the standard order of operations obvious!
Evaluating Expressions with Variables

- When an expression includes variables, they are first replaced with their current value.

- Example: recall our code fragment:

```java
quarters = 10;
dimes = 3;
nickels = 7;
pennies = 6;
cents = 25*quarters + 10*dimes + 5*nickels + pennies;
= 25* 10 + 10* 3 + 5* 7 + 6
= 250 + 30 + 35 + 6
= 315 + 6
= 321
```

println Statements Revisited

- Recall our earlier syntax for println statements:

```java
System.out.println("<text>");
```

- Here is a more complete version:

```java
System.out.println(<expression>);
```

- Examples:

```java
System.out.println(3.1416);
System.out.println(2 + 10 / 5);
System.out.println(cents); // a variable
System.out.println("cents"); // a string
```
println Statements Revisited (cont.)

- The expression is first evaluated, and then the value is printed.
  
  ```java
  System.out.println(2 + 10 / 5);
  System.out.println(4); // output: 4
  System.out.println(cents);
  System.out.println(321); // output: 321
  System.out.println("cents");
  System.out.println("cents"); // output: cents
  ```

- Note that the surrounding quotes are *not* displayed when a string is printed.

println Statements Revisited (cont.)

- Another example:
  
  ```java
  System.out.println(10*dimes + 5*nickels);
  System.out.println(10*3 + 5*7);
  System.out.println(65);
  ```
Expressions in DrJava

- If you enter an expression in the Interactions Pane, DrJava evaluates it and displays the result.
  - examples:
    ```java
    > "Hello world!"    // do not put a semi-colon!
    "Hello world!"
    > 5 + 10
    15
    > 10 * 4 + 5 * 2
    50
    > 10 * (4 + 5 * 2)
    140
    ```
  - Note: This type of thing does not work inside a program.
    ```java
    public static void main(String[] args) {
      5 + 10                      // not allowed!
      System.out.println(5 + 10); // do this instead
    }
    ```

Data Types

- A data type is a set of related data values.
  - examples:
    - integers
    - strings
    - characters
  - Every data type in Java has a name that we can use to identify it.
Commonly Used Data Types for Numbers

- **int**
  - used for integers
  - examples: 25, -2

- **double**
  - used for real numbers (ones with a fractional part)
  - examples: 3.1416, -15.2
  - used for *any* numeric literal with a decimal point, even if it's an integer:
    - 5.0
  - also used for *any* numeric literal written in scientific notation
    - 3e8, -1.60e-19
  - more generally:
    - \(<n> x 10^{<p>}\) is written \(<n>e<p>\)

Incorrect Change-Adder Program

```java
/*
 * ChangeAdder.java
 * Dave Sullivan (dgs@cs.bu.edu)
 * This program determines the value of some coins.
 */

public class ChangeAdder {
    public static void main(String[] args) {
        quarters = 10;
        dimes = 3;
        nickels = 7;
        pennies = 6;

        // compute and print the total value
        cents = 25*quarters + 10*dimes + 5*nickels + pennies;
        System.out.print("total in cents is: ");
        System.out.println(cents);
    }
}
```
Declaring a Variable

• Java requires that we specify the type of a variable before attempting to use it.

• This is called declaring the variable.
  • syntax:
    
    \[
    \text{<type> <name>;} \]
  
  • examples:
    
    int count;       // will hold an integer
    double area;     // will hold a real number

• A variable declaration can also include more than one variable of the same type:
  
    int quarters, dimes;

Assignment Statements

• Used to give a value to a variable.

• Syntax:
  
  \[
  \text{<variable> = <expression>;} \]

  = is known as the assignment operator.

• Examples:
  
    int quarters = 10;     // declaration plus assignment
    
    \text{// declaration first, assignment later}
    \text{int cents;}
    \text{cents = 25*quarters + 10*dimes + 5*nickels + pennies;}

    \text{// can also use to change the value of a variable}
    \text{quarters = 15;}
Corrected Change-Adder Program

/* ChangeAdder.java
 * Dave Sullivan (dgs@cs.bu.edu)
 * This program determines the value of some coins.
 */
public class ChangeAdder {
    public static void main(String[] args) {
        int quarters = 10;
        int dimes = 3;
        int nickels = 7;
        int pennies = 6;
        int cents;

        // compute and print the total value
        cents = 25*quarters + 10*dimes + 5*nickels + pennies;
        System.out.print("total in cents is: ");
        System.out.println(cents);
    }
}

Assignment Statements (cont.)

- Steps in executing an assignment statement:
  1) evaluate the expression on the right-hand side of the =
  2) assign the resulting value to the variable on the left-hand side of the =

- Examples:
  int quarters = 10;
  int quarters = 10;   // 10 evaluates to itself!
  int quartersValue = 25 * quarters;
  int quartersValue = 25 * 10;
  int quartersValue = 250;
Assignment Statements (cont.)

• An assignment statement does not create a permanent relationship between variables.

• Example from the DrJava Interactions Pane:
  > int x = 10;
  > int y = x + 2;
  > y
  12
  > x = 20;
  > y
  12

  • changing the value of x does not change the value of y!

• You can only change the value of a variable by assigning it a new value.

Assignment Statements (cont.)

• As the values of variables change, it can be helpful to picture what's happening in memory.

• Examples:

<table>
<thead>
<tr>
<th>int num1;</th>
<th>num1</th>
<th>num2</th>
</tr>
</thead>
<tbody>
<tr>
<td>int num2 = 120;</td>
<td>?</td>
<td>120</td>
</tr>
<tr>
<td>num1 = 50;</td>
<td>50</td>
<td>120</td>
</tr>
<tr>
<td>num1 = num2 * 2;</td>
<td>240</td>
<td>120</td>
</tr>
<tr>
<td>num2 = 60;</td>
<td>240</td>
<td>60</td>
</tr>
</tbody>
</table>

after the assignment at left, we get:

The value of num1 is unchanged!
Assignment Statements (cont.)

- A variable can appear on both sides of the assignment operator!

- Example (fill in the missing values):
  
  ```
  int sum = 13;
  int val = 30;
  
  sum = sum + val;
  
  val = val * 2;
  ```

Operators and Data Types

- Each data type has its own set of operators.
  - the `int` version of an operator produces an `int` result
  - the `double` version produces a `double` result
  - etc.

- Rules for numeric operators:
  - if the operands are both of type `int`, the `int` version of the operator is used.
    - examples: 15 + 30
      1 / 2
      25 * quarters
  - if at least one of the operands is of type `double`, the `double` version of the operator is used.
    - examples: 15.5 + 30.1
      1 / 2.0
      25.0 * quarters
Incorrect Extended Change-Adder Program

/*
  * ChangeAdder2.java
  * Dave Sullivan (dgs@cs.bu.edu)
  * This program determines the value of some coins.
  */

public class ChangeAdder2 {
    public static void main(String[] args) {
        int quarters = 10;
        int dimes = 3;
        int nickels = 7;
        int pennies = 6;
        int cents;

        // compute and print the total value
        cents = 25*quarters + 10*dimes + 5*nickels + pennies;
        System.out.print("total in cents is: ");
        System.out.println(cents);
        double dollars = cents / 100;
        System.out.print("total in dollars is: ");
        System.out.println(dollars);
    }
}

Two Types of Division

- The int version of the / operator performs integer division, which discards the fractional part of the result (i.e., everything after the decimal).
  - examples:
    > 5 / 3
    1
    > 11 / 5
    2

- The double version of the / operator performs floating-point division, which keeps the fractional part.
  - examples:
    > 5.0 / 3.0
    1.6666666666666667
    > 11 / 5.0
    2.2
How Can We Fix Our Program?

/*
 * ChangeAdder2.java
 * Dave Sullivan (dgs@cs.bu.edu)
 * This program determines the value of some coins.
 */

public class ChangeAdder2 {
   public static void main(String[] args) {
      int quarters = 10;
      int dimes = 3;
      int nickels = 7;
      int pennies = 6;
      int cents;

      // compute and print the total value
      cents = 25*quarters + 10*dimes + 5*nickels + pennies;
      System.out.print("total in cents is: ");
      System.out.println(cents);
      double dollars = cents / 100;
      System.out.print("total in dollars is: ");
      System.out.println(dollars);
   }
}

String Concatenation

• The meaning of the + operator depends on the types of the operands.

• When at least one of the operands is a string, the + operator performs string concatenation.
  • combines two or more strings into a single string
  • example:
    System.out.println(\"hello \" + \"world\");
    is equivalent to
    System.out.println(\"hello world\");
String Concatenation (cont.)

- If one operand is a string and the other is a number, the number is converted to a string and then concatenated.
  - example: instead of writing
    ```java
    System.out.print("total in cents: ");
    System.out.println(cents);
    ```
  - we can write
    ```java
    System.out.println("total in cents: " + cents);
    ```
  - Here's how the evaluation occurs:
    ```java
    int cents = 321;
    System.out.println("total in cents: " + cents);
                // "total in cents: " + 321
                // "total in cents: " + "321"
                // "total in cents: 321"
    ```

Change-Adder Using String Concatenation

```java
/*
 * ChangeAdder2.java
 * Dave Sullivan (dgs@cs.bu.edu)
 * This program determines the value of some coins.
 */

class ChangeAdder2 {
    public static void main(String[] args) {
        int quarters = 10;
        int dimes = 3;
        int nickels = 7;
        int pennies = 6;
        int cents;

        // compute and print the total value
        cents = 25*quarters + 10*dimes + 5*nickels + pennies;
        System.out.println("total in cents is: " + cents);
        double dollars = cents / 100.0;
        System.out.println("total in dollars is: " + dollars);
    }
}
```
An Incorrect Program for Computing a Grade

/*
 * ComputeGrade.java
 * Dave Sullivan (dgs@cs.bu.edu)
 * This program computes a grade as a percentage.
 */

public class ComputeGrade {
    public static void main(String[] args) {
        int pointsEarned = 13;
        int possiblePoints = 15;

        // compute and print the grade as a percentage
        double grade = pointsEarned / possiblePoints * 100;
        System.out.println("The grade is: " + grade);
    }
}

• What is the output?

Will This Fix Things?

/*
 * ComputeGrade.java
 * Dave Sullivan (dgs@cs.bu.edu)
 * This program computes a grade as a percentage.
 */

public class ComputeGrade {
    public static void main(String[] args) {
        int pointsEarned = 13;
        int possiblePoints = 15;

        // compute and print the grade as a percentage
        double grade = pointsEarned / possiblePoints * 100.0;
        System.out.println("The grade is: " + grade);
    }
}
Type Casts

• To compute the percentage, we need to tell Java to treat at least one of the operands as a `double`.

• We do so by performing a type cast:
  
  grade = (double)pointsEarned / possiblePoints * 100;
  
  or
  
  grade = pointsEarned / (double)possiblePoints * 100;
  
• General syntax for a type cast:
  
  (<type>)<variable>

Corrected Program for Computing a Grade

```
/*
 * ComputeGrade.java
 * Dave Sullivan (dgs@cs.bu.edu)
 * This program computes a grade as a percentage.
 */

public class ComputeGrade {
    public static void main(String[] args) {
        int pointsEarned = 13;
        int possiblePoints = 15;

        // compute and print the grade as a percentage double grade;
        grade = (double)pointsEarned / possiblePoints * 100;
        System.out.println("The grade is: " + grade);
    }
}
```
Evaluating a Type Cast

• Example of evaluating a type cast:
  
  ```java
  pointsEarned = 13;
  possiblePoints = 15;
  grade = (double)pointsEarned / possiblePoints * 100;
  (double)13 / 15 * 100;
  13.0 / 15 * 100;
  0.8666666666666667 * 100;
  86.66666666666667;
  ```

• Note that the type cast occurs after the variable is replaced by its value.

• It does not change the value that is actually stored in the variable.
  • in the example above, pointsEarned is still 13

Type Conversions

• Java will automatically convert values from one type to another provided there is no potential loss of information.

• Example: we can perform the following assignment without a type cast:
  ```java
  double d = 3;
  variable of type double value of type int
  ```

• the JVM will convert the integer value 3 to the floating-point value 3.0 and assign that value to d
• any int can be assigned to a double without losing any information
Type Conversions (cont.)

- The compiler will complain if the necessary type conversion could (at least in some cases) lead to a loss of information:
  
  ```java
  int i = 7.5;    // won't compile
  ```

  ![Variable of type int](variable.png)

  ![Value of type double](value.png)

- This is true regardless of the actual value being converted:
  
  ```java
  int i = 5.0;    // won't compile
  ```

- To make the compiler happy in such cases, we need to use a type cast:
  
  ```java
  double area = 5.7;
  int approximateArea = (int)area;
  System.out.println(approximateArea);
  ```

- what would the output be?

Type Conversions (cont.)

- When an automatic type conversion is performed as part of an assignment, the conversion happens after the evaluation of the expression to the right of the `=.`

- Example:
  
  ```java
  double d = 1 / 3;
  = 0;    // uses integer division. why?
  = 0.0;
  ```
A Block of Code

- A block of code is a set of statements that is treated as a single unit.
- In Java, a block is typically surrounded by curly braces.
- Examples:
  - each class is a block
  - each method is a block

```java
public class MyProgram {
    public static void main(String[] args) {
        int i = 5;
        System.out.println(i * 3);
        int j = 10;
        System.out.println(j / i);
    }
}
```

Variable Scope

- The scope of a variable is the portion of a program in which the variable can be used.
- By default, the scope of a variable in Java:
  - begins at the point at which it is declared
  - ends at the end of the innermost block that encloses the declaration

```java
public class MyProgram2 {
    public static void main(String[] args) {
        System.out.println("Welcome!");
        System.out.println("Let's do some math!");
        int j = 10;
        System.out.println(j / 5); // scope of j
    }
}
```
- Because of these rules, a variable cannot be used outside of the block in which it is declared.
Another Example

```java
public class MyProgram3 {
    public static void method1() {
        int i = 5;
        System.out.println(i * 3);
        int j = 10;
        System.out.println(j / i);
    }
    public static void main(String[] args) {
        // The following line won't compile.
        System.out.println(i + j);
        int i = 4;
        System.out.println(i * 6);
        method1();
    }
}
```

Local Variables vs. Global Variables

```java
public class MyProgram {
    static int x = 10;    // a global variable
    public static void method1() {
        int y = 5;        // a local variable
        System.out.println(x + y);
    }
    ...
}
```

• Variables that are declared inside a method are *local variables*.
  • they cannot be used outside that method.

• In theory, we can define *global variables* that are available throughout the program.
  • they are declared outside of any method,
    using the keyword static

• However, we generally avoid global variables.
  • can lead to problems in which one method accidentally affects the behavior of another method
Yet Another Change-Adder Program!

• Let’s change it to print the result in dollars and cents.
  • 321 cents should print as 3 dollars, 21 cents

```java
public class ChangeAdder3 {
    public static void main(String[] args) {
        int quarters = 10;
        int dimes = 3;
        int nickels = 7;
        int pennies = 6;
        int dollars, cents;
        cents = 25*quarters + 10*dimes + 5*nickels + pennies;
        // what should go here?
        System.out.println("dollars = " + dollars);
        System.out.println("cents = " + cents);
    }
}
```

The Need for Conditional Execution

• What if the user has 121 cents?
  • will print as 1 dollars, 21 cents
  • would like it to print as 1 dollar, 21 cents

• We need a means of choosing what to print at runtime.
Recall: Conditional Execution in Scratch

![Diagram of Scratch conditional execution blocks]

If the condition is true:
1. The statement(s) in the true block are executed
2. The statement(s) in the false block are skipped

If the condition is false:
1. The statement(s) in the false block are executed
2. The statement(s) in the true block are skipped

Conditional Execution in Java

```java
if (<condition>) {
    <true block>
} else {
    <false block>
}
```

- If the condition is true:
  1. The statement(s) in the true block are executed
  2. The statement(s) in the false block (if any) are skipped

- If the condition is false:
  1. The statement(s) in the false block (if any) are executed
  2. The statement(s) in the true block are skipped
Expressing Simple Conditions

• Java provides a set of operators called *relational operators* for expressing simple conditions:

<table>
<thead>
<tr>
<th>operator</th>
<th>name</th>
<th>examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;</td>
<td>less than</td>
<td>5 &lt; 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>num &lt; 0</td>
</tr>
<tr>
<td>&gt;</td>
<td>greater than</td>
<td>40 &gt; 60 (which is false!)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>count &gt; 10</td>
</tr>
<tr>
<td>&lt;=</td>
<td>less than or equal to</td>
<td>average &lt;= 85.8</td>
</tr>
<tr>
<td>&gt;=</td>
<td>greater than or equal to</td>
<td>temp &gt;= 32</td>
</tr>
<tr>
<td>==</td>
<td>equal to</td>
<td>sum == 10</td>
</tr>
<tr>
<td></td>
<td>(don't confuse with =)</td>
<td>firstChar == 'P'</td>
</tr>
<tr>
<td>!=</td>
<td>not equal to</td>
<td>age != myAge</td>
</tr>
</tbody>
</table>

Change Adder With Conditional Execution

```java
public class ChangeAdder3 {
    public static void main(String[] args) {
        ...
        System.out.print(dollars);
        if (dollars == 1) {
            System.out.print(" dollar, ");
        } else {
            System.out.print(" dollars, ");
        }
        // Add statements to correctly print cents.
        // Try to use only an if, not an else.
    }
}
```

```
Classifying Bugs

• Syntax errors
  • found by the compiler
  • occur when code doesn't follow the rules of the programming language
  • examples?

• Logic errors
  • the code compiles, but it doesn’t do what you intended it to do
  • may or may not cause the program to crash
    • called runtime errors if the program crashes
  • often harder to find!
Common Syntax Errors Involving Variables

- Failing to declare the type of the variable.

- Failing to initialize a variable before you use it:
  ```java
  int radius;
  double area = 3.1416 * radius * radius;
  ```

- Trying to declare a variable when there is already a variable with that same name in the current scope:
  ```java
  int val1 = 10;
  System.out.print(val1 * 2);
  int val1 = 20;
  ```

Will This Compile?

```java
public class ChangeAdder {
    public static void main(String[] args) {
        ...
        int cents;
        cents = 25*quarters + 10*dimes + 5*nickels + pennies;
        if (cents % 100 == 0) {
            int dollars = cents / 100;
            System.out.println(dollars + " dollars");
        } else {
            int dollars = cents / 100;
            cents = dollars % 100;
            System.out.println(dollars + " dollars and " + cents + " cents");
        }
    }
}
```
Representing Integers

• Like all values in a computer, integers are stored as binary numbers – sequences of bits (0s and 1s).

• With n bits, we can represent $2^n$ different values.
  • examples:
    • 2 bits give $2^2 = 4$ different values
      00, 01, 10, 11
    • 3 bits give $2^3 = 8$ different values
      000, 001, 010, 011, 100, 101, 110, 111

• When we allow for negative integers (which Java does) n bits can represent any integer from $-2^{n-1}$ to $2^{n-1} – 1$.
  • there's one fewer positive value to make room for 0

Java’s Integer Types

• Java’s actually has four primitive types for integers, all of which represent signed integers.

<table>
<thead>
<tr>
<th>type</th>
<th># of bits</th>
<th>range of values</th>
</tr>
</thead>
<tbody>
<tr>
<td>byte</td>
<td>8</td>
<td>$-2^7$ to $2^7 – 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($-128$ to 127)</td>
</tr>
<tr>
<td>short</td>
<td>16</td>
<td>$-2^{15}$ to $2^{15} – 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($-32768$ to 32767)</td>
</tr>
<tr>
<td>int</td>
<td>32</td>
<td>$-2^{31}$ to $2^{31} – 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(approx. +/-2 billion)</td>
</tr>
<tr>
<td>long</td>
<td>64</td>
<td>$-2^{63}$ to $2^{63} – 1$</td>
</tr>
</tbody>
</table>

• We typically use int, unless there's a good reason not to.
Java’s Floating-Point Types

- Java has two primitive types for floating-point numbers:

<table>
<thead>
<tr>
<th>type</th>
<th># of bits</th>
<th>approx. range</th>
<th>approx. precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>float</td>
<td>32</td>
<td>+/-10^{-45} to +/-10^{38}</td>
<td>7 decimal digits</td>
</tr>
<tr>
<td>double</td>
<td>64</td>
<td>+/-10^{-324} to +/-10^{308}</td>
<td>15 decimal digits</td>
</tr>
</tbody>
</table>

- We typically use double because of its greater precision.

Binary to Decimal

- Number the bits from right to left
  - example:
    - b7 b6 b5 b4 b3 b2 b1 b0
    - 0 1 0 1 1 1 0 1

- For each bit that is 1, add 2^n, where n = the bit number
  - example:
    - b7 b6 b5 b4 b3 b2 b1 b0
    - 0 1 0 1 1 1 0 1
    - decimal value = 2^6 + 2^4 + 2^3 + 2^2 + 2^0
    - 64 + 16 + 8 + 4 + 1 = 93

- another example: what is the integer represented by 01001011?
Decimal to Binary

• Go in the reverse direction: determine which powers of 2 need to be added together to produce the decimal number.
  • example: \(42 = 32 + 8 + 2\)
    \[= 2^5 + 2^3 + 2^1\]
  • thus, bits 5, 3, and 1 are all 1s: \(42 = 00101010\)

• Start with the largest power of 2 less than or equal to the number, and work down from there.
  • example: what is 21 in binary?
    \[16 \text{ is the largest power of 2 } \leq 21: \quad 21 = 16 + 5\]
    now, break the 5 into powers of 2:
    \[21 = 16 + 4 + 1\]
    1 is a power of 2 \((2^0)\), so we’re done:
    \[21 = 16 + 4 + 1 = 2^4 + 2^2 + 2^0 = 00010101\]

Decimal to Binary (cont.)

• Another example: what is 90 in binary?
printf: Formatted Output

• When printing a decimal number, you may want to limit yourself to a certain number of places after the decimal.

• You can do so using the `System.out.printf` method.
  • example:
    ```java
    System.out.printf("%.2f", 1.0/3);
    ```
    will print
    0.33
  • the number after the decimal point in the first parameter indicates how many places after the decimal should be used

• There are other types of formatting that can also be performed using this method.
  • see Table 4.6 in the textbook for more examples

Review

• Consider the following code fragments
  1) 1000
  2) 10 * 5
  3) `System.out.println("Hello");`
  4) `hello`
  5) `num1 = 5;`
  6) `2*width + 2*length`
  7) `main`

• Which of them are examples of:
  • literals?
  • identifiers?
  • expressions?
  • statements?
Definite Loops

Computer Science S-111
Harvard University
David G. Sullivan, Ph.D.

Using a Variable for Counting

• Let's say that we're using a variable \( i \) to count the number of times that something has been done:

\[
\text{int } i = 0; \quad i \quad 0
\]

• To increase the count, we can do this:

\[
i = i + 1;
\]

\[
0 + 1 \quad 1
\]

\[
i \quad 1
\]

• To increase the count again, we repeat the same assignment:

\[
i = i + 1;
\]

\[
1 + 1 \quad 2
\]

\[
i \quad 2
\]
Increment and Decrement Operators

- Instead of writing
  \[ i = i + 1; \]
  we can use a shortcut and just write
  \[ i++; \]
- \[ ++ \] is known as the increment operator.
  - increment = increase by 1
- Java also provides a decrement operator (\[ -- \]).
  - decrement = decrease by 1
  - example:
    \[ i--; \]

Review: Flow of Control

- Flow of control = the order in which instructions are executed
- By default, instructions are executed in sequential order.

\[
\begin{align*}
\text{instructions} & \quad \text{flowchart} \\
\text{int sum = 0;} & \quad \text{int sum = 0;} \\
\text{int num1 = 5;} & \quad \text{int num1 = 5;} \\
\text{int num2 = 10;} & \quad \text{int num2 = 10;} \\
\text{sum = num1 + num2;} & \quad \text{sum = num1 + num2;} \\
\end{align*}
\]

- When we make a method call, the flow of control "jumps" to the method, and it "jumps" back when the method completes.
Altering the Flow of Control: Repetition

• To solve many types of problems, we need to be able to modify the order in which instructions are executed.

• One reason for doing this is to allow for repetition.

• We saw this in Scratch:

Example of the Need for Repetition

• Here’s a method for writing a large block letter L:

```java
public static void writeL() {
    System.out.println("|");
    System.out.println("|");
    System.out.println("|");
    System.out.println("|");
    System.out.println("|");
    System.out.println("|");
    System.out.println("+----------");
}
```

• Rather than duplicating the statement
  ```java
  System.out.println("|");
  ```
  seven times, we’d like to have this statement appear just once and execute it seven times.
for Loops

• To repeat one or more statements multiple times, we can use a construct known as a for loop.

• Here’s a revised version of our writeL method that uses one:

```java
public static void writeL() {
    for (int i = 0; i < 7; i++) {
        System.out.println("|");
    }
    System.out.println("+----------");
}
```

for Loops

• Syntax:

```
for (<initialization>; <continuation test>; <update>) {
    <one or more statements>
}
```

• In our example:

```
for (int i = 0; i < 7; i++) {
    System.out.println("|");
}
```

• The statements inside the loop are known as the body of the loop.

• In our example, we use the variable i to count the number of times that the body has been executed.
Executing a for Loop

for (<initialization>; <continuation test>; <update>) {
   <body of the loop>
}

Notes:
- the initialization is only performed once
- the body is only executed if the test is true
- we repeatedly do: test body update until the test is false

Executing Our for Loop

for (int i = 0; i < 7; i++) {
   System.out.println("|");
}

<table>
<thead>
<tr>
<th>i</th>
<th>i &lt; 7</th>
<th>action</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>true</td>
<td>print 1st &quot;</td>
</tr>
<tr>
<td>1</td>
<td>true</td>
<td>print 2nd &quot;</td>
</tr>
<tr>
<td>2</td>
<td>true</td>
<td>print 3rd &quot;</td>
</tr>
<tr>
<td>3</td>
<td>true</td>
<td>print 4th &quot;</td>
</tr>
<tr>
<td>4</td>
<td>true</td>
<td>print 5th &quot;</td>
</tr>
<tr>
<td>5</td>
<td>true</td>
<td>print 6th &quot;</td>
</tr>
<tr>
<td>6</td>
<td>true</td>
<td>print 7th &quot;</td>
</tr>
<tr>
<td>7</td>
<td>false</td>
<td>execute stmt. after the loop</td>
</tr>
</tbody>
</table>
Definite Loops

- For now, we'll limit ourselves to definite loops – which repeat actions a fixed number of times.

- To repeat the body of a loop \(<N>\) times, we typically take one of the following approaches:
  
  ```java
  for (int i = 0; i < <N>; i++) {
      <body of the loop>
  }
  OR
  for (int i = 1; i <= <N>; i++) {
      <body of the loop>
  }
  ```

- Each time that the body of a loop is executed is known as an iteration of the loop.
  - the loops shown above perform \(<N>\) iterations

Other Examples of Definite Loops

- What does this loop do?
  ```java
  for (int i = 0; i < 3; i++) {
      System.out.println("Hip! Hip!");
      System.out.println("Hooray!");
  }
  ```

- What does this loop do?
  ```java
  for (int i = 0; i < 10; i++) {
      System.out.println(i);
  }
  ```
Using Different Initializations, Tests, and Updates

• The second loop from the previous page would be clearer if we expressed it like this:

```java
for (int i = 0; i <= 9; i++) {
    System.out.println(i);
}
```

• Different problems may require different initializations, continuation tests, and updates.

• What does this code fragment do?

```java
for (int i = 2; i <= 10; i = i + 2) {
    System.out.println(i * 10);
}
```

Tracing a for Loop

• Let's trace through the final code fragment from the last slide:

```java
for (int i = 2; i <= 10; i = i + 2) {
    System.out.println(i * 10);
}
```

<table>
<thead>
<tr>
<th>i</th>
<th>i &lt;= 10</th>
<th>value printed</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>F</td>
<td></td>
</tr>
</tbody>
</table>
Common Mistake

- You should not put a semi-colon after the for-loop header:
  
  ```java
  for (int i = 0; i < 7; i++) { 
      System.out.println("|");
  }
  ```

- The semi-colon ends the for statement.
  - thus, it doesn't repeat anything!

- The println is independent of the for statement, and only executes once.

Practice

- Fill in the blanks below to print the integers from 1 to 10:
  ```java
  for (____________; ____________; ____________) {
      System.out.println(i);
  }
  ```

- Fill in the blanks below to print the integers from 10 to 20:
  ```java
  for (____________; ____________; ____________) {
      System.out.println(i);
  }
  ```

- Fill in the blanks below to print the integers from 10 down to 1:
  ```java
  for (____________; ____________; ____________) {
      System.out.println(i);
  }
  ```
Other Java Shortcuts

- Recall this code fragment:
  ```java
  for (int i = 2; i <= 10; i = i + 2) {
    System.out.println(i * 10);
  }
  ```

- Instead of writing
  ```java
  i = i + 2;
  ```
  we can use a shortcut and just write
  ```java
  i += 2;
  ```

- In general
  ```java
  <variable> += <expression>;
  ```
  is equivalent to
  ```java
  <variable> = <variable> + (<expression>);
  ```

Java Shortcuts

- Java offers other shortcut operators as well.

- Here's a summary of all of them:
  ```
  shortcut     equivalent to
  ---------     --------------------
  <var> ++;    <var> = <var> + 1;
  <var> --;    <var> = <var> - 1;
  <var> += <expr>;    <var> = <var> + (<expr>);
  <var> -= <expr>;    <var> = <var> - (<expr>);
  <var> *= <expr>;    <var> = <var> * (<expr>);
  <var> /= <expr>;    <var> = <var> / (<expr>);
  <var> %= <expr>;    <var> = <var> % (<expr>);
  ```

- Important: the = must come after the mathematical operator.
  ```
  += is correct
  //= is not!
  ```
More Practice

• Fill in the blanks below to print the even integers in reverse order from 20 down to 6:

```java
for (____________; __________; __________) {
    System.out.println(i);
}
```

Find the Error

• Let's say that we want to print the numbers from 1 to n.

• Where is the error in the following code?

```java
for (int i = 1; i < n; i++) {
    System.out.println(i);
}
```

• This is an example of an off-by-one error. Beware of these when writing your loop conditions!
Example Problem: Printing a Pattern, version 1

• Ask the user for a positive integer (call it n), and print a pattern containing n asterisks.
  • example:
    Enter a positive integer: 3
    ***
  
• Let's use a for loop to do this:
  // code to read n goes here...
  for ( ) {
    System.out.print("*" acompanents);
  }
  System.out.println();

Example Problem: Printing a Pattern, version 2

• Print a pattern containing n lines of n asterisks.
  • example:
    Enter a positive integer: 3
    ***
    ***
    ***
  
• One way to do this is to use a nested loop – one loop inside another:
  // code to read in n goes here...
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      System.out.print("*" acompanents);
    }
    System.out.println();
  }
  
• This makes it easier to create a similar box of a different size.
Nested Loops

• When you have a nested loop, the inner loop is executed to completion for every iteration of the outer loop.

• Recall our Scratch drawing program:

![Scratch drawing program]

• How many times is the move statement executed?

Nested Loops (cont.)

• How many times is the println statement executed below?

```java
for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 7; j++) {
        System.out.println(i + " " + j);
    }
}
```

• How many times is the println statement executed below?

```java
for (int i = 0; i < 5; i++) {
    for (int j = 0; j < i; j++) {
        System.out.println(i + " " + j);
    }
}
```
Tracing a Nested for Loop

```
for (int i = 0; i < 5; i++) {
    for (int j = 0; j < i; j++) {
        System.out.println(i + " " + j);
    }
}
```

Value printed:
- 

Recall: Variable Scope

- The scope of a variable is the portion of a program in which the variable can be used.

- By default, the scope of a variable in Java:
  - begins at the point at which it is declared
  - ends at the end of the innermost block that encloses the declaration

```
public class MyProgram2 {
    public static void main(String[] args) {
        System.out.println("Welcome!");
        System.out.println("Let's do some math!");
        int j = 10;
        System.out.println(j / 5);  // scope of j
    }
}
```
Special Case: for Loops and Variable Scope

- When a variable is declared in the initialization clause of a for loop, its scope is limited to the loop.

- Example:

```java
public static void myMethod() {
    for (int i = 0; i < 5; i++) {
        int j = i * 3;
        System.out.println(j);
    }
    // the following line won't compile
    System.out.print(i);
    System.out.println(" values were printed.");
}
```

Special Case: for Loops and Variable Scope (cont.)

- To allow i to be used outside the loop, we need to declare it outside the loop:

- Example:

```java
public static void myMethod() {
    int i;
    for (i = 0; i < 5; i++) {
        int j = i * 3;
        System.out.println(j);
    }
    // now this will compile
    System.out.print(i);
    System.out.println(" values were printed.");
}
```
Special Case: for Loops and Variable Scope (cont.)

- Limiting the scope of a loop variable allows us to use the standard loop templates multiple times in the same method.

- Example:

```java
public static void myMethod() {
    for (int i = 0; i < 5; i++) {
        int j = i * 3;
        System.out.println(j);
    }

    for (int i = 0; i < 7; i++) {
        System.out.println("Go Crimson!");
    }
}
```

Review: Simple Repetition Loops

- Recall our two templates for performing `<N>` repetitions:

```java
for (int i = 0; i < `<N>`; i++) {
    // code to be repeated
}
```

```java
for (int i = 1; i <= `<N>`; i++) {
    // code to be repeated
}
```

- How many repetitions will each of the following perform?

```java
for (int i = 1; i <= 15; i++) {
    System.out.println("Hello");
    System.out.println("How are you?");
}
for (int i = 0; i < 2*j; i++) {
    ...
}
More Practice: Tracing a Nested for Loop

```java
for (int i = 1; i <= 3; i++) {
    for (int j = 0; j < 2*i + 1; j++) {
        System.out.print("*");
    }
    System.out.println();
}
```

output

Case Study: Drawing a Complex Figure

- Here's the figure:

```
()  
((()))
(((()))
========
|::::::|
|::::|
|::|
|::|
|::|
+=+=
```

- To begin with, we'll focus on creating this exact figure.
- Then we'll modify our code so that the size of the figure can easily be changed.
  - we'll use for loops to allow for this
Problem Decomposition

• We begin by breaking the problem into subproblems, looking for groups of lines that follow the same pattern:

\[
\begin{align*}
() & \quad \leftarrow \text{flame} \\
(() & \quad \leftarrow \text{rim of torch} \\
((()) & \quad \leftarrow \text{top of torch} \\
((())) & \quad \leftarrow \text{handle of torch} \\
((())()) & \quad \leftarrow \text{bottom of torch}
\end{align*}
\]

Problem Decomposition (cont.)

• This gives us the following initial pseudocode:

\[
\begin{align*}
() & \quad \text{draw the flame} \\
(() & \quad \text{draw the rim of the torch} \\
((()) & \quad \text{draw the top of the torch} \\
((())) & \quad \text{draw the handle of the torch} \\
((())()) & \quad \text{draw the bottom of the torch}
\end{align*}
\]

• This is a high-level description of what needs to be done.

• We'll gradually expand the pseudocode into more and more detailed instructions – until we're able to implement them in Java.
Drawing the Flame

• Let's begin by refining our specification for drawing the flame.

• Here's our initial pseudocode for this task:

```plaintext
for (each of 4 lines) {
    print some spaces (possibly 0)
    print some left parentheses
    print some right parentheses
    go to a new line
}
```

• We need formulas for how many spaces and parens should be printed on a given line.

Finding the Formulas

• To begin with, we:
  • number the lines in the flame
  • form a table of the number of spaces and parentheses on each line:

<table>
<thead>
<tr>
<th>line</th>
<th>spaces</th>
<th>parens (each type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

• Then we find the formulas.
  • assume the formulas are linear functions of the line number:
    \[ c_1 \times \text{line} + c_2 \]
    where \( c_1 \) and \( c_2 \) are constants
  • parens = ?
  • spaces = ?
Refining the Pseudocode

• Given these formulas, we can refine our pseudocode:

```plaintext
for (each of 4 lines) {
    print some spaces (possibly 0)
    print some left parentheses
    print some right parentheses
    go to a new line
}
```

```plaintext
for (line going from 1 to 4) {
    print 4 – line spaces
    print line left parentheses
    print line right parentheses
    go to a new line
}
```

Implementing the Pseudocode in Java

• We use nested for loops:

```java
for (line going from 1 to 4) {
    print 4 – line spaces
    print line left parentheses
    print line right parentheses
    go to a new line
}
```

```java
for (int line = 1; line <= 4; line++) {
    for (int i = 0; i < 4 – line; i++) {
        System.out.print(" ");
    }
    for (int i = 0; i < line; i++) {
        System.out.print("(");
    }
    for (int i = 0; i < line; i++) {
        System.out.print(")");
    }
    System.out.println();
}
```
A Method for Drawing the Flame

• We put the code in its own static method, and add some explanatory comments:

```java
public static void drawFlame() {
    for (int line = 1; line <= 4; line++) {
        // spaces to the left of the current line
        for (int i = 0; i < 4 - line; i++) {
            System.out.print(" ");
        }
        // left and right parens on the current line
        for (int i = 0; i < line; i++) {
            System.out.print("(");
        }
        for (int i = 0; i < line; i++) {
            System.out.print(")");
        }
        System.out.println();
    }
}
```

Drawing the Top of the Torch

• What's the initial pseudocode for this task?

```plaintext
for (each of 2 lines) {
    1|::::::|
    2 |::::|
}
```

• Here's a table for the number of spaces and number of colons:

<table>
<thead>
<tr>
<th>line</th>
<th>spaces</th>
<th>colons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

• spaces = ?
• colons decreases by 2 as line increases by 1
  ➔ colons = -2*line + c2 for some number c2
• try different values, and eventually get: colons = ?
Refining the Pseudocode

• Once again, we use the formulas to refine our pseudocode:

```
for (each of 2 lines) {
    print some spaces (possibly 0)
    print a single vertical bar
    print some colons
    print a single vertical bar
    go to a new line
}
```

```
for (line going from 1 to 2) {
    print line - 1 spaces
    print a single vertical bar
    print \(-2*line + 8\) colons
    print a single vertical bar
    go to a new line
}
```

---

A Method for Drawing the Top of the Torch

```
public static void drawTop() {
    for (int line = 1; line <= 2; line++) {
        // spaces to the left of the current line
        for (int i = 0; i < line - 1; i++) {
            System.out.print(" ");
        }
        // bars and colons on the current line
        System.out.print("|");
        for (int i = 0; i < \(-2*line + 8\); i++) {
            System.out.print(":");
        }
        System.out.print("|");
        System.out.println();
    }
}
```
Drawing the Rim

- This always has only one line, so we don't need nested loops.

- However, we still need a single loop, because we want to be able to scale the size of the figure.

- What should the code look like?

```java
for (            ;             ;            ) {

}
```

- This code also goes in its own method, called `drawRim()`

Incremental Development

- We take similar steps to implement methods for the remaining subtasks.

- After completing a given method, we test and debug it.

- The `main` method just calls the methods for the subtasks:

```java
public static void main(String[] args) {
    drawFlame();
    drawRim();
    drawTop();
    drawHandle();
    drawBottom();
}
```

- See the example program `DrawTorch.java`
Using Class Constants

- To make the torch larger or smaller, we'd need to make many changes.
  - the size of the figure is hard-coded into most methods

- To make the program more flexible, we can store info. about the figure's dimensions in one or more class constants.
  - like variables, but their values are fixed
  - can be used throughout the program

Using Class Constants (cont.)

- We only need one constant for the torch.
  - for the default size, it equals 2
  - its connection to some of the dimensions is shown at right

- We declare it at the very start of the class:
  ```java
  public class DrawTorch2 {
    public static final int SCALE_FACTOR = 2;
    ...
  }
  ```

- General syntax:
  ```java
  public static final <type> <name> = <expression>;
  ```

  - conventions:
    - capitalize all letters in the name
    - put an underscore ('_') between multiple words
Scaling the Figure

- Here are some other versions of the figure:

```
()  ()
((()))
((()))
|::|
|:
|:
|:
|:
|:
|:
+====+

SCALE_FACTOR = 3
```

```
 SCALE_FACTOR = 1
```

Revised Method for Drawing the Flame

- We replace the two 4s with 2*SCALE_FACTOR:

```java
public static void drawFlame() {
    for (int line = 1; line <= 2*SCALE_FACTOR; line++) {
        // spaces to the left of the flame
        for (int i = 0; i < 2*SCALE_FACTOR - line; i++) {
            System.out.print(" ");
        }
        // the flame itself, both left and right halves
        for (int i = 0; i < line; i++) {
            System.out.print("(");
        }
        for (int i = 0; i < line; i++) {
            System.out.print(")");
        }
        System.out.println();
    }
}
```
Making the Rim Scaleable

• How does the width of the rim depend on SCALE_FACTOR?

  ()   ()   ()
(()()) (()()) (()())
(())())  (())())  (())())  ====
((()))  (((()))) ====
((((())))((((()))))  ===========

  
• Use a table!

<table>
<thead>
<tr>
<th>SCALE_FACTOR</th>
<th>width of rim</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

width of rim = ?

Revised Method for Drawing the Rim

• Original version (for the default size):

```java
public static void drawRim() {
    for (int i = 0; i < 8; i++) {
        System.out.print("=");
    }
    System.out.println();
}
```

• Scaleable version:

```java
public static void drawRim() {
    for (int i = 0; i < 4*SCALE_FACTOR; i++) {
        System.out.print("=");
    }
    System.out.println();
}
```
Making the Top of the Torch Scaleable

• For SCALE_FACTOR = 2, we got:
  
  number of lines = 2  
  spaces = line – 1  
  colons = -2 * line + 8
  
  • What about SCALE_FACTOR = 3?

<table>
<thead>
<tr>
<th>line</th>
<th>spaces</th>
<th>colons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>
  
  number of lines = 3
  spaces = ?
  colons = ?

• in general, number of lines = ?

Making the Top of the Torch Scaleable (cont.)

• Compare the two sets of formulas:

  | SCALE_FACTOR = 2 | SCALE_FACTOR = 3 |
  | spaces = line – 1 | spaces = line – 1 |
  | colons = -2 * line + 8 | colons = -2 * line + 12 |

• There’s no change in:
  • the formula for spaces
  • the first constant in the formula for colons

• Use a table for the second constant:

<table>
<thead>
<tr>
<th>SCALE_FACTOR</th>
<th>constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
  constant = ?

• Scaleable formulas:
  spaces = line – 1
  colons = ?
Revised Method for Drawing the Top of the Torch

```java
public static void drawTop() {
    for (int line = 1; line <= SCALE_FACTOR; line++) {
        // spaces to the left of the current line
        for (int i = 0; i < line - 1; i++) {
            System.out.print(" ");
        }

        // bars and colons on the current line
        System.out.print("|");
        for (int i = 0; i < -2*line + 4*SCALE_FACTOR; i++) {
            System.out.print(":");
        }
        System.out.print("|");
        System.out.println();
    }
}
```

Practice: The Torch Handle

- Pseudocode for default size:

```
(()
(()))
((()))
(((())))
========
|::::::|
|::::::|
1 |:::|
2 |:::|
3 |:::|
4 |:::|
```+==+

- Java code for default size:

```java
public static void drawHandle() {
}
```
Practice: Making the Handle Scaleable

• We again compare two different sizes.

<table>
<thead>
<tr>
<th>SCALE_FACTOR</th>
<th># lines</th>
<th>spaces</th>
<th>colons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

• number of lines = ?
spaces = ?
colons = ?

Revised Method for Drawing the Handle

• What changes do we need to make?

```java
public static void drawHandle() {
    for (int line = 1; line <= 4; line++) {
        for (int i = 0; i < 2; i++) {
            System.out.print(" ");
        }
        System.out.print("|");
        for (int i = 0; i < 2; i++) {
            System.out.print(":");
        }
        System.out.println("|");
    }
}
```
Extra Practice: Printing a Pattern, version 3

• Print a triangular pattern with lines containing n, n – 1, ..., 1 asterisks.

  • example:
    
    Enter a positive integer: 3
    
    ***
    **
    *

• How would we use a nested loop to do this?

  for ( ) {
    for ( ) {
      System.out.print("*");
    }
    System.out.println();
  }
Review: Static Methods

- We've seen how we can use static methods to:
  1. capture the structure of a program – breaking a task into subtasks
  2. eliminate code duplication
- Thus far, our methods have been limited in their ability to accomplish these tasks.
A Limitation of Simple Static Methods

• For example, in our DrawTorch program, there are several for loops that each print a series of spaces, such as:
  
  ```java
  for (int i = 0; i < 4 - line; i++) {
    System.out.print(" ");
  }
  
  for (int i = 0; i < line - 1; i++) {
    System.out.print(" ");
  }
  ```

• However, despite the fact that all of these loops print spaces, we can't replace them with a method that looks like this:
  
  ```java
  public static void printSpaces() {
    ...
  }
  ```

  Why not?

Parameters

• In order for a method that prints spaces to be useful, we need one that can print an arbitrary number of spaces.

• Such a method would allow us to write commands like these:
  
  ```java
  printSpaces(5);
  printSpaces(4 - line);
  ```

  where the number of spaces to be printed is specified between the parentheses.

• To do so, we write a method that has a parameter:
  
  ```java
  public static void printSpaces(int numSpaces) {
    for (int i = 0; i < numSpaces; i++) {
      System.out.print(" ");
    }
  }
  ```
Parameters (cont.)

- A parameter is a special type of variable that allows us to pass information into a method.

- Consider again this method:

```java
public static void printSpaces(int numSpaces) {
    for (int i = 0; i < numSpaces; i++) {
        System.out.print(" ");
    }
}
```

- When we execute a method call like

```java
printSpaces(10);
```

the expression specified between the parentheses:

- is evaluated
- is assigned to the parameter
- can thereby be used by the code inside the method

Parameters (cont.)

```java
public static void printSpaces(int numSpaces) {
    for (int i = 0; i < numSpaces; i++) {
        System.out.print(" ");
    }
}
```

- Here's an example with a more complicated expression:

```java
int line = 2;
printSpaces(4 - line);
```

```java
4 - 2
2
```
A Note on Terminology

- The term parameter is used for both:
  - the variable specified in the method header
    - known as a formal parameter
  - the value that you specify when you make the method call
    - known as an actual parameter
    - also known as an argument

```java
public static void printSpaces(int numSpaces) {
    for (int i = 0; i < numSpaces; i++) {
        System.out.print(" ");
    }
}

printSpaces(10);
```

Parameters and Generalization

- Parameters allow us to generalize a task.

- They allow us to write one method that can perform a family of related tasks – instead of writing a separate method for each separate task.

  print5Spaces()
  print10Spaces()
  print20Spaces()
  print100Spaces()
  ...

  printSpaces(parameter)
Representing Individual Characters

- So far we've learned about two data types:
  - int
  - double

- The char type is used to represent individual characters.

- To specify a char literal, we surround the character by single quotes:
  - examples: 'a' 'Z' '0' '7' '?' '\\'
  - can only represent single characters
  - don't use double-quotes!
    "a" is a string, not a character

Methods with Multiple Parameters

- Here's a method with more than one parameter:
  ```java
  public static void printChars(char ch, int num) {
      for (int i = 0; i < num; i++) {
          System.out.print(ch);
      }
  }
  ```

- Example of calling this method:
  ```java
  printChars(' ', 10);
  ```

- Notes:
  - the parameters (both formal and actual) are separated by commas
  - each formal parameter must be preceded by its type
  - the actual parameters are evaluated and assigned to the corresponding formal parameters
Example of Using a Method with Parameters

```java
public static void drawFlame() {
    for (int line = 1; line <= 4; line++) {
        for (int i = 0; i < 4 - line; i++) {
            System.out.print(" ");
        }
        for (int i = 0; i < line; i++) {
            System.out.print("(");
        }
        for (int i = 0; i < line; i++) {
            System.out.print(")");
        }
        System.out.println();
    }
}
```

```java
public static void drawFlame() {
    for (int line = 1; line <= 4; line++) {
        printChars(' ', 4 - line);
        printChars('(', line);
        printChars(')', line);
        System.out.println();
    }
}
```

replace nested loops with method calls

Recall: Variable Scope

- The scope of a variable is the portion of a program in which the variable can be used.
- By default, the scope of a variable in Java:
  - begins at the point at which it is declared
  - ends at the end of the innermost block that encloses the declaration

```java
public static void printResults(int a, int b) {
    System.out.println("Here are the stats:");
    int sum = a + b;
    System.out.print("sum = ");
    System.out.println(sum);
    double avg = (a + b) / 2.0;
    System.out.print("average = ");
    System.out.println(avg);
}
```
Special Case: Parameters and Variable Scope

- What about the parameters of a method?
  - they do not follow the default scope rules!
  - their scope is limited to their method

```java
public class MyClass {
    public static void printResults(int a, int b) {
        System.out.println("Here are the stats:");
        int sum = a + b;
        System.out.print("sum = ");
        System.out.println(sum);
        double avg = (a + b) / 2.0;
        System.out.print("average = ");
        System.out.println(avg);
    }

    static int c = a + b;  // does not compile!
}
```

Practice with Scope

```java
public static void drawRectangle(int height) {
    for (int i = 0; i < height; i++) {
        // which variables could be used here?
        int width = height * 2;
        for (int j = 0; j < width; j++) {
            System.out.print("*");
            // what about here?
        }
        // what about here?
        System.out.println();
    }
    // what about here?
}

public static void repeatMessage(int numTimes) {
    // what about here?
    for (int i = 0; i < numTimes; i++) {
        System.out.println("What is your scope?");
    }
}
```
Practice with Parameters

```java
public static void printValues(int a, int b) {
    System.out.println(a + " " + b);
    b = 2 * a;
    System.out.println("b" + b);
}

public static void main(String[] args) {
    int a = 2;
    int b = 3;
    printValues(b, a);
    printValues(7, b * 3);
    System.out.println(a + " " + b);
}

• What's the output?
```

A Limitation of Parameters

• Parameters allow us to pass values into a method.

• They don't allow us to get a value out of a method.
A Limitation of Parameters (cont.)

- Example: using a method to compute the opposite of a number

  - This won't work:
    ```java
    public static void opposite(int number) {
        number = number * -1;
    }
    public static void main(String[] args) {
        // read in points from the user
        opposite(points);
        ...
    }
    ```

  - the opposite method changes the value of number, but number can't be used outside of that method
  - the method doesn't change the value of points

Methods That Return a Value

- To compute the opposite of a number, we need a method that's able to return a value.

  - Such a method would allow us to write statements like this:
    ```java
    int penalty = opposite(points);
    ```

  - The value returned by the method would replace the method call in the original statement.

  - Example:
    ```java
    int points = 10;
    int penalty = opposite(points);
    
    int penalty = -10;  // after the method completes
    ```
Defining a Method that Returns a Value

• Here's a method that computes and returns the opposite of a number:

```java
public static int opposite(int number) {
    return number * -1;
}
```

• In the header of the method, `void` is replaced by `int`, which is the type of the returned value.

• The returned value is specified using a `return` statement.

Syntax:

```
return <expression>;
```

• `<expression>` is evaluated
• the resulting value replaces the method call in
  the statement that called the method

Defining a Method that Returns a Value (cont.)

• The complete syntax for the header of a static method is:

```
public static <return type> <name>(<type1> <param1>,
    <type2> <param2>, ...)
```

• Note: a method call is a type of expression!
  • it evaluates to its return value
    ```java
    int opp = opposite(10);
    int opp = -10;
    ```

• In our earlier methods, the return type was always `void`:

```
public static void printSpaces(int numSpaces) {
    ...
```

This is a special return type that indicates that no value is returned.
Flow of Control with Methods That Return a Value

- The flow of control jumps to a method until it returns.
- The flow jumps back, and the returned value replaces the call.

Example:
```java
int num = 10;
int opp = opposite(num);
System.out.println(opp);
```

Flow of Control with Methods That Return a Value

- The flow of control jumps to a method until it returns.
- The flow jumps back, and the returned value replaces the call.

Example:
```java
int num = 10;
int opp = opposite(num);
System.out.println(opp);
```

Example (corrected):  
```java
int num = 10;
int opp = opposite(num);
System.out.println(opp);
```
Returning vs. Printing

• Instead of returning a value, we could write a method that prints the value:

```java
public static void printOpposite(int number) {
    System.out.println(number * -1);
}
```

• However, a method that returns a value is typically more useful.

• With such a method, you can still print the value by printing what the method returns:

```java
System.out.println(opposite(num));
```

• the return value replaces the method call and is printed

• In addition, you can do other things besides printing:

```java
int penalty = opposite(num);
```

Practice: Computing the Volume of a Cone

• volume of a cone = base * height

  \[
  \frac{3}{3}
  \]

• Let's write a method named `coneVol` for computing it.

  • parameters and their types?
  • return type?

  • method definition:

```java
public static _______ coneVol(___________________________) {

}
```
The Math Class

- Java's built-in Math class contains static methods for mathematical operations.

- These methods return the result of applying the operation to the parameters.

- Examples:
  - `round(double value)` – returns the result of rounding value to the nearest integer
  - `abs(double value)` – returns the absolute value of value
  - `pow(double base, double expon)` – returns the result of raising base to the expon power
  - `sqrt(double value)` – returns the square root of value

- Table 3.2 in the textbook includes other examples.

The Math Class (cont.)

- To use a static method defined in another class, we need to use the name of the class when we call it.

- We use what's known as dot notation.

- Syntax:
  ```
  <class name>.<method name>(<param1>, <param2>, ...)
  ```

- Example:
  ```java
  double maxVal = Math.pow(2, numBits - 1) - 1;
  ```
*** Common Mistake ***

- Consider this alternative opposite method:

  ```java
  public static int opposite(int number) {
      number = number * -1;
      return number;
  }
  ```

- What's wrong with the following code that uses it?

  ```java
  public class OppositeFinder {
      public static void main(String[] args) {
          int number = 10;
          opposite(number);
          System.out.print("opposite = ");
          System.out.println(number);
      }
  }
  ```

Keeping Track of Variables

- Consider again the alternative opposite method:

  ```java
  public static int opposite(int number) {
      number = number * -1;
      return number;
  }
  ```

- Here's some code that uses it correctly:

  ```java
  public class OppositeFinder {
      public static void main(String[] args) {
          int number = 10;
          int otherNumber = opposite(number);
          ...
      }
  }
  ```

- There are two different variables named number. How does the runtime system distinguish between them?

- More generally, how does it keep track of variables?
Keeping Track of Variables (cont.)

• When you make a method call, the Java runtime sets aside a block of memory known as the *frame* of that method call.

  
  ![Diagram](image.png)

  *note: we're ignoring main's parameter for now*

  main
  number  otherNumber

• The frame is used to store:
  • the formal parameters of the method
  • any local variables – variables declared within the method

• A given frame can only be accessed by statements that are part of the corresponding method call.

---

Keeping Track of Variables (cont.)

• When a method (method1) calls another method (method2), the frame of method1 is set aside temporarily.

  • method1's frame is "covered up" by the frame of method2
  
  • example: after main calls opposite, we get:

    ![Diagram](image.png)

    main
    opposite
    number

• When the runtime system encounters a variable, it uses the one from the current frame (the one on top).

• When a method returns, its frame is removed, which "uncovers" the frame of the method that called it.
public class OppositeFinder {
    public static void main(String[] args) {
        int number = 10;
        int otherNumber = opposite(number);
        System.out.print("opposite = ");
        System.out.println(otherNumber);
    }
    public static int opposite(int number) {
        number = number * -1;
        return number;
    }
}

Example: Tracing Through a Program

public class OppositeFinder {
    public static void main(String[] args) {
        int number = 10;
        int otherNumber = opposite(number);
        System.out.print("opposite = ");
        System.out.println(otherNumber);
    }
    public static int opposite(int number) {
        number = number * -1;
        return number;
    }
}
Example: Tracing Through a Program

```java
public class OppositeFinder {
    public static void main(String[] args) {
        int number = 10;
        int otherNumber = opposite(number);
        System.out.print("opposite = ");
        System.out.println(otherNumber);
    }

    public static int opposite(int number) {
        number = number * -1;
        return number;
    }
}
```

• A frame is created for the opposite method, and that frame "covers up" the frame for main.
Example: Tracing Through a Program

```java
public class OppositeFinder {
    public static void main(String[] args) {
        int number = 10;
        int otherNumber = opposite(10);
        System.out.print("opposite = ");
        System.out.println(otherNumber);
    }
    public static int opposite(int number) {
        number = number * -1;
        return number;
    }
}
```

- The actual parameter is passed in and is assigned to the formal parameter.
Example: Tracing Through a Program

```
public class OppositeFinder {
    public static void main(String[] args) {
        int number = 10;
        int otherNumber = opposite(10);
        System.out.print("opposite = ");
        System.out.println(otherNumber);
    }
    public static int opposite(int number) {
        number = -10;
        return number;
    }
}
```

- opposite returns, which removes its frame.
- The variable number in main's frame hasn't been changed!
public class OppositeFinder {
    public static void main(String[] args) {
        int number = 10;
        int otherNumber = opposite(10);
        System.out.print("opposite = ");
        System.out.println(otherNumber);
    }

    public static int opposite(int number) {
        number = -10;
        return -10;
    }
}

• The returned value replaces the method call.

Example: Tracing Through a Program

public class OppositeFinder {
    public static void main(String[] args) {
        int number = 10;
        int otherNumber = opposite(10);
        System.out.print("opposite = ");
        System.out.println(otherNumber);
    }

    public static int opposite(int number) {
        number = -10;
        return -10;
    }
}
public class OppositeFinder {
    public static void main(String[] args) {
        int number = 10;
        int otherNumber = -10;
        System.out.print("opposite = ");
        System.out.println(otherNumber);
    }
    public static int opposite(int number) {
        number = -10;
        return -10;
    }
}

Example: Tracing Through a Program

- main returns, which removes its frame.
Practice

- What is the output of the following program?

```java
public class MethodPractice {
    public static int triple(int x) {
        x = x * 3;
        return x;
    }

    public static void main(String[] args) {
        int y = 2;
        y = triple(y);
        System.out.println(y);
        triple(y);
        System.out.println(y);
    }
}
```

More Practice

```java
public class Mystery {
    public static int foo(int x, int y) {
        y = y + 1;
        x = x + y;
        System.out.println(x + " " + y);
        return x;
    }

    public static void main(String[] args) {
        int x = 2;
        int y = 0;
        y = foo(y, x);
        System.out.println(x + " " + y);
        foo(x, x);
        System.out.println(x + " " + y);
        System.out.println(foo(x, y));
        System.out.println(x + " " + y);
    }
}
```
public class TwoTriangles {
    public static void main(String[] args) {
        char ch = '*';      // character used in printing
        int smallBase = 5;  // base length of smaller triangle

        // Print the small triangle.
        for (int line = 1; line <= smallBase; line++) {
            for (int i = 0; i < line; i++) {
                System.out.print(ch);
            }
            System.out.println();
        }

        // Print the large triangle.
        for (int line = 1; line <= 2 * smallBase; line++) {
            for (int i = 0; i < line; i++) {
                System.out.print(ch);
            }
            System.out.println();
        }
    }
}

public static void printTriangle(_______________________) {
}

From Unstructured to Structured (cont.)

public class TwoTriangles {
    public static void main(String[] args) {
        char ch = '*';      // character used in printing
        int smallBase = 5;  // base length of smaller triangle

        // Print the small triangle.
        printTriangle(_______________________);

        // Print the large triangle.
        printTriangle(_______________________);
    }
    public static void printTriangle(_______________________) {
    }
}
Using Objects from Existing Classes

Computer Science S-111
Harvard University
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Combining Data and Operations

- The data types that we've seen thus far are referred to as *primitive* data types.
  - `int`, `double`, `char`
  - several others
- Java allows us to use another kind of data known as an *object*.
- An object groups together:
  - one or more data values (the object's *fields*)
  - a set of operations (the object's *methods*)
- Objects in a program are often used to model real-world objects.
Combining Data and Operations (cont.)

- Example: an Address object
  - possible fields: street, city, state, zip
  - possible operations: get the city, change the city, check if two addresses are equal

- Here are two ways to visualize an Address object:

```plaintext
street "111 Cummington St."  fields
city "Boston"
state "MA"
zip "02215"
getCity() methods
changeCity()
...
```

Classes as Blueprints

- We've been using classes as containers for our programs.
- A class can also serve as a blueprint – as the definition of a new type of object.
- The objects of a given class are built according to its blueprint.
- Another analogy:
  - class = cookie cutter
  - objects = cookies
- The objects of a class are also referred to as instances of the class.
Class vs. Object

• The Address class is a blueprint:

```java
public class Address {
    // definitions of the fields
    ...
    // definitions of the methods
    ...
}
```

• Address objects are built according to that blueprint:

```
street    "111 Cummington St."
city      "Boston"
state     "MA"
zip       "02215"

street    "240 West 44th Street"
city      "New York"
state     "NY"
zip       "10036"

street    "1600 Pennsylvania Ave."
city      "Washington"
state     "DC"
zip       "20500"
```

Using Objects from Existing Classes

• Later in the course, you'll learn how to create your own classes that act as blueprints for objects.

• For now, we'll focus on learning how to use objects from existing classes.
String Objects

- In Java, a string (like "Hello, world!") is actually represented using an object.
  - data values: the characters in the string
  - operations: get the length of the string, get a substring, etc.

- The `String` class defines this type of object:

```java
public class String {
    // definitions of the fields
    ... 
    // definitions of the methods
    ... 
}
```

- Individual `String` objects are instances of the `String` class:

| Perry | Hello | object |

Variables for Objects

- When we use a variable to represent an object, the type of the variable is the name of the object's class.

- Here's a declaration of a variable for a `String` object:

```java
String name;
```

- we capitalize `String`, because it's a class name
Creating String Objects

- One way to create a String object is to specify a string literal:
  ```java
  String name = "Perry Sullivan";
  ```

- We create a new String from existing Strings when we use the + operator to perform concatenation:
  ```java
  String firstName = "Perry";
  String lastName = "Sullivan";
  String fullName = firstName + " " + lastName;
  ```

- Recall that we can concatenate a String with other types of values:
  ```java
  String msg = "Perry is " + 6;
  // msg now represents "Perry is 6"
  ```

Using an Object's Methods

- An object's methods are different from the static methods that we've seen thus far.
  - they're called non-static or instance methods

- An object's methods belong to the object. They specify the operations that the object can perform.

- To use a non-static method, we have to specify the object to which the method belongs.
  - use dot notation, preceding the method name with the object's variable:
    ```java
    String firstName = "Perry";
    int len = firstName.length();
    ```

- Using an object's method is like sending a message to the object, asking it to perform that operation.
The API of a Class

- The methods defined within a class are known as the API of that class.
  - API = application programming interface

- We can consult the API of an existing class to determine which operations are supported.

- The API of all classes that come with Java is available here:
  https://docs.oracle.com/javase/8/docs/api/

  - there's a link on the resources page of the course website

Consulting the Java API

select the package name (optional)

String is in java.lang
Consulting the Java API

- Scroll down to see a summary of the available methods:
Consulting the Java API (cont.)

• Clicking on a method name gives you more information:

```
public int length()
```

**method header**

Returns the length of this string. The length is equal to the number of Unicode code units in the string.

**behavior**

Specified by:
length in interface CharSequence

Returns:
the length of the sequence of characters represented by this object.

• From the header, we can determine:
  • the return type: `int`
  • the parameters we need to supply:
    the empty () indicates that `length` has no parameters

Numbering the Characters in a String

• The characters are numbered from left to right, starting from 0.

```
0 1 2 3 4
```

**Perry**

• The position of a character in a string is known as its index.
  • 'P' has an index of 0 in "Perry"
  • 'y' has an index of 4
substring Method

<table>
<thead>
<tr>
<th>substring</th>
</tr>
</thead>
<tbody>
<tr>
<td>public String substring(int beginIndex, int endIndex)</td>
</tr>
</tbody>
</table>

Returns a new string that is a substring of this string. The substring begins at the specified `beginIndex` and extends to the character at index `endIndex - 1`. Thus the length of the substring is `endIndex - beginIndex`.

**String substring(int beginIndex, int endIndex)**

- return type: `String`
- parameters: `<int beginIndex, int endIndex>`
- behavior: returns the substring that:
  - begins at `beginIndex`
  - ends at `endIndex - 1`

substring Method (cont.)

- To extract a substring of length $N$, you can just figure out `beginIndex` and do:
  ```java
  substring(beginIndex, beginIndex + N)
  ```
- example: consider again this string:
  ```java
  String name = "Perry Sullivan";
  ```
  To extract a substring containing the first 5 characters, we can do this:
  ```java
  String first = name.substring(0, 5);
  ```
Review: Calling a Method

• Consider this code fragment:
  String name = "Perry Sullivan";
  int start = 6;
  String last = name.substring(start, start + 8);

• Steps for executing the method call:
  1. the actual parameters are evaluated to give:
     String last = name.substring(6, 14);
  2. a frame is created for the method, and the actual parameters are assigned to the formal parameters
  3. flow of control jumps to the method, which creates and returns the substring "Sullivan"
  4. flow of control jumps back, and the returned value replaces the method call:
     String last = "Sullivan";

How should we fill in the blank?

```java
charAt

public char charAt(int index)

Returns the char value at the specified index. An index ranges from 0 to length() - 1.

String s = "Strings have methods inside them!";
int len = s.length();
_______________ // get the last character in s
```
charAt Method

The `charAt()` method that we use for indexing returns a character, not a String.

We have to be careful when we use its return value!

- example: what does this print?
  ```java
  String name = "Perry Sullivan";
  System.out.println(name.charAt(0) + name.charAt(6));
  System.out.println('P' + 'S');
  System.out.println("PS");
  ```

Here’s how we can fix this:

```java
String name = "Perry Sullivan";
System.out.println(name.charAt(0) + "" + name.charAt(6));
System.out.println('P' + 'S');
System.out.println("PS");
```
Another String Method

String **toUpperCase()**
returns a new String in which all of the letters in the original String are converted to upper-case letters

- Example:
  ```java
  String warning = "Start the problem set ASAP!";
  System.out.println(warning.toUpperCase());
  System.out.println("START THE PROBLEM SET ASAP!");
  ```
- **toUpperCase()** creates and returns a new String. It does not change the original String.
- In fact, it's *never* possible to change an existing String object.
- We say that strings are **immutable** objects.

indexOf Method

int **indexOf(char ch)**
- return type: int
- parameter list: (char ch)
- returns:
  - the index of the first occurrence of ch in the string
  - -1 if the ch does not appear in the string
- examples:
  ```java
  String name = "Perry Sullivan";
  System.out.println(name.indexOf('r'));
  System.out.println(name.indexOf('x'));
  ```
The Signature of a Method

• The signature of a method consists of:
  • its name
  • the number and types of its parameters

  public String substring(int beginIndex, int endIndex)

• A class cannot include two methods with the same signature.

Two Methods with the Same Name

• There are actually two String methods named substring:
  String substring(int beginIndex, int endIndex)
  String substring(int beginIndex)
  • returns the substring that begins at beginIndex and continues to the end of the string

• Do these two methods have the same signature?

• Giving two methods the same name is known as method overloading.

• When you call an overloaded method, the compiler uses the number and types of the actual parameters to figure out which version to use.
Console Input Using a Scanner Object

- We’ve been printing text in the console window.
- You can also ask the user to enter a value in that window.
  - known as console input
- To do so, we use a type of object known as a Scanner.
  - recall PS 2

Packages

- Java groups related classes into packages.
- Many classes are part of the java.lang package.
  - examples: String, Math
  - We don't need to tell the compiler where to find these classes.
- If a class is in another package, we need to use an import statement so that the compiler will be able to find it.
  - put it before the definition of the class
- The Scanner class is in the java.util package, so we do this:

```java
import java.util.*;
public class MyProgram {
  ...
```
Creating an Object

- String objects are different from other objects, because we're able to create them using literals.

- To create an object, we typically use a special method known as a constructor.

- Syntax:
  
  ```java
  <variable> = new <ClassName>(<parameters>);
  
  or
  
  <type> <variable> = new <ClassName>(<parameters>);
  ```

- To create a Scanner object for console input:

  ```java
  Scanner console = new Scanner(System.in);
  ```

  the parameter tells the constructor that we want the Scanner to read from the standard input (i.e., the keyboard)

Scanner Methods: A Partial List

- String next()
  - read in a single "word" and return it

- int nextInt()
  - read in an integer and return it

- double nextDouble()
  - read in a floating-point value and return it

- String nextLine()
  - read in a "line" of input (could be multiple words) and return it
Example of Using a Scanner Object

- To read an integer from the user:
  ```java
  Scanner console = new Scanner(System.in);
  int numGrades = console.nextInt();
  ```
- The second line causes the program to pause until the user types in an integer followed by the [ENTER] key.
- If the user only hits [ENTER], it will continue to pause.
- If the user enters an integer, it is returned and assigned to `numGrades`.
- If the user enters a non-integer, an exception is thrown and the program crashes.

Example Program: GradeCalculator

```java
import java.util.*;
public class GradeCalculator {
    public static void main(String[] args) {
        Scanner console = new Scanner(System.in);
        System.out.print("Points earned: ");
        int points = console.nextInt();
        System.out.print("Possible points: ");
        int possiblePoints = console.nextInt();

        double grade = points/(double)possiblePoints;
        grade = grade * 100.0;

        System.out.println("grade is " + grade);
    }
}
```
Important Note About Console Input

• When writing an interactive program that involves user input in methods other than main, you should:
  • *create a single scanner object in the first line of the main method*
  • pass that object into any other method that needs it

• This allows you to avoid creating multiple objects that all do the same thing.

• It also facilitates our grading, because it allows us to provide a series of inputs using a file instead of the keyboard.

---

Important Note About Console Input (cont.)

• Example:

```java
public class MyProgram {
    public static void main(String[] args) {
        Scanner console = new Scanner(System.in);
        String str1 = getString(console);
        String str2 = getString(console);
        System.out.println(str1 + " " + str2);
    }

    public static String getString(Scanner console){
        System.out.print("Enter a string: ");
        String str = console.next();
        return str;
    }
}
```
What's Wrong with the Following?

```java
import java.util.*;

public class LengthConverter {
    public static void main(String[] args) {
        Scanner console = new Scanner(System.in);
        int cm = (int)(getInches(console) * 2.54);
        System.out.println(getInches(console) + " inches = " + cm + " cm");
    }
    
    public static int getInches(Scanner console) {
        System.out.print("Enter a length in inches: ");
        int inches = console.nextInt();
        return inches;
    }
}
```

Exercise: Analyzing a Name: First Version

```java
public class NameAnalyzer {
    public static void main(String[] args) {
        String name = "Perry Sullivan";
        System.out.println("full name = " + name);
        int length = name.length();
        System.out.println("length = " + length);
        String first = name.substring(0, 5);
        System.out.println("first name = " + first);
        String last = name.substring(6);
        System.out.println("last name = " + last);
    }
}
```
Making the Program More General

- Would the code work if we used a different name?

```java
import java.util.*;

public class NameAnalyzer {
    public static void main(String[] args) {
        Scanner console = new Scanner(System.in);
        String name = console.nextLine();
        System.out.println("full name = " + name);

        int length = name.length();
        System.out.println("length = " + length);

        String first = name.substring(0, 5);
        System.out.println("first name = " + first);

        String last = name.substring(6);
        System.out.println("last name = " + last);
    }
}
```

Breaking Up a Name

- Given a string of the form "firstName  lastName", how can we get the first and last names, without knowing how long it is?

- Pseudocode for what we need to do:

- What string methods can we use? Consult the API!

- Code:
Static Methods for Breaking Up a Name

• How could we rewrite our name analyzer to use separate methods for extracting the first and last names?

```java
public static String firstName(String name) {
    // implementation
}

public static String lastName(String name) {
    // implementation
}
```

Using the Static Methods

• Given the methods from the previous slide, what would the main method now look like?

```java
public static void main(String[] args) {
    Scanner console = new Scanner(System.in);
    String name = console.nextLine();
    System.out.println("full name = " + name);
    int length = name.length();
    System.out.println("length = " + length);
}
```
Processing a String One Character at a Time

- Write a method for printing the name vertically, one char per line.

```java
import java.util.*;
public class NameAnalyzer {
    public static void main(String[] args) {
        Scanner console = new Scanner(System.in);
        String name = console.nextLine();
        System.out.println("full name = " + name);
        ...
        printVertical(name);
    }
    public static _____ printVertical(_______________){
        for (int i = 0; i < _____________; i++) {
        }
    }
}
```

Scanner Objects and Tokens

- Most Scanner methods read one token at a time.

- Tokens are separated by whitespace (spaces, tabs, newlines).
  - example: if the user enters the line
    ```
    wow, I slept for 9 hours!
    ```
    there are six tokens:
    - wow,
    - I
    - slept
    - for
    - 9
    - hours!
    ```
    newline character, which you get when you hit [ENTER]```
Scanner Objects and Tokens (cont.)

- Consider the following lines of code:

  ```java
  System.out.print("Enter the length and width: ");
  int length = console.nextInt();
  int width = console.nextInt();
  ```

- Because the `nextInt()` method reads one token at a time, the user can either:
  - enter the two numbers on the same line, separated by one or more whitespace characters
    ```java
    Enter the length and width: 30 15
    ```
  - enter the two numbers on different lines
    ```java
    Enter the length and width: 30
    15
    ```

nextLine Method

- The `nextLine()` method does not just read a single token.

- Using `nextLine` can lead to unexpected behavior, for reasons that we'll discuss later on.

- Avoid it for now!
Additional Terminology

• To avoid having too many new terms at once, I’ve limited the terminology introduced in these notes.

• Here are some additional terms related to classes, objects, and methods:
  • *invoking* a method = calling a method
  • method *invocation* = method call
  • the *called object* = the object used to make a method call
  • *instantiate* an object = create an object
  • *members* of a class = the fields and methods of a class
Conditional Execution

Review: Simple Conditional Execution in Java

if (<condition>) {
    <true block>
} else {
    <false block>
}

- If the condition is true:
  - the statement(s) in the true block are executed
  - the statement(s) in the false block (if any) are skipped

- If the condition is false:
  - the statement(s) in the false block (if any) are executed
  - the statement(s) in the true block are skipped
Example: Analyzing a Number

```java
scanner console = new Scanner(System.in);
System.out.print("Enter an integer: ");
int num = console.nextInt();

if (num % 2 == 0) {
    System.out.println(num + " is even.");
} else {
    System.out.println(num + " is odd.");
}
```

Flowchart for an if-else Statement
Common Mistake

• You should not put a semi-colon after an if-statement header:
  
  ```java
  if (num % 2 == 0); {
    System.out.println(...);
  }
  ```

• The semi-colon ends the if statement.
  • thus, it has an empty true block

• The println and other statements are independent of the if statement, and always execute.

Choosing at Most One of Several Options

• Consider this code:
  
  ```java
  if (num < 0) {
    System.out.println("The number is negative.");
  }
  if (num > 0) {
    System.out.println("The number is positive.");
  }
  if (num == 0) {
    System.out.println("The number is zero.");
  }
  ```

• All three conditions are evaluated, but at most one of them can be true (in this case, exactly one).
Choosing at Most One of Several Options (cont.)

- We can do this instead:

```java
if (num < 0) {
    System.out.println("The number is negative.");
} else if (num > 0) {
    System.out.println("The number is positive.");
} else if (num == 0) {
    System.out.println("The number is zero.");
}
```

- If the first condition is true, it will skip the second and third.
- If the first condition is false, it will evaluate the second, and if the second condition is true, it will skip the third.
- If the second condition is false, it will evaluate the third, etc.

Choosing at Most One of Several Options (cont.)

- We can also make things more compact as follows:

```java
if (num < 0) {
    System.out.println("The number is negative.");
} else if (num > 0) {
    System.out.println("The number is positive.");
} else if (num == 0) {
    System.out.println("The number is zero.");
}
```

- This emphasizes that the entire thing is one compound statement.
if-else if Statements

- Syntax:
  
  ```
  if (<condition1>) {
    <true block for condition1>
  } else if (<condition2>) {
    <true block for condition2>
  }
  ...
  } else {
    <false block for all of the conditions>
  }
  ```

- The conditions are evaluated in order. The true block of the first true condition is executed. 
  *All of the remaining conditions and their blocks are skipped.*

- If no condition is true, the false block (if any) is executed.

Flowchart for an if-else if Statement

```
<table>
<thead>
<tr>
<th>Condition</th>
<th>True Block</th>
<th>False Block</th>
</tr>
</thead>
<tbody>
<tr>
<td>condition1</td>
<td>true block 1</td>
<td>false block</td>
</tr>
<tr>
<td></td>
<td>true block 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>...</td>
<td></td>
</tr>
<tr>
<td></td>
<td>false</td>
<td>next statement</td>
</tr>
</tbody>
</table>
```
Choosing Exactly One Option

• Consider again this code fragment:
  ```java
  if (num < 0) {
    System.out.println("The number is negative.");
  } else if (num > 0) {
    System.out.println("The number is positive.");
  } else if (num == 0) {
    System.out.println("The number is zero.");
  }
  ```

• One of the conditions must be true, so we can omit the last one:
  ```java
  if (num < 0) {
    System.out.println("The number is negative.");
  } else if (num > 0) {
    System.out.println("The number is positive.");
  } else {
    System.out.println("The number is zero.");
  }
  ```

Types of Conditional Execution

• If it want to execute any number of several conditional blocks, use sequential if statements:
  ```java
  if (num < 0) {
    System.out.println("The number is negative.");
  }
  if (num % 2 == 0) {
    System.out.println("The number is even.");
  }
  ```

• If you want to execute at most one (i.e., 0 or 1) of several blocks, use an if-else if statement ending in else if:
  ```java
  if (num < 0) {
    System.out.println("The number is negative.");
  } else if (num > 0) {
    System.out.println("The number is positive.");
  }
  ```

• If you want to execute exactly one of several blocks, use an if-else if ending in just else (see bottom of last slide).
Find the Logic Error

```java
Scanner console = new Scanner(System.in);

System.out.print("Enter the student's score: ");
int score = console.nextInt();
String grade;
if (score >= 90) {
    grade = "A";
}
if (score >= 80) {
    grade = "B";
}
if (score >= 70) {
    grade = "C";
}
if (score >= 60) {
    grade = "D";
}
if (score < 60) {
    grade = "F";
}
```

Review: Variable Scope

- Recall: the scope of a variable is the portion of a program in which the variable can be used.
- By default, the scope of a variable:
  - begins at the point at which it is declared
  - ends at the end of the innermost block that encloses the declaration
- Because of these rules, a variable cannot be used outside of the block in which it is declared.
Variable Scope and if-else statements

• The following program will produce compile-time errors:
  ```java
  public static void main(String[] args) {
      Scanner console = new Scanner(System.in);
      System.out.print("enter a positive int: ");
      int num = console.nextInt();
      if (num < 0) {
          System.out.println("number is negative; " + " using its absolute value");
          double sqrt = Math.sqrt(num * -1);
      } else {
          sqrt = Math.sqrt(num);
      }
      System.out.println("square root = " + sqrt);
  }
  ```

• Why?

Variable Scope and if-else statements (cont.)

• To eliminate the errors, declare the variable outside of the true block:
  ```java
  public static void main(String[] args) {
      Scanner console = new Scanner(System.in);
      System.out.print("enter a positive int: ");
      int num = console.nextInt();
      double sqrt;
      if (num < 0) {
          System.out.println("number is negative; " + " using its absolute value");
          sqrt = Math.sqrt(num * -1);
      } else {
          sqrt = Math.sqrt(num);
      }
      System.out.println("square root = " + sqrt);
  }
  ```

• What is the scope of sqrt now?
Review: Loop Patterns for n Repetitions

- Thus far, we've mainly used for loops to repeat something a definite number of times.

- We've seen two different patterns for this:
  - pattern 1:
    ```java
    for (int i = 0; i < n; i++) {
        <statements to repeat>
    }
    ```
  - pattern 2:
    ```java
    for (int i = 1; i <= n; i++) {
        <statements to repeat>
    }
    ```

Another Loop Pattern: Cumulative Sum

- We can also use a for loop to add up a set of numbers.

- Basic pattern (using pseudocode):
  ```java
  sum = 0
  for (all of the numbers that we want to sum) {
    num = the next number
    sum = sum + num
  }
  ```
```java
public class GradeAverager {
    public static void main(String[] args) {
        Scanner console = new Scanner(System.in);
        System.out.print("number of grades? ");
        int numGrades = console.nextInt();
        if (numGrades <= 0) {
            System.out.println("nothing to average");
        } else {
            int sum = 0;
            for (int i = 1; i <= numGrades; i++) {
                System.out.print("grade #" + i + ": ");
                int grade = console.nextInt();
                sum = sum + grade;
            }
            System.out.println("The average is " +
                    (double)sum / numGrades);
        }
    }
}
```

- Note the use of an if-else statement to handle invalid user inputs.

---

**Example of Using a Cumulative Sum**

```java
TRACING THROUGH A CUMULATIVE SUM

- Let's trace through this code.
  ```java
  int sum = 0;
  for (int i = 1; i <= numGrades; i++) {
      System.out.print("grade #" + i + ": ");
      int grade = console.nextInt();
      sum = sum + grade;
  }
  ```
  
  assuming that the user enters these grades: 80, 90, 84.
  ```java
  numGrades = 3
  ```
  ```java
  i    i <= numGrades    grade    sum
  1  80
  2  90
  3  84
  ```
  ````
Conditional Execution and Return Values

• With conditional execution, it's possible to write a method with more than one return statement.
  • example:
    ```java
    public static int min(int a, int b) {
        if (a < b) {
            return a;
        } else {
            return b;
        }
    }
    ```
  • Only one of the return statements is executed.
  • As soon as you reach a return statement, the method's execution stops and the specified value is returned.
    • the rest of the method is not executed

Conditional Execution and Return Values (cont.)

• Instead of writing the method this way:
  ```java
  public static int min(int a, int b) {
      if (a < b) {
          return a;
      } else {
          return b;
      }
  }
  ```
  we could instead write it like this, without the else:
  ```java
  public static int min(int a, int b) {
      if (a < b) {
          return a;
      } return b;
  }
  ```
  • Why is this equivalent?
Conditional Execution and Return Values (cont.)

• Consider this method, which has a compile-time error:

```java
public static int compare(int a, int b) {
    if (a < b) {
        return -1;
    } else if (a > b) {
        return 1;
    } else if (a == b) {
        return 0;
    }
}
```

• Because all of the return statements are connected to conditions, the compiler worries that no value will be returned.

• Here’s one way to fix it:

```java
public static int compare(int a, int b) {
    if (a < b) {
        return -1;
    } else if (a > b) {
        return 1;
    } else {
        return 0;
    }
}
```
Conditional Execution and Return Values (cont.)

• Here's another way:
  ```java
  public static int compare(int a, int b) {
    if (a < b) {
      return -1;
    } else if (a > b) {
      return 1;
    }
    return 0;
  }
  ```

• Both fixes allow the compiler to know for certain that a value will always be returned.

Returning From a void Method

```
public static void repeat(String msg, int n) {
  if (n <= 0) {   // special cases
    return;
  }
  for (int i = 0; i < n; i++) {
    System.out.println(msg);
  }
}
```

• Note that this method has a return type of void.
  • it doesn't return a value.

• However, it still has a return statement.
  • used to break out of the method
  • note that there's nothing between the return and the ;
Testing for Equivalent Primitive Values

- The == and != operators are used when comparing primitives.
  - int, double, char, etc.

- Example:
  ```java
  Scanner console = new Scanner(System.in);
  ...
  System.out.print("Do you have another (y/n)? ");
  char choice = console.next().charAt(0);
  if (choice == 'y') {  // this works just fine
    processItem();
  } else if (choice == 'n') {
    return;
  } else {
    System.out.println("invalid input");
  }
  ```

Testing for Equivalent Objects

- The == and != operators do not typically work when comparing objects. (We'll see why this is later.)

- Example:
  ```java
  Scanner console = new Scanner(System.in);
  System.out.print("regular or diet? ");
  String choice = console.next();
  if (choice == "regular") {  // doesn't work
    processRegular();
  } else {
    ...  
  }
  ```

- choice == "regular" compiles, but it evaluates to false, even when the user does enter "regular"!
Testing for Equivalent Objects (cont.)

• We use a special method called the equals method to test if two objects are equivalent.
  • example:
    ```java
    Scanner console = new Scanner(System.in);
    System.out.print("regular or diet? ");
    String choice = console.next();
    if (choice.equals("regular")) {
      processRegular();
    } else {
      ...
    }
    ```
  • choice.equals("regular") compares the string represented by the variable choice with the string "regular"
    • returns true when they are equivalent
    • returns false when they are not

equalsIgnoreCase()

• We often want to compare two strings without paying attention to the case of the letters.
  • example: we want to treat as equivalent:
    "regular"
    "Regular"
    "REGULAR"
    etc.
  • The String class has a method called equalsIgnoreCase that can be used for this purpose:
    ```java
    if (choice.equalsIgnoreCase("regular")) {
      ...
    }
    ```
Example Problem: Ticket Sales

- Different prices for balcony seats and orchestra seats

- Here are the rules:
  - persons younger than 25 receive discounted prices:
    - $20 for balcony seats
    - $35 for orchestra seats
  - everyone else pays the regular prices:
    - $30 for balcony seats
    - $50 for orchestra seats

- Assume only valid inputs.

Ticket Sales Program: main method

```java
Scanner console = new Scanner(System.in);
System.out.print("Enter your age: ");
int age = console.nextInt();
if (age < 25) {
    // handle people younger than 25
    System.out.print("orchestra or balcony? ");
    String choice = console.next();
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 35;
    } else {
        price = 20;
    }
    System.out.println("The price is $" + price);
} else {
    // handle people 25 and older
    ...
}
```
Ticket Sales Program: main method (cont.)

...}
} else {
    // handle people 25 and older
    System.out.print("orchestra or balcony? ");
    String choice = console.next();
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 50;
    } else {
        price = 30;
    }
    System.out.println("The price is $" + price);
}

Where Is the Code Duplication?

...}
} else {
    System.out.print("orchestra or balcony? ");
    String choice = console.next();
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 50;
    } else {
        price = 30;
    }
    System.out.println("The price is $" + price);
Factoring Out Code Common to Multiple Cases

```java
scanner console = new Scanner(System.in);
System.out.print("Enter your age: ");
int age = console.nextInt();
System.out.print("orchestra or balcony? ");
String choice = console.next();
if (age < 25) {
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 35;
    } else {
        price = 20;
    }
} else {
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 50;
    } else {
        price = 30;
    }
}
System.out.println("The price is "+ price);
```

What Other Change Is Needed?

```java
scanner console = new Scanner(System.in);
System.out.print("Enter your age: ");
int age = console.nextInt();
System.out.print("orchestra or balcony? ");
String choice = console.next();
if (age < 25) {
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 35;
    } else {
        price = 20;
    }
} else {
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 50;
    } else {
        price = 30;
    }
}
System.out.println("The price is "+ price);
```
Now Let's Make It Structured

```java
public static void main(String[] args) {
    int age = console.nextInt();
    System.out.print("orchestra or balcony? ");
    String choice = console.next();
    int price;
    if (age < 25) {
        ________________________________;
    } else {
        ...
    }
    System.out.println("The price is $" + price);
}
```

```java
public static ______ discountPrice(______________) {
}
```

Expanded Ticket Sales Problem

- One additional case:
  - **persons younger than 13 cannot buy a ticket**
  - persons whose age is **13-24** receive discounted prices:
    - $20 for balcony seats
    - $35 for orchestra seats
  - everyone else pays the regular prices:
    - $30 for balcony seats
    - $50 for orchestra seats
Here's the Unfactored Version

```java
if (age < 13) {
    System.out.println("You cannot buy a ticket.");
} else if (age < 25) {
    System.out.print("orchestra or balcony? ");
    String choice = console.next();
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 35;
    } else {
        price = 20;
    }
    System.out.println("The price is "+ price);
} else {
    System.out.print("orchestra or balcony? ");
    String choice = console.next();
    int price;
    if (choice.equalsIgnoreCase("orchestra")) {
        price = 50;
    } else {
        price = 30;
    }
    System.out.println("The price is "+ price);
}
```

We now have code common to the 2nd and 3rd cases, but not the 1st.

Group the Second and Third Cases Together

```java
if (age < 13) {
    System.out.println("You cannot buy a ticket.");
} else {
    if (age < 25) {
        System.out.print("orchestra or balcony? ");
        String choice = console.next();
        int price;
        if (choice.equalsIgnoreCase("orchestra")) {
            price = 35;
        } else {
            price = 20;
        }
        System.out.println("The price is "+ price);
    } else {
        System.out.print("orchestra or balcony? ");
        String choice = console.next();
        ... System.out.println("The price is "+ price);
    }
}
```
Then Factor Out the Common Code

```java
... if (age < 13) {
    System.out.println("You cannot buy a ticket.");
} else {
    System.out.print("orchestra or balcony? ");
    String choice = console.next();
    int price;
    if (age < 25) {
        if (choice.equalsIgnoreCase("orchestra")) {
            price = 35;
        } else {
            price = 20;
        }
    } else {
        if (choice.equalsIgnoreCase("orchestra")) {
            price = 50;
        } else {
            price = 30;
        }
    }
    System.out.println("The price is $" + price);
}
```

Case Study: Coffee Shop Price Calculator

- Relevant info:
  - brewed coffee prices by size:
    - tiny: $1.60
    - medio: $1.80
    - gigundo: $2.00
  - latte prices by size:
    - tiny: $2.80
    - medio: $3.20
    - gigundo: $3.60
  - plus, add 50 cents for a latte with flavored syrup
  - sales tax:
    - students: no tax
    - non-students: 6.25% tax
Case Study: Coffee Shop Price Calculator (cont.)

- Developing a solution:
  1. Begin with an *unstructured* solution.
     - everything in the *main* method
     - use if-else-if statement(s) to handle the various cases
  2. Next, *factor out* code that is common to multiple cases.
     - put it either before or after the appropriate
       if-else-if statement
  3. Finally, create a fully *structured* solution.
     - use procedural decomposition to capture logical pieces of the solution

Case Study: Coffee Shop Price Calculator (cont.)
Optional: Comparing Floating-Point Values

- Because the floating-point types have limited precision, it's possible to end up with roundoff errors.

- Example:
  ```java
double sum = 0.1 + 0.1 + 0.1 + 0.1 + 0.1;
sum = sum + 0.1 + 0.1 + 0.1 + 0.1 + 0.1;
System.out.println(sum);
// get 0.9999999999999999!
```

- Thus when trying to determine if two floating-point values are equal, we usually do not use the == operator.

- Instead, we test if the difference between the two values is less than some small threshold value:
  ```java
if (Math.abs(sum - 1.0) < 0.0000001) {
    System.out.println(sum + " == 1.0");
}
```

Optional: Another Cumulative Computation

- The same pattern can be used for other types of computations.

- Example: counting the occurrences of a character in a string.

- Let's write a static method called numOccur that does this.

  ```java
public static ___ numOccur(_____________________) {
    EXAMPLES:
    numOccur('l', "hello") should return 2
    numOccur('s', "Mississippi") should return 4
    public static ___ numOccur(_____________________) {
```

```
Unit 3, Part 4

Indefinite Loops and Boolean Expressions

Computer Science S-111
Harvard University
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Review: Definite Loops

- The loops that we’ve seen thus far have been *definite loops*.  
  - we know exactly how many iterations will be performed before the loop even begins

- In an *indefinite loop*, the number of iterations is either:
  - not as obvious
  - impossible to determine before the loop begins
Sample Problem: Finding Multiples

- Problem: Print all multiples of a number (call it num) that are less than 100.
  - output for num = 9:
    9 18 27 36 45 54 63 72 81 90 99

- Pseudocode for one possible algorithm:
  ```
  mult = num
  repeat as long as mult < 100:
    print mult + "  
    mult = mult + num
  print a newline
  ```

Sample Problem: Finding Multiples (cont.)

- Pseudocode:
  ```
  mult = num
  repeat as long as mult < 100:
    print mult + "  
    mult = mult + num
  print a newline
  ```

- Here's how we would write this in Java:
  ```java
  int mult = num;
  while (mult < 100) {
    System.out.print(mult + " ");
    mult = mult + num;
  }
  System.out.println();
  ```
while Loops

• In general, a while loop has the form

```java
while (<test>) {
    <one or more statements>
}
```

• As with for loops, the statements in the block of a while loop are known as the **body** of the loop.

---

Evaluating a while Loop

**Steps:**
1. evaluate the test
2. if it's false, skip the statements in the body
3. if it's true, execute the statements in the body, and go back to step 1
Tracing a while Loop

• Let's trace through our code when num has the value 15:

```java
int mult = num;
while (mult < 100) {
    System.out.print(mult + " ");
    mult = mult + num;
}
```

<table>
<thead>
<tr>
<th>output thus far</th>
<th>mult</th>
</tr>
</thead>
<tbody>
<tr>
<td>before entering the loop</td>
<td>15</td>
</tr>
<tr>
<td>after the first iteration</td>
<td>15  30</td>
</tr>
<tr>
<td>after the second iteration</td>
<td>15  30  45</td>
</tr>
<tr>
<td>after the third iteration</td>
<td>15  30  45  60</td>
</tr>
<tr>
<td>after the fourth iteration</td>
<td>15  30  45  60  75</td>
</tr>
<tr>
<td>after the fifth iteration</td>
<td>15  30  45  60  75  90</td>
</tr>
<tr>
<td>after the sixth iteration</td>
<td>15  30  45  60  75  90  105</td>
</tr>
</tbody>
</table>

and now (mult < 100) is false, so we exit the loop

Comparing if and while

• The true block of an if statement is evaluated at most once.
• The body of a while statement can be evaluated multiple times, provided the test remains true.
Typical while Loop Structure

- Typical structure:

  ```java
  initialization statement(s)
  while (test) {
    other statements
    update statement(s)
  }
  ```

- In our example:

  ```java
  int mult = num;                    // initialization
  while (mult < 100) {
    System.out.print(mult + " ");
    mult = mult + num;               // update
  }
  ```

Comparing for and while loops

- while loop (typical structure):

  ```java
  initialization
  while (test) {
    other statements
    update
  }
  ```

- for loop:

  ```java
  for (initialization; test; update) {
    one or more statements
  }
  ```
Infinite Loops

- Let's say that we change the condition for our while loop:
  ```java
  int mult = num;
  while (mult != 100) {    // replaced < with !=
      System.out.print(mult + " ");
      mult = mult + num;
  }
  ```

- When num is 15, the condition will always be true.
  - why?

  - an infinite loop – the program will hang (or repeatedly output something), and needs to be stopped manually
  - what class of error is this (syntax or logic)?

- It's generally better to use <, <=, >, >= in a loop condition, rather than == or !=

Infinite Loops (cont.)

- Another common source of infinite loops is forgetting the update statement:
  ```java
  int mult = num;
  while (mult < 100) {
      System.out.print(mult + " ");
      // update should go here
  }
  ```
A Need for Error-Checking

• Let's return to our original version:

```java
int mult = num;
while (mult < 100) {
    System.out.print(mult + " ");
    mult = mult + num;
}
```

• This could still end up in an infinite loop! How?

Using a Loop When Error-Checking

• We need to check that the user enters a positive integer.

• If the number is <= 0, ask the user to try again.

• Here's one way of doing it using a while loop:

```java
Scanner console = new Scanner(System.in);
System.out.print("Enter a positive integer: ");
int num = console.nextInt();
while (num <= 0) {
    System.out.print("Enter a positive integer: ");
    num = console.nextInt();
}
```

• Note that we end up duplicating code.
Error-Checking Using a do-while Loop

- Java has a second type of loop statement that allows us to eliminate the duplicated code in this case:

```java
Scanner console = new Scanner(System.in);
int num;
do {
    System.out.print("Enter a positive integer: ");
    num = console.nextInt();
} while (num <= 0);
```

- The code in the body of a do-while loop is always executed at least once.

do-while Loops

- In general, a do-while statement has the form

```java
do {
    <one or more statements>
} while (<test>);
```

- Note the need for a semi-colon after the condition.
- We do not need a semi-colon after the condition in a while loop.
  - beware of using one – it can actually create an infinite loop!
Evaluating a do-while Loop

Steps:
1. execute the statements in the body
2. evaluate the test
3. if it's true, go back to step 1
   (if it's false, continue to the next statement)

Formulating Loop Conditions

- We often need to repeat actions until a condition is met.
  - example: keep reading a value until the value is positive
  - such conditions are termination conditions – they indicate when the repetition should stop

- However, loops in Java repeat actions while a condition is met.
  - they use continuation conditions

- As a result, you may need to convert a termination condition into a continuation condition.
Which Type of Loop Should You Use?

- Use a for loop when the number of repetitions is known in advance – i.e., for a definite loop.

- Otherwise, use a while loop or do-while loop:
  - use a while loop if the body of the loop may not be executed at all
    - i.e., if the condition may be false at the start of the loop
  - use a do-while loop if:
    - the body will always be executed at least once
    - doing so will allow you to avoid duplicating code

Find the Error...

- Where is the syntax error below?

```java
Scanner console = new Scanner(System.in);
do {
    System.out.print("Enter a positive integer: ");
    int num = console.nextInt();
} while (num <= 0);
System.out.println("\nThe multiples of " + num + " less than 100 are:");
int mult = num;
while (mult < 100) {
    System.out.print(mult + " ");
    mult = mult + num;
}
System.out.println();
```
Practice with while loops

- What does the following loop output?

```java
int a = 10;
while (a > 2) {
    a = a - 2;
    System.out.println(a * 2);
}
```

<table>
<thead>
<tr>
<th>a &gt; 2</th>
<th>a</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>before loop</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st iteration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd iteration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd iteration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4th iteration</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

boolean Data Type

- A condition like `mult < 100` has one of two values: `true` or `false`

- In Java, these two values are represented using the `boolean` data type.
  - one of the primitive data types (like `int`, `double`, and `char`)
  - `true` and `false` are its two literal values

- This type is named after the 19th-century mathematician George Boole, who developed the system of logic called `boolean algebra`. 
boolean Expressions

- We have seen a number of constructs that use a "test".
  - loops
  - if statements

- A more precise term for a "test" is a *boolean expression*.

- A boolean expression is any expression that evaluates to true or false.
  - examples: `num > 0`
    `false`
    `firstChar == 'P'`
    `score != 20`

boolean Expressions (cont.)

- Recall this line from our ticket-price program:
  ```java
  if (choice.equals("orchestra")) ...  
  ```
  a boolean expression, because it evaluates to true or false

- if we look at the String class in the Java API, we see that the equals method has this header:
  ```java
  public boolean equals(...)  
  ```
  it returns either true or false
Forming More Complex Conditions

• We often need to make a decision based on more than one condition – or based on the opposite of a condition.
  
  • examples in pseudocode:
    if the number is even AND it is greater than 100…
    if it is NOT the case that your grade is > 80…

• Java provides three logical operators for this purpose:

<table>
<thead>
<tr>
<th>operator</th>
<th>name</th>
<th>example</th>
</tr>
</thead>
<tbody>
<tr>
<td>&amp;&amp;</td>
<td>and</td>
<td>age &gt;= 18 &amp;&amp; age &lt;= 35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>!</td>
<td>not</td>
<td>!(grade &gt; 80)</td>
</tr>
</tbody>
</table>

Truth Tables

• The logical operators operate on boolean expressions.
  • let a and b represent two such expressions

• We can define the logical operators using truth tables.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a &amp;&amp; b</th>
</tr>
</thead>
<tbody>
<tr>
<td>false</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
<td>true</td>
</tr>
<tr>
<td>true</td>
<td>true</td>
<td>true</td>
</tr>
</tbody>
</table>

  | a   | b   | a || b |
  |-----|-----|-------|
  | false | false | false |
  | false | true  | true  |
  | true  | false | true  |
  | true  | true  | true  |

<table>
<thead>
<tr>
<th>a</th>
<th>!a</th>
</tr>
</thead>
<tbody>
<tr>
<td>false</td>
<td>true</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
</tr>
</tbody>
</table>
Truth Tables (cont.)

• Example: evaluate the following expression:
  
  \[(20 \geq 0) \land (30 \mod 4 = 1)\]

• First, evaluate each of the operands:
  
  \[(20 \geq 0) \land (30 \mod 4 = 1)\]
  
  \[\text{true} \land \text{false}\]

• Then, consult the appropriate row of the truth table:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a \land b</th>
</tr>
</thead>
<tbody>
<tr>
<td>false</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>true</td>
<td>true</td>
<td>true</td>
</tr>
</tbody>
</table>

• Thus, \[(20 \geq 0) \land (30 \mod 4 = 1)\] evaluates to \text{false}

Practice with Boolean Expressions

• Let’s say that we wanted to express the following English condition in Java:
  "num is not equal to either 0 or 1"

• Which of the following boolean expression(s) would work?
  
  a) \(\text{num} \neq 0 \lor 1\)
  b) \(\text{num} \neq 0 \lor \text{num} \neq 1\)
  c) \(!(\text{num} = 0 \lor \text{num} = 1)\)

• Is there a different boolean expression that would work here?
**boolean Variables**

- We can declare variables of type boolean, and assign the values of boolean expressions to them:
  ```
  int num = 10;
  boolean isPos = (num > 0);
  boolean isDone = false;
  ```

- these statements give us the following picture in memory:
  - `isPos`: true
  - `isDone`: false

- Using a boolean variable can make your code more readable:
  ```
  if (value % 2 == 0) {
      ...
  }
  ```

- Alternatively:
  ```
  boolean isEven = (value % 2 == 0);
  if (isEven == true) {
      ...
  }
  ```

  The extra comparison isn't necessary!

**boolean Variables (cont.)**

- Instead of doing this:
  ```
  boolean isEven = (num % 2 == 0);
  if (isEven == true) {
      ...
  }
  ```

  you could just do this:
  ```
  boolean isEven = (num % 2 == 0);
  if (isEven) {
      ...
  }
  ```

  The extra comparison isn't necessary!

- Similarly, instead of writing:
  ```
  if (isEven == false) {
      ...
  }
  ```

  you could just write this:
  ```
  if (!isEven) {
      ...
  }
Input Using a Sentinel

- Example problem: averaging an arbitrary number of grades.

- Instead of having the user tell us the number of grades in advance, we can let the user indicate that there are no more grades by entering a special sentinel value.

- When we encounter the sentinel, we break out of the loop
  
  - example interaction:
    
    Enter grade (-1 to end): 10
    Enter grade (-1 to end): 8
    Enter grade (-1 to end): 9
    Enter grade (-1 to end): 5
    Enter grade (-1 to end): -1
    The average is: 8.0

Input Using a Sentinel (cont.)

- Here’s one way to do this:

  ```java
  Scanner console = new Scanner(System.in);
  int total = 0;
  int numGrades = 0;

  System.out.print("Enter grade (or -1 to quit): ");
  int grade = console.nextInt();
  while (grade != -1) {
      total += grade;
      numGrades++;
      System.out.print("Enter grade (or -1 to quit): ");
      grade = console.nextInt();
  }

  if (numGrades > 0) {
      System.out.print("The average is ");
      System.out.println((double)total/numGrades);
  }
  ```
Input Using a Sentinel and a Boolean Flag

- Here's another way, using what is known as a boolean flag, which is a variable that keeps track of some condition:

```java
Scanner console = new Scanner(System.in);
int total = 0;
int numGrades = 0;
boolean done = false;

while (!done) {
    System.out.print("Enter grade (or -1 to quit): ");
    int grade = console.nextInt();
    if (grade == -1) {
        done = true;
    } else {
        total += grade;
        numGrades++;
    }
}
if (numGrades > 0) {
    ...
```

Input Using a Sentinel and a break Statement

- Here's another way, using what is known as a break statement, which "breaks out" of the loop:

```java
Scanner console = new Scanner(System.in);
int total = 0;
int numGrades = 0;

while (true) {
    System.out.print("Enter grade (or -1 to quit): ");
    int grade = console.nextInt();
    if (grade == -1) {
        break;
    } else {
        total += grade;
        numGrades++;
    }
}
// after the break statement, the flow of control
// resumes here...
if (numGrades > 0) {
    ...
```
Arrays

Computer Science S-111
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Collections of Data

• Recall our program for averaging quiz grades:
public static void main(String[] args) {
    Scanner console = new Scanner(System.in);
    int total = 0;
    int numGrades = 0;
    while (true) {
        System.out.print("Enter a grade (or -1 to quit): ");
        int grade = console.nextInt();
        if (grade == -1) {
            break;
        }
        total += grade;
        numGrades++;
    }
    if (numGrades > 0) {
        ...
    }
}

• What if we wanted to store the individual grades?
  • an example of a collection of data
Arrays

• An array is a collection of data values of the same type.

• In the same way that we think of a variable as a single box, an array can be thought of as a sequence of boxes:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>6</td>
<td>10</td>
<td>7</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

  ← indices
  ← elements

• Each box contains one of the data values in the collection
  • referred to as the elements of the array

• Each element has a numeric index
  • the first element has an index of 0,
    the second element has an index of 1,
    etc.
  • example: the value 6 above has an index of 3
  • like the index of a character in a String

Declaring and Creating an Array

• We use a variable to represent the array as a whole.

• Example of declaring an array variable:

```java
int[] grades;
```
  • the [] indicates that it will represent an array
  • the int indicates that the elements will be ints

• Declaring the array variable does not create the array.

• Example of creating an array:

```java
grades = new int[8];
```
  the length of the array –
  i.e., the number of elements
Declaring and Creating an Array (cont.)

- We often declare and create an array in the same statement:
  ```java
  int[] grades = new int[8];
  ```

- General syntax:
  ```java
  <type>[] <array> = new <type>[<length>];
  ```
  where
  - `<type>` is the type of the individual elements
  - `<array>` is the name of the variable used for the array
  - `<length>` is the number of elements in the array

The Length of an Array

- The `length` of an array is the number of elements in the array.

- The length of an array can be obtained as follows:
  ```java
  <array>.length
  ```
  - example:
    ```java
    grades.length
    ```

  - note: it is not a method
    ```java
    grades.length() won't work!
    ```
Auto-Initialization

- When you create an array in this way:
  ```java
  int[] grades = new int[8];
  ```
  the runtime system gives the elements default values:
  
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- The value used depends on the type of the elements:
  ```java
  int 0
  double 0.0
  char '\0'
  boolean false
  objects null
  ```

Accessing an Array Element

- To access an array element, we use an expression of the form
  ```java
  <array>[<index>]
  ```

- Examples:
  - grades[0] accesses the first element
  - grades[1] accesses the second element
  - grades[5] accesses the sixth element

- Here's one way of setting up the array we showed earlier:
  ```java
  int[] grades = new int[8];
  grades[0] = 7;  grades[1] = 8;  grades[2] = 9;
  ```
Accessing an Array Element (cont.)

- Acceptable index values:
  integers from 0 to \(<array>.length - 1\)

- If we specify an index outside that range, we'll get an
  ArrayIndexOutOfBoundsException at runtime.
  - example:
    ```java
    int[] grades = int[8];
    grades[8] = 5;
    ```
    ![Array Index Example](image)

- The index can be any integer expression.
  - example:
    ```java
    int lastGrade = grades[grades.length - 1];
    ```

- We can operate on an array element in the same way that
  we operate on any other variable of that type.
  - example: applying a 10% late penalty to the grade
    at index \(i\)
    ```java
    grades[i] = (int)(grades[i] * 0.9);
    ```
  - example: adding 5 points of extra credit to the grade
    at index \(i\)
    ```java
    grades[i] += 5;
    ```
Another Way to Create an Array

• If we know that we want an array to contain specific values, we can specify them when create the array.

• Example: here's another way to create and initialize our grades array:

  int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};

• The list of values is known as an initialization list.
  • it can only be specified when the array is declared
  • we don't use the new operator in this case
  • we don't specify the length of the array – it is determined from the number of values in the initialization list

• Other examples:

  double[] heights = {65.2, 72.0, 70.6, 67.9};
  boolean[] isPassing = {true, true, false, true};

Storing Grades Entered by the User

• We need to know how big to make the array.
  • one way: ask the user for the maximum number of values

```java
public static void main(String[] args) {
    Scanner console = new Scanner(System.in);
    System.out.print("How many grades? ");
    int maxNumGrades = console.nextInt();
    int[] grades = new int[maxNumGrades];
    int total = 0;
    int numGrades = 0;
    while (numGrades < maxNumGrades) {
        System.out.print("Enter a grade (or -1 to quit): ");
        grades[numGrades] = console.nextInt();
        if (grades[numGrades] == -1) {
            break;
        }
        total += grades[numGrades];
        numGrades++;
    }
    ...
}
```
Processing the Values in an Array

• We often use a for loop to process the values in an array.

• Example: print out all of the grades
  ```java
  int[] grades = new int[maxNumGrades];
  ...
  for (int i = 0; i < grades.length; i++) {
    System.out.println("grade "+i+:"+grades[i]);
  }
  ```

• General pattern:
  ```java
  for (int i = 0; i < <array>.length; i++) {
    do something with <array>[i];
  }
  ```

• Processing array elements sequentially from first to last is known as *traversing* the array.
  • noun = *traversal*

Another Example of Traversing an Array

• Let's write code to find the highest quiz grade in the array:
  ```java
  int max = __________________;
  for (_________; _________________; ______) {
    }
  ```
Another Example of Traversing an Array (cont.)

grades array: 7 8 9 6 10 7 9 5

• Let's trace through our code:
  ```java
  int max = grades[0];
  for (int i = 1; i < grades.length; i++) {
    if (grades[i] > max) {
      max = grades[i];
    }
  }
  ```

i grades[i] max
7 8 8
2 9 9
3 6 9
4 10 10
5 7 10
...

Review: What Is a Variable?

• We've seen that a variable is like a named "box" in memory that can be used to store a value.
  ```java
  int count = 10;
  count = 10
  ```

• If a variable represents a primitive-type value, the value is stored in the variable itself, as shown above.
Reference Variables

• If a variable represents an object, the object itself is not stored inside the variable.

• Rather, the object is located somewhere else in memory, and the variable holds the memory address of the object.
  • we say that the variable stores a reference to the object
  • such variables are called reference variables

Arrays and References

• An array is a type of object.

• Thus, an array variable is a reference variable.
  • it stores a reference to the array

• Example:
  
  ```java
  int[] grades = new int[8];
  ```

  might give the following picture:

  ```
  memory location: 2000
  ```

  ```
  grades [2000] 0 0 0 0 0 0 0 0
  ```

• We usually use an arrow to represent a reference:

  ```
  grades [ ] 0 0 0 0 0 0 0 0
  ```
Printing an Array

• What is the output of the following lines?
  ```java
  int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};
  System.out.println(grades);
  ```

• To print the contents of the array, we can use a for loop as we showed earlier.

• We can also use the `Arrays.toString()` method, which is part of Java's built in `Arrays` class.
  ```java
  int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};
  System.out.println(Arrays.toString(grades));
  ```

• doing so produces the following output:
  ```java
  [7, 8, 9, 6, 10, 7, 9, 5]
  ```

• To use this method, we need to import the `java.util` package.

What is the output of the full program?

```java
import java.util.*;
public class FunWithArrays {
    public static void main(String[] args) {
        int[] temps = {51, 50, 36, 29, 30};
        int first = temps[0];
        int numTemps = temps.length;
        int last = temps[numTemps - 1];
        temps[2] = 40;
        temps[3] += 5;
        System.out.println(temps[3]);
        System.out.println(Arrays.toString(temps));
    }
}
```

| temps | first | numTemps | last | output: |
Copying References

• When we assign the value of one reference variable to another, we copy the reference to the object. We do not copy the object itself.

• Example involving objects:

```java
String s1 = "hello, world";
String s2 = s1;
```

```
s1             "hello, world"
               
s2
```

Copying References (cont.)

• An example involving an array:

```java
int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};
int[] other = grades;
```

```
grades             7   8   9   6   10   7   9   5
other
```

• Given the lines of code above, what will the lines below print?

```java
other[2] = 4;
System.out.println(grades[2] + " " + other[2]);
```
Changing the Internals vs. Changing a Variable

• When two variables hold a reference to the same array...

```
int[] list1 = {7, 8, 9};
int[] list2 = list1;
```

- ...if we change the internals of the array, both variables will "see" the change:

```
list2[2] = 4;
System.out.println(Arrays.toString(list1));
```

• ...if we change the variables itself, that does not change the other variable:

```
list2 = new int[3];
System.out.println(Arrays.toString(list1));
```

---

Changing the Internals vs. Changing a Variable (cont.)

• When two variables hold a reference to the same array...

```
int[] list1 = {7, 8, 9};
int[] list2 = list1;
```

- ...if we change one of the variables itself, that does not change the other variable:

```
list2 = new int[3];
System.out.println(Arrays.toString(list1));
```

---

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Null References

• To indicate that a reference variable doesn't yet refer to any object, we can assign it a special value called null.

```java
int[] grades = null;
String s = null;
```

grades null s null

• Attempting to use a null reference to access an object produces a NullPointerException.
  • "pointer" is another name for reference
  • examples:
    ```java
    int[] grades = null;
    String s = null;
    grades[3] = 10;        // NullPointerException!
    char ch = s.charAt(5); // NullPointerException!
    ```

Copying an Array

• To actually create a copy of an array, we can:
  • create a new array of the same length as the first
  • traverse the arrays and copy the individual elements

• Example:
  ```java
  int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};
  int[] other = new int[grades.length];
  for (int i = 0; i < grades.length; i++) {
    other[i] = grades[i];
  }
  ```

• What do the following lines print now?
  ```java
  other[2] = 4;
  System.out.println(grades[2] + " " + other[2]);
  ```
Programming Style Point

• Here's how we copied the array:

```
int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};
int[] other = new int[grades.length];
for (int i = 0; i < grades.length; i++) {
    other[i] = grades[i];
}
```

• This would also work:

```
int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};
int[] other = new int[8];
for (int i = 0; i < 8; i++) {
    other[i] = grades[i];
}
```

• Why is the first way better?

Passing an Array to a Method

• Let's put our code for finding the highest grade into a method:

```java
public class GradeAnalyzer {
    public static _______ maxGrade(int[] grades) {
        int max = grades[0];
        for (int i = 1; i < grades.length; i++) {
            if (grades[i] > max) {
                max = grades[i];
            }
        }
        ____________________;
    }

    public static void main(String[] args) {
        ...
        int maxNumGrades = console.nextInt();
        int[] grades = new int[maxNumGrades];
        ... // code to read in the values
        System.out.println("max grade = " +
                           ____________________);
    }
```
Passing an Array to a Method (cont.)

• What's wrong with this alternative approach?

```java
public class GradeAnalyzer {
    public static int maxGrade(int[] grades) {
        int max = grades[0];
        for (int i = 1; i < grades.length; i++) {
            if (grades[i] > max) {
                max = grades[i];
            }
        }
        return max;
    }
    public static void main(String[] args) {
        ...
        int maxNumGrades = console.nextInt();
        int[] grades = new int[maxNumGrades];
        ... // code to read in the values
        maxGrade(grades);
        System.out.println("max grade = " + max);
    }
}
```

Passing an Array to a Method (cont.)

• We could do this instead:

```java
public class GradeAnalyzer {
    public static int maxGrade(int[] grades) {
        int max = grades[0];
        for (int i = 1; i < grades.length; i++) {
            if (grades[i] > max) {
                max = grades[i];
            }
        }
        return max;
    }
    public static void main(String[] args) {
        ...
        int maxNumGrades = console.nextInt();
        int[] grades = new int[maxNumGrades];
        ... // code to read in the values
        int max = maxGrade(grades);
        System.out.println("max grade = " + max);
    }
}
```
Finding the Average Value in an Array

• Here's a method that computes the average grade:

```java
public static double averageGrade(int[] grades) {
    int total = 0;
    for (int i = 0; i < grades.length; i++) {
        total += grades[i];
    }
    return (double)total / grades.length;
}
```

Testing If An Array Meets Some Condition

• Let's say that we need to be able to determine if there are any grades below a certain cutoff value.
  • e.g., to determine if a retest should be given

• Does this method work?

```java
public static boolean anyGradesBelow(int[] grades, int cutoff) {
    for (int i = 0; i < grades.length; i++) {
        if (grades[i] < cutoff) {
            return true;
        } else {
            return false;
        }
    }
    return false;
}
```
Testing If An Array Meets Some Condition (cont.)

• We can return true as soon as we find a grade that is below the threshold.

• We can only return false if none of the grades is below.

• Here is a corrected version:

```java
public static boolean anyGradesBelow(int[] grades, int cutoff) {
    for (int i = 0; i < grades.length; i++) {
        if (grades[i] < cutoff) {
            return true;
        }
    }
    // if we get here, none of the grades is below.
    return false;
}
```

Testing If An Array Meets Some Condition (cont.)

• Here’s a similar problem: write a method that determines if all of the grades are perfect (assume perfect = 100).

```java
public static boolean allPerfect(int[] grades) {
}
```
Using an Array to Count Things

• Let's say that we want to count how many times each of the possible grade values appears in a collection of grades.

• We can use an array to store the counts.
  • counts[i] will store the number of times that the grade i appears
  • for this grades array

grades: 7 8 9 6 10 7 9 5

we would have this array of counts:

counts: 0 0 0 0 0 1 1 2 1 2 1

• The size of the counts array should be one more than the maximum value being counted:

```java
int max = maxGrade(grades);
int[] counts = new int[max + 1];
```

• Given the array, here's how to do the actual counting:

```java
for (int i = 0; i < grades.length; i++) {
    counts[grades[i]]++;
}
```
Using an Array to Count Things (cont.)

Let's trace through this code for the grades array shown above:

```java
for (int i = 0; i < grades.length; i++) {
    counts[grades[i]]++;
}
```

- `i` represents the index of the current element in the `grades` array.
- `grades[i]` is the current grade.
- The operation performed is to increment the count at the index corresponding to the current grade.

A Method That Returns an Array

We can write a method to create and return the array of counts:

```java
public static int[] getCounts(int[] grades, int maxGrade) {
    int[] counts = new int[maxGrade + 1];
    for (int i = 0; i < grades.length; i++) {
        counts[grades[i]]++;
    }
    return counts;
}
```

```java
public static void main(String[] args) {
    ...
    int max = maxGrade(grades);
    int[] counts = getCounts(grades, max);
    ...
}
```
Using a Method to Change an Array's Contents

```java
public static void main(String[] args) {
    int[] a = {1, 2, 3};
    triple(a);
    System.out.println(Arrays.toString(a));
}

public static void triple(int[] n) {
    for (int i = 0; i < n.length; i++) {
        n[i] = n[i] * 3;
    }
}
```

- When a method is passed an array as a parameter, it gets a copy of the reference, not a copy of the array.
- If the method changes the internals of the array, those changes will be there after the method returns.

Using a Method to Change an Array's Contents (cont.)
Changing the Internals vs. Changing a Variable

```java
public static void main(String[] args) {
    int[] a = {1, 2, 3};
    triple(a);
    System.out.println(Arrays.toString(a));
}

public static void triple(int[] n) {
    for (int i = 0; i < n.length; i++) {
        n[i] = n[i] * 3;  // changes internals
    }
}
```

• If the method changes the **internals** of the array, those changes will be there after the method returns.

![Diagram showing changes in `a` array]

### Changing the Internals vs. Changing a Variable (cont.)

```java
public static void main(String[] args) {
    int[] a = {1, 2, 3};
    triple(a);
    System.out.println(Arrays.toString(a));
}

public static void method2(int[] n) {
    n = new int[3];  // changes the variable
}
```

• However, if the method changes its **variable** for the array, that change does **not** affect the original array.

• Changing what's in one variable doesn't affect any other variable!

![Diagram showing changes in `n` array]
Swapping Elements in an Array

- We sometimes need to be able to swap two elements in an array.

- Example:

```c
arr[2] = arr[5];
arr[5] = arr[2];
```

- What's wrong with this code for swapping the two values?

```c
arr[2] = arr[5];
arr[5] = arr[2];
```

- it gives this:

```c
35 6 19 23 3 47 9 15
```

Swapping Elements in an Array (cont.)

- To perform a swap, we need to use a temporary variable:

```c
int temp = arr[2];
arr[2] = arr[5];
arr[5] = temp;
```

```c
35 6 19 23 3 47 9 15
```

```c
35 6 47 23 3 19 9 15
```

CSCI S-111

David G. Sullivan, Ph.D.
A Method for Swapping Elements

• Here's a method for swapping the elements at positions \(i\) and \(j\) in the array \(arr\):

```java
public static void swap(int[] arr, int i, int j) {
    int temp = arr[i];
    arr[i] = arr[j];
    arr[j] = temp;
}
```

• We don't need to return anything, because the method changes the internals of the array that is passed in.

• Here's an example of how we would use it:

```java
int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};
swap(grades, 2, 5);
System.out.println(Arrays.toString(grades));
```

• What would the output be?

---

Recall: A Method That Returns an Array

• We can write a method to create and return the array of counts:

```java
public static int[] getCounts(int[] grades, int maxGrade) {
    int[] counts = new int[maxGrade + 1];
    for (int i = 0; i < grades.length; i++) {
        counts[grades[i]]++;
    }
    return counts;
}
```

```java
public static void main(String[] args) {
    ... // main method begins as in the earlier versions
    int max = maxGrade(grades);
    int[] counts = getCounts(grades, max);
    ...
}
```
An Alternative Approach for the Array of Counts

• Create the array ahead of time and pass it into the method:

```java
public static void getCounts(int[] grades, int[] counts) {
    for (int i = 0; i < grades.length; i++) {
        counts[grades[i]]++;
    }
}
```

```java
public static void main(String[] args) {
    // main method begins as in the earlier versions
    int max = maxGrade(grades);
    int[] counts = new int[max];
    getCounts(grades, counts);
    ...
}
```

• Because the method changes the internals of the array, those changes will be there after the method returns.

Shifting Values in an Array

• Let's say a small business is using an array to store the number of items sold over a 10-day period.

```plaintext
numSold: 15 8 19 2 5 8 11 18 7 16
```

- `numSold[0]` gives the number of items sold today
- `numSold[1]` gives the number of items sold 1 day ago
- `numSold[2]` gives the number of items sold 2 days ago
- ...
- `numSold[9]` gives the number of items sold 9 days ago
Shifting Values in an Array (cont.)

• At the start of each day, it's necessary to shift the values over to make room for the new day's sales.

  numSold ← 15 8 19 2 5 8 11 18 7 16
  numSold ← 0 15 8 19 2 5 8 11 18 7

• the last value is lost, since it's now 10 days old

• In order to shift the values over, we need to perform assignments like the following:

  numSold[9] = numSold[8];
  numSold[6] = numSold[5];
  numSold[2] = numSold[1];

• what is the general form (the pattern) of these assignments?

Shifting Values in an Array (cont.)

• Here's one attempt at code for shifting all of the elements:

  for (int i = 0; i < numSold.length; i++) {
    numSold[i] = numSold[i - 1];
  }

• If we run this, we get an ArrayIndexOutOfBoundsException. Why?
Shifting Values in an Array (cont.)

• This version of the code eliminates the exception:
  ```java
  for (int i = 1; i < numSold.length; i++) {
    numSold[i] = numSold[i - 1];
  }
  ```

• Let’s trace it to see what it does:

  - when `i == 1`, we perform `numSold[1] = numSold[0]` to get:
    
    ![Array with values 15 15 19 2 5 8 11 18 7 16]

  - when `i == 2`, we perform `numSold[2] = numSold[1]` to get:
    
    ![Array with values 15 15 15 25 8 1 1 1 8 7 1 6]

  this obviously doesn’t work!

Shifting Values in an Array (cont.)

• How can we fix this code so that it does the right thing?
  ```java
  for (int i = 1; i < numSold.length; i++) {
    numSold[i] = numSold[i - 1];
  }
  ```

  - After performing all of the shifts, we would do: `numSold[0] = 0;`

    ![Array with values 0 15 8 19 2 5 8 11 18 7]
"Growing" an Array

- Once we have created an array, we can't increase its size.

- Instead, we need to do the following:
  - create a new, larger array (use a temporary variable)
  - copy the contents of the original array into the new array
  - assign the new array to the original array variable

- Example for our grades array:

  ```java
  int[] grades = {7, 8, 9, 6, 10, 7, 9, 5};
  ...
  int[] temp = new int[16];
  for (int i = 0; i < grades.length; i++) {
      temp[i] = grades[i];
  }
  grades = temp;
  ```

Arrays of Objects

- We can use an array to represent a collection of objects.

- In such cases, the cells of the array store references to the objects.

- Example:

  ```java
  String[] suitNames = {"clubs", "spades", "hearts", "diamonds"};
  ```
Two-Dimensional Arrays

- Thus far, we've been looking at single-dimensional arrays.
- We can also create multi-dimensional arrays.
- The most common type is a two-dimensional (2-D) array.
- We can visualize it as a matrix consisting of rows and columns:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
<td>8</td>
<td>3</td>
<td>16</td>
<td>12</td>
<td>7</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>11</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>3</td>
<td>5</td>
<td>18</td>
<td>10</td>
<td>6</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>14</td>
<td>13</td>
<td>6</td>
<td>13</td>
<td>12</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>9</td>
<td>5</td>
<td>16</td>
<td>20</td>
<td>2</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

columns: row, column indices

2-D Array Basics

- Example of declaring and creating a 2-D array:
  ```java
  int[][] scores = new int[5][8];
  ```
- To access an element, we use an expression of the form
  `<array>[<row>][<column>]`
  - example: `scores[3][4]` gives the score at row 3, column 4
Example Application: Maintaining a Game Board

- For a Tic-Tac-Toe board, we could use a 2-D array to keep track of the state of the board:
  
  ```java
  char[][] board = new char[3][3];
  ```

- Alternatively, we could create and initialize it as follows:
  
  ```java
  char[][] board = {{' ', ' ', ' '},
                   {' ', ' ', ' '},
                   {' ', ' ', ' '}};
  ```

- If a player puts an X in the middle square, we could record this fact by making the following assignment:
  
  ```java
  board[1][1] = 'X';
  ```

An Array of Arrays

- A 2-D array is really an array of arrays!

- `scores[0]` represents the entire first row.
- `scores[1]` represents the entire second row, etc.

- `<array>.length` gives the number of rows.
- `<array>[<row>].length` gives the number of columns in that row.
Processing All of the Elements in a 2-D Array

- To perform some operation on all of the elements in a 2-D array, we typically use a nested loop.
- Example: finding the maximum value in a 2-D array.

```java
public static int maxValue(int[][] arr) {
    int max = arr[0][0];
    for (int r = 0; r < arr.length; r++) {
        for (int c = 0; c < arr[r].length; c++) {
            if (arr[r][c] > max) {
                max = arr[r][c];
            }
        }
    }
    return max;
}
```

Optional: Other Multi-Dimensional Arrays

- It's possible to have a "ragged" 2-D array in which different rows have different numbers of columns:

```java
int[][] foo = {{11, 22, 33},
              {7, 20, 30, 40},
              {1, 2}};
```

- We can also create arrays of higher dimensions.
- Example: a three-dimensional matrix:

```java
double[][][] matrix = new double[2][5][4];
```
A Class for Representing a File

- The `File` class in Java is used to represent a file on disk.
- To use it, we need to import the `java.io` package:
  ```java
  import java.io.*;
  ```
- Here's how we typically create a `File` object:
  ```java
  File f = new File("<filename>");
  ```
- Here are some useful methods from this class:
  ```java
  public boolean exists()
  public boolean canRead()
  public boolean canWrite()
  public boolean delete()
  public long length()
  public String getName()
  public String getPath()
  ```

See the Java API documentation for more info.
Review: Scanner Objects

- We've been using a Scanner object to read from the console:
  ```java
  Scanner console = new Scanner(System.in);
  ```
  tells the constructor to construct a Scanner object that reads from the console

- Scanner methods:
  ```java
  next()
  nextInt()
  nextDouble()
  nextLine()
  ```

Reading from a Text File

- We can also use a Scanner object to read from a text file:
  ```java
  File f = new File("<filename>");
  Scanner input = new Scanner(f);
  ```
  tells the constructor to construct a Scanner object that reads from the file

- We can combine the two lines above into a single line:
  ```java
  Scanner input = new Scanner(new File("<filename>"));
  ```

- We use a different name for the Scanner (input), to stress that we’re reading from an input file.

- All of the same Scanner methods can be used.
Scanner Lookahead and Files

- When reading a file, we often don't know how big the file is.
- Solution: use an indefinite loop and a scanner "lookahead" method.
- Basic structure:
  ```java
  Scanner input = new Scanner(new File(<filename>));
  while (input.hasNextLine()) {
      String line = input.nextLine();
      // code to process the line goes here...
  }
  ```
- hasNextLine() returns:
  - true if there's at least one more line of the file to be read
  - false if we've reached the end of the file

Sample Problem: Printing the Contents of a File

- Assume that we've already created a scanner called input that is connected to a file.
- Here's the code for printing its contents:
  ```java
  while (input.hasNextLine()) {
      String line = input.nextLine();
      System.out.println(line);
  }
  ```
File-Processing Exceptions

- Recall: An exception is an error that occurs at runtime as a result of some type of "exceptional" circumstance.

- We've seen several examples:
  - StringIndexOutOfBoundsException
  - IllegalArgumentException
  - TypeMismatchException

- When using a Scanner to process a file, we can get a 
  - FileNotFoundException
  - if the file that we specify isn't there
  - if the file is inaccessible for some reason

Checked vs. Unchecked Exceptions

- Most of the exceptions we've seen thus far have been unchecked exceptions.
  - we do not need to handle them
  - instead, we usually take steps to avoid them

- FileNotFoundException is a checked exception. The compiler checks that we either:
  1) handle it
  2) declare that we don't handle it

- For now, we'll take option 2. We do this by adding a
  - throws clause to the header of any method in which a Scanner for a file is created:

    ```java
    public static void main(String[] args) throws FileNotFoundException {
    ```
Sample Program: Counting the Lines in a File

```java
import java.util.*;  // needed for Scanner
import java.io.*;    // needed for File

class CountLines {
    public static void main(String[] args)
        throws FileNotFoundException {
        Scanner input = new Scanner(new File("romeo.txt"));
        int count = 0;
        while (input.hasNextLine()) {
            input.nextLine();  // read line and throw away
            count++;
        }
        System.out.println("The file has " + count + " lines.");
    }
}
```

Counting Lines in a File, version 2

```java
import java.util.*;  // needed for Scanner
import java.io.*;    // needed for File

class CountLines {
    public static void main(String[] args)
        throws FileNotFoundException {
        Scanner console = new Scanner(System.in);
        System.out.print("Name of file: ");
        String fileName = console.next();

        Scanner input = new Scanner(new File(fileName));
        int count = 0;
        while (input.hasNextLine()) {
            input.nextLine();  // read line and throw away
            count++;
        }
        System.out.println("The file has " + count + " lines.");
    }
}
```
public static void main(String[] args)
    throws FileNotFoundException {
    Scanner console = new Scanner(System.in);
    System.out.print("Name of file: ");
    String fileName = console.next();
    System.out.println("The file has "+
        numLines(fileName) + " lines.");
}

public static int numLines(String fileName)
    throws FileNotFoundException {
    Scanner input = new Scanner(new File(fileName));
    int count = 0;
    while (input.hasNextLine()) {
        input.nextLine(); // read line and throw away
        count++;
    }
    return count;
}

• We put the counting code in a separate method (numLines).
• Both numLines and main need a throws clause.

Extracting Data from a File

• Collections of data are often stored in a text file.

• Example: the results of a track meet might be summarized in a text file that looks like this:

    Mike Mercury, BU, mile, 4:50:00
    Steve Slug, BC, mile, 7:30:00
    Fran Flash, BU, 800m, 2:15:00
    Tammy Turtle, UMass, 800m, 4:00:00

• Each line of the file represents a record.

• Each record is made up of multiple fields.

• In this case, the fields are separated by commas.
  • known as a CSV file – comma separated values
  • the commas serve as delimiters
  • could also use spaces or tabs (\"\t\") instead of commas
Extracting Data from a File (cont.)

Mike Mercury, BU, mile, 4:50:00
Steve Slug, BC, mile, 7:30:00
Fran Flash, BU, 800m, 2:15:00
Tammy Turtle, UMass, 800m, 4:00:00

• We want a program that:
  • reads in a results file like the one above
  • extracts and prints only the results for a particular school
    • with the name of the school omitted

• Basic approach:
  • ask the user for the school of interest (the target school)
  • read one line at a time from the file
  • split the line into fields
  • if the field corresponding to the school name matches
    the target school, print out the other fields in that record

Splitting a String

• The String class includes a method named split().
  • breaks a string into component strings
  • takes a parameter indicating what delimiter should be
    used when performing the split
  • returns a String array containing the components

• Example:
  > String sentence = "How now brown cow?";
  > String[] words = sentence.split(" ");
  > words[0]
    "How"
  > words[1]
    "now"
  > words[3]
    "cow?"
  > words.length
    4
Extracting Data from a File (cont.)

import java.util.*;  // needed for Scanner
import java.io.*;    // needed for File
public class ExtractResults {
    public static void main(String[] args)
        throws FileNotFoundException {
        Scanner console = new Scanner(System.in);
        System.out.print("School to extract: ");
        String targetSchool = console.nextLine();
        Scanner input = new Scanner(new File("results.txt");
        while (input.hasNextLine()) {
            String record = input.nextLine();
            String[] fields = record.split(",");
            if (fields[1].equals(targetSchool)) {
                System.out.print(fields[0] + ",");
            }
        }
    }
}

• How can we modify it to print a message when no results are found for the target school?

Example Problem: Averaging Enrollments

• Let's say that we have a file showing how course enrollments have changed over time:

    cs111 90 100 120 115 140 170 130 135 125
    cs105 14 8
    cs108 40 35 30 42 38 26
    cs101 180 200 175 190 200 230 160 154 120

• For each course, we want to compute the average enrollment.
  • different courses have different numbers of values

• Initial pseudocode:

    while (there is another course in the file) {
        read the line corresponding to the course
        split it into an array of fields
        average the fields for the enrollments
        print the course name and average enrollment
    }
Example Problem: Averaging Enrollments (cont.)

```
cs108 40 35 30 42 38 26
cs111 90 100 120 115 140 170 130 135 125
cs105 14 8
cs101 180 200 175 190 200 230 160 154 120
```

- When we split a line into fields, we get an array of strings.
  - example for the first line above:
    `{"cs108", "40", "35", "30", "42", "38", "26"}`

- We can convert the enrollments from strings to integers using a method called `Integer.parseInt()`
  - example:
    ```java
    String[] fields = record.split(" ");
    String courseName = fields[0];
    int firstEnrollment = Integer.parseInt(fields[1]);
    ```
  - note: `parseInt()` is a static method, so we call it using its class name (`Integer`)
Other Details About Reading Text Files

- Although we think of a text file as being two-dimensional (like a piece of paper), the computer treats it as a one-dimensional string of characters.
  - example: the file containing these lines
    
    Hello, world.
    How are you?
    I'm tired.
  
  is represented like this:
  
  Hello, world.
  How are you?
  I'm tired.

- When reading a file using a scanner, you are limited to sequential accesses in the forward direction.
  - you can't back up
  - you can't jump to an arbitrary location
  - to go back to the beginning of the file, you need to create a new scanner object.

Optional Extra Topic: Writing to a Text File

- To write to a text file, we can use a PrintStream object, which has the same methods that we've used with System.out:
  - print(), println()

- Actually, System.out is a PrintStream that has been constructed to print to the console.

- To instantiate a PrintStream for a file:
  ```java
  File f = new File("<filename>");
  PrintStream output = new PrintStream(f);
  ```

- We can also combine these two steps:
  ```java
  PrintStream output = new PrintStream(
    new File("<filename>"));
  ```

- If there’s an existing file with the same name, it will be overwritten.
Copying a Text File

import java.util.*;  // needed for Scanner
import java.io.*;    // needed for File
public class CopyFile {
    public static void main(String[] args)
        throws FileNotFoundException {
        Scanner console = new Scanner(System.in);
        System.out.print("Name of original file: ");
        String original = console.next();
        System.out.print("Name of copy: ");
        String copy = console.next();

        Scanner input = new Scanner(new File(original));
        PrintStream output = new PrintStream(new File(copy));
        while (input.hasNextLine()) {
            String line = input.nextLine();
            output.println(line);
        }
    }
}

• How could we combine the two lines in the body of the while loop?

Our Track-Meet Program Revisited

import java.util.*;  // needed for Scanner
import java.io.*;    // needed for File
public class ExtractResults {
    public static void main(String[] args)
        throws FileNotFoundException {
        Scanner console = new Scanner(System.in);
        System.out.print("School to extract: ");
        String targetSchool = console.nextLine();
        Scanner input = new Scanner(new File("results.txt"));
        while (input.hasNextLine()) {
            String record = input.nextLine();
            String[] fields = record.split(",");
            if (fields[1].equals(targetSchool)) {
                System.out.print(fields[0] + ",");
            }
        }
    }
}

• How can we modify it to print the extracted results to a separate file?
Optional Extra Topic: Binary Files

• Not all files are text files.

• Binary files don't store the string representation of non-string values.
  • instead, they store their binary representation – the way they are stored in memory

• Example: 125
  • the text representation of 125 stores the string "125" – i.e., the characters for the individual digits in the number
    
    | '1' | '2' | '5' |
    |-----|-----|-----|
    | 49  | 50  | 53  |

  • the binary representation of 125 stores the four-byte binary representation of the integer 125
    
    | 0   | 0   | 0   | 125 |
    |-----|-----|-----|-----|
Recursion

Review: Method Frames

- When you make a method call, the Java runtime sets aside a block of memory known as the *frame* of that method call.

```
main
  number    otherNumber
```

- The frame is used to store:
  - the formal parameters of the method
  - any *local variables* - variables declared within the method

- A given frame can only be accessed by statements that are part of the corresponding method call.
Frames and the Stack

- The frames we've been speaking about are stored in a region of memory known as the stack.

- For each method call, a new frame is added to the top of the stack.

```java
public class Foo {
    public static int y(int i) {
        int j = i * 3;
        return j;
    }
    public static int x(int i) {
        int j = i - 2;
        return y(i + j);
    }
    public static void main(String[] args) {
        System.out.println(x(5));
    }
}
```

- When a method completes, its stack frame is removed.

Iteration

- Whenever we've encountered a problem that requires repetition, we've used iteration - i.e., some type of loop.

- Sample problem: printing the series of integers from n1 to n2, where n1 <= n2.
  - example: printSeries(5, 10) should print the following:
    5, 6, 7, 8, 9, 10

- Here's an iterative solution to this problem:

```java
public static void printSeries(int n1, int n2) {
    for (int i = n1; i < n2; i++) {
        System.out.print(i + " ");
    }
    System.out.println(n2);
}
```
Recursion

- An alternative approach to problems that require repetition is to solve them using a method that calls itself.

- Applying this approach to the print-series problem gives:
  
  ```java
  public static void printSeries(int n1, int n2) {
      if (n1 == n2) {
          System.out.println(n2);
      } else {
          System.out.print(n1 + "", "");
          printSeries(n1 + 1, n2);
      }
  }
  ```

- A method that calls itself is a recursive method.

- This approach to problem-solving is known as recursion.

Tracing a Recursive Method

```java
public static void printSeries(int n1, int n2) {
    if (n1 == n2) {
        System.out.println(n2);
    } else {
        System.out.print(n1 + "", "");
        printSeries(n1 + 1, n2);
    }
}
```

- What happens when we execute `printSeries(5, 7)`?

  ```java
  printSeries(5, 7):
      System.out.print(5 + ", ");
  printSeries(6, 7):
      System.out.print(6 + ", ");
  printSeries(7, 7):
      System.out.println(7);
  return
  return
  return
  ```
Recursive Problem-Solving

• When we use recursion, we solve a problem by reducing it to a simpler problem of the same kind.

• We keep doing this until we reach a problem that is simple enough to be solved directly.

• This simplest problem is known as the base case.

```java
public static void printSeries(int n1, int n2) {
    if (n1 == n2) {           // base case
        System.out.println(n2);
    } else {
        System.out.print(n1 + ", ");
        printSeries(n1 + 1, n2);
    }
}
```

• The base case stops the recursion, because it doesn't make another call to the method.

Recursive Problem-Solving (cont.)

• If the base case hasn't been reached, we execute the recursive case.

```java
public static void printSeries(int n1, int n2) {
    if (n1 == n2) {           // base case
        System.out.println(n2);
    } else {                  // recursive case
        System.out.print(n1 + ", ");
        printSeries(n1 + 1, n2);
    }
}
```

• The recursive case:
  • reduces the overall problem to one or more simpler problems of the same kind
  • makes recursive calls to solve the simpler problems
Structure of a Recursive Method

```java
recursiveMethod(parameters) {
    if (stopping condition) {
        // handle the base case
    } else {
        // recursive case:
        // possibly do something here
        recursiveMethod(modified parameters);
        // possibly do something here
    }
}
```

- There can be multiple base cases and recursive cases.
- When we make the recursive call, we typically use parameters that bring us closer to a base case.

Tracing a Recursive Method: Second Example

```java
public static void mystery(int i) {
    if (i <= 0) {     // base case
        return;
    }
    // recursive case
    System.out.println(i);
    mystery(i - 1);
    System.out.println(i);
}
```

- What happens when we execute `mystery(2)`?
Printing a File to the Console

• Here's a method that prints a file using iteration:

```java
public static void print(Scanner input) {
    while (input.hasNextLine()) {
        System.out.println(input.nextLine());
    }
}
```

• Here's a method that uses recursion to do the same thing:

```java
public static void printRecursive(Scanner input) {
    // base case
    if (!input.hasNextLine()) {
        return;
    }
    // recursive case
    System.out.println(input.nextLine());
    printRecursive(input);  // print the rest
}
```

Printing a File in Reverse Order

• What if we want to print the lines of a file in reverse order?

• It's not easy to do this using iteration. Why not?

• It's easy to do it using recursion!

• How could we modify our previous method to make it print the lines in reverse order?

```java
public static void printRecursive(Scanner input) {
    if (!input.hasNextLine()) {  // base case
        return;
    }
    String line = input.nextLine();
    System.out.println(line);
    printRecursive(input);  // print the rest
}
```
Printing a File in Reverse Order (cont.)

• An iterative approach to reversing the file would need to:
  • read all of the lines in the file and store them in a temporary data structure (e.g., an array)
  • retrieve the lines from the data structure and print them in reverse order

• The recursive method doesn't need a separate data structure.
  • the lines are stored in the stack frames for the recursive method calls!

A Recursive Method That Returns a Value

• Simple example: summing the integers from 1 to n
  public static int sum(int n) {
    if (n <= 0) {
      return 0;
    }
    int rest = sum(n - 1);
    return n + rest;
  }

• Example of this approach to computing the sum:
  sum(6) = 6 + sum(5)
  = 6 + 5 + sum(4)
  ...

Tracing a Recursive Method

```java
public static int sum(int n) {
    if (n <= 0) {
        return 0;
    }
    int rest = sum(n - 1);
    return n + rest;
}
```

- What happens when we execute `int x = sum(3);` from inside the `main()` method?

---

**Tracing a Recursive Method on the Stack**

```java
public static int sum(int n) {
    if (n <= 0) {
        return 0;
    }
    int rest = sum(n - 1);
    return n + rest;
}
```

**Example: sum(3)**

- `sum(3)`
  - `n 3`
  - `rest 3`
  - `base case`  
  - `n 2`
  - `rest 2`
  - `return 1 + 0`
  - `n 1`
  - `rest 1`
  - `return 2 + 1`
  - `n 0`
  - `rest 0`
  - `return 3 + 3`

The final result gets built up on the way back from the base case!
Another Option for Tracing a Recursive Method

```java
public static int sum(int n) {
    if (n <= 0) {
        return 0;
    }
    int rest = sum(n - 1);
    return n + rest;
}
```

Infinite Recursion

- We have to ensure that a recursive method will eventually reach a base case, regardless of the initial input.

- Otherwise, we can get infinite recursion.
  - produces stack overflow - there's no room for more frames on the stack!

- Example: here's a version of our `sum()` method that uses a different test for the base case:

```java
public static int sum(int n) {
    if (n == 0) {
        return 0;
    }
    int rest = sum(n - 1);
    return n + rest;
}
```

- what values of n would cause infinite recursion?
Designing a Recursive Method

1. Start by programming the base case(s).
   - What instance(s) of this problem can I solve directly (without looking at anything smaller)?

2. Find the recursive substructure.
   - How could I use the solution to any smaller version of the problem to solve the overall problem?

3. Solve the smaller problem using a recursive call!
   - store its result in a variable

4. Do your one step.
   - build your solution from the result of the recursive call
   - use concrete cases to figure out what you need to do

Processing a String Recursively

• A string is a recursive data structure. It is either:
  • empty ("")
  • a single character, followed by a string

• Thus, we can easily use recursion to process a string.
  • process one or two of the characters ourselves
  • make a recursive call to process the rest of the string

• Example: print a string vertically, one character per line:
  
  public static void printVertical(String str) {
    if (str == null || str.equals("")) {
      return;
    }

    System.out.println(str.charAt(0)); // first char
    printVertical(str.substring(1)); // rest of string
  }
Short-Circuited Evaluation

- The second operand of both the && and || operators will not be evaluated if the result can be determined on the basis of the first operand alone.

- $\text{expr1} || \text{expr2}$
  - if $\text{expr1}$ evaluates to true, $\text{expr2}$ is not evaluated, because we already know that $\text{expr1} || \text{expr2}$ is true
  - example from the last slide:
    ```java
    if (str == null || str.equals("")) {
        return;
    }
    // if str is null, we won't check for empty string. // This prevents a null pointer exception!
    ```

- $\text{expr1} && \text{expr2}$
  - if $\text{expr1}$ evaluates to ______, $\text{expr2}$ is not evaluated, because we already know that $\text{expr1} && \text{expr2}$ is ______.

Counting Occurrences of a Character in a String

- $\text{numOccur}(\text{c}, \text{s})$ should return the number of times that the character $\text{c}$ appears in the string $\text{s}$
  - $\text{numOccur}(\text{'n'}, \text{"banana"})$ should return 2
  - $\text{numOccur}(\text{'a'}, \text{"banana"})$ should return 3

- Take the approach outlined earlier:
  - base case: empty string (or null)
  - delegate $\text{s.substring(1)}$ to the recursive call
  - we're responsible for handling $\text{s.charAt(0)}$
Applying the String-Processing Template

public static int numOccur(char c, String s) {
    if (s == null || s.equals("")) { // base case
        return __________;
    } else { // recursive case
        int rest = ________________;
        // do our one step!
    }
}

Determining Our One Step

public static int numOccur(char c, String s) {
    if (s == null || s.equals("")) {
        return 0;
    } else {
        int rest = numOccur(c, s.substring(1));
        // do our one step!
        
        • In our one step, we take care of s.charAt(0).
        • we build the solution to the larger problem on the
          solution to the smaller problem (in this case, rest)
        • does what we do depend on the value of s.charAt(0)?

        • Use concrete cases to figure out the logic!
    }
}

Consider this concrete case...

public static int numOccur(char c, String s) {
    if (s == null || s.equals("")) {
        return 0;
    } else {
        int rest = numOccur(c, s.substring(1));
        // do our one step!
        ...
        numOccur('r', "recurse")

text: numOccur('r', "recurse")
c = 'r', s = "recurse"
Consider Concrete Cases

numOccur('r', "recurse") # first char is a match
• what is its solution?
• what is the next smaller subproblem?
• what is the solution to that subproblem?
• how can we use the solution to the subproblem?
   What is our one step?

numOccur('a', "banana") # first char is not a match
• what is its solution?
• what is the next smaller subproblem?
• what is the solution to that subproblem?
• how can we use the solution to the subproblem?
   What is our one step?

Now complete the method!

public static int numOccur(char c, String s) {
    if (s == null || s.equals("")) {
        return 0;
    } else {
        int rest = numOccur(c, s.substring(1));
        if (s.charAt(0) == c) {
            return __________________;
        } else {
            return __________________;
        }
    }
}

**Tracing a Recursive Method on the Stack**

public static int numOccur(char c, String s) {
    if (s == null || s.equals("") {  
        return 0;
    } else {  
        int rest = numOccur(c, s.substring(1));  
        if (s.charAt(0) == c) {  
            return 1 + rest;  
        } else {  
            return rest;  
        }
    }
}

numOccur('a', "aha")

The final result gets built up on the way back from the base case!

**Common Mistake**

- This version of the method does not work:

```java
public static int numOccur(char c, String s) {
    if (s == null || s.equals("")) {  
        return 0;
    }
    int count = 0;  
    if (s.charAt(0) == c) {  
        count++;  
    }
    numOccur(c, s.substring(1));  
    return count;
}
```
Another Faulty Approach

• Some people make count "global" to fix the prior version:
  ```java
  public static int count = 0;
  public static int numOccur(char c, String s) {
    if (s == null || s.equals("") ) {
      return 0;
    }
    if (s.charAt(0) == c) {
      count++;
    }
    numOccur(c, s.substring(1));
    return count;
  }
  ```

• Not recommended, and not allowed on the problem sets!

• Problems with this approach?

Recursion vs. Iteration

• Some problems are much easier to solve using recursion.

• Other problems are just as easy to solve using iteration.

• Recursion is a bit more costly because of the overhead involved in invoking a method.
  • also: in some cases, there may not be room on the stack

• Rule of thumb:
  • if it's easier to formulate a solution recursively, use recursion, unless the cost of doing so is too high
  • otherwise, use iteration
Extra Practice: A Recursive Palindrome Checker

- A palindrome is a string that reads the same forward and backward.
  - examples: "radar", "mom", "abcddcba"

- `isPal(s)` should return true if s is a palindrome, and false otherwise.

- We need more than one base case. What are they?

- How should we reduce the problem in the recursive call?

Consider Concrete Cases!

`isPal("radar")`
- what is its solution?
- what is the next smaller subproblem?
- what is the solution to that subproblem?
- how can we use the solution to the subproblem...?  
  *What is our one step?*

`isPal("modem")`
- what is its solution?
- what is the next smaller subproblem?
- what is the solution to that subproblem?
- how can we use the solution to the subproblem...?  
  *What is our one step?*
A Recursive Palindrome Checker (cont.)

- Method definition (assuming no nulls):

```java
public static boolean isPal(String s) {
    int len = s.length();
    if (len <= 1) {
        return __________;
    } else if (_________________________________) {
        return __________;
    } else {
        boolean isPalRest = ______________________;
        // do our one step!
    }
}
```
Classes as Blueprints: How to Define New Types of Objects

Computer Science S-111
Harvard University
David G. Sullivan, Ph.D.

Types of Decomposition

- When writing a program, it's important to decompose it into manageable pieces.
- We've already seen how to use procedural decomposition.
  - break a task into smaller subtasks, each of which gets its own method
- Another way to decompose a program is to view it as a collection of objects.
  - referred to as object-oriented programming
Review: What is an Object?

- An object groups together:
  - one or more data values (the object's fields)
  - a set of operations that the object can perform (the object's methods)

Review: Using an Object's Methods

- An object's methods are different from the static methods that we've been writing thus far.
  - they're called non-static or instance methods

- When using an instance method, we specify the object to which the method belongs by using dot notation:

  ```java
  String firstName = "Perry";
  int len = firstName.length();
  ```

- Using an instance method is like sending a message to an object, asking it to perform an operation.

- We refer to the object on which the method is invoked as either:
  - the called object
  - the current object
Review: Classes as Blueprints

• We've been using classes as containers for our programs.

• A class can also serve as a blueprint – as the definition of a new type of object.
  • specifying the fields and methods that objects of that type will have

• The objects of a given class are built according to its blueprint.

• Objects of a class are referred to as instances of the class.

Rectangle Objects

• Java comes with a built-in Rectangle class.
  • in the java.awt package

• Each Rectangle object has the following fields:
  • x – the x coordinate of its upper left corner
  • y – the y coordinate of its upper left corner
  • width
  • height

• Here's an example of one:

<table>
<thead>
<tr>
<th></th>
<th>x 200</th>
<th>y 150</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>height</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>
**Rectangle Methods**

- A Rectangle's methods include:
  
  ```java
  void grow(int h, int v)
  void translate(int x, int y)
  double getWidth()
  double getHeight()
  double getX()
  double getY()
  ```

**Writing a "Blueprint Class"**

- To illustrate how to define a new type of object, let's write our own class for Rectangle objects.
  ```java
  public class Rectangle {
      ...
  }
  ```

- As always, the class definition goes in an appropriately named text file.
  - in this case: `Rectangle.java`
Using Fields to Capture an Object's State

• Here's the first version of our Rectangle class:

```java
public class Rectangle {
    int x;
    int y;
    int width;
    int height;
}
```

• it declares four fields, each of which stores an int
• each Rectangle object gets its own set of these fields

• Another name for a field is an *instance variable*.

Using Fields to Capture an Object's State (cont.)

• For now, we'll create Rectangle objects like this:

```java
Rectangle r1 = new Rectangle();
```

• The fields are initially filled with the default values for their types.
  • just like array elements

• Fields can be accessed using dot notation:

```java
r1.x = 10;
r1.y = 20;
r1.width = 100;
r1.height = 50;
```
Client Programs

- Our Rectangle class is not a program.
  - it has no main method

- Instead, it will be used by code defined in other classes.
  - referred to as client programs or client code

- More generally, when we define a new type of object, we create a building block that can be used in other code.
  - just like the objects from the built-in classes:
    - String, Scanner, File, etc.
  - our programs have been clients of those classes

```
Initial Client Program

public class RectangleClient {
    public static void main(String[] args) {
        Rectangle r1 = new Rectangle();
        r1.x = 10;       r1.y = 20;
        r1.width = 100;  r1.height = 50;
        Rectangle r2 = new Rectangle();
        r2.x = 50;       r2.y = 100;
        r2.width = 20;   r2.height = 80;
        System.out.println("r1: "+ r1.width + "x" + r1.height);
        int area1 = r1.width * r1.height;
        System.out.println("area = " + area1);
        System.out.println("r2: "+ r2.width + "x" + r2.height);
        int area2 = r2.width * r2.height;
        System.out.println("area = " + area2);
        // grow both rectangles
        r1.width += 50;  r1.height += 10;
        r2.width += 5;   r2.height += 30;
        System.out.println("r1: "+ r1.width + "x" + r1.height);
        System.out.println("r2: "+ r2.width + "x" + r2.height);
    }
}
```
Using Methods to Capture an Object's Behavior

- It would be useful to have a method for growing a Rectangle.

- One option would be to define a static method:

```java
public static void grow(Rectangle r, int dWidth, int dHeight) {
    r.width += dWidth;
    r.height += dHeight;
}
```

- This would allow us to replace the statements

```java
r1.width += 50;
r1.height += 10;
```

with the method call

```java
Rectangle.grow(r1, 50, 10);
```

(Note: We need to use the class name, because we're calling the method from outside the Rectangle class.)
Using Methods to Capture an Object's Behavior (cont.)

• A better approach is to give each Rectangle object the ability to grow itself.

• We do so by defining a non-static or instance method.

• We'll use dot notation to call the instance method:

  ```java
  r1.grow(50, 10);
  instead of Rectangle.grow(r1, 50, 10);
  ```

• This is like sending a message to r1, asking it to grow itself.

Using Methods to Capture an Object's Behavior (cont.)

• Here's our grow instance method:

  ```java
  public void grow(int dWidth, int dHeight) { // no static
    this.width += dWidth;
    this.height += dHeight;
  }
  ```

• We don't pass the Rectangle object as an explicit parameter.

• Instead, the Java keyword this gives us access to the called object.
  • every instance method has this special variable
  • referred to as the implicit parameter

• Example: r1.grow(50, 10)
  • r1 is the called object
  • this.width gives us access to r1's width field
  • this.height gives us access to r1's height field
Comparing the Static and Non-Static Versions

- Static:
  ```java
  public static void grow(Rectangle r, int dWidth, int dHeight) {
    r.width += dWidth;
    r.height += dHeight;
  }
  ```
  - sample method call: Rectangle.grow(r1, 50, 10);

- Non-static:
  ```java
  public void grow(int dWidth, int dHeight) {
    this.width += dWidth;
    this.height += dHeight;
  }
  ```
  - there's no keyword static in the method header
  - the Rectangle object is not an explicit parameter
  - the implicit parameter this gives access to the object
  - sample method call: r1.grow(50, 10);

Omitting the Keyword this

- The use of this to access the fields is optional.
  - example:
    ```java
    public void grow(int dWidth, int dHeight) {
      width += dWidth;
      height += dHeight;
    }
    ```
Another Example of an Instance Method

- Here's an instance method for getting the area of a Rectangle:
  ```java
  public int area() {
      return this.width * this.height;
  }
  ```

- Sample method calls:
  ```java
  int area1 = r1.area();
  int area2 = r2.area();
  ```
  - We're asking r1 and r2 to give us their areas
  - No explicit parameters are needed because the necessary info. is in the objects' fields!

Types of Instance Methods

- There are two main types of instance methods:
  - **mutators** – methods that change an object's internal state
  - **accessors** – methods that retrieve information from an object without changing its state

- Examples of mutators:
  - `grow()` in our `Rectangle` class

- Examples of accessors:
  - `area()` in our `Rectangle` class
  - String methods: `length()`, `substring()`, `charAt()`
Which method call increases r's height by 5?

```java
public class Rectangle {
    int x;
    int y;
    int width;
    int height;

    public void grow(int dWidth, int dHeight) {
        this.width += dWidth;
        this.height += dHeight;
    }

    public int area() {
        return this.width * this.height;
    }
}

• Consider this client code:
  Rectangle r = new Rectangle();
  r.width = 10;
  r.height = 15;
  ______???
```
Initial Client Program

```java
public class RectangleClient {
    public static void main(String[] args) {
        Rectangle r1 = new Rectangle();
        r1.x = 10;       r1.y = 20;
        r1.width = 100;  r1.height = 50;
        Rectangle r2 = new Rectangle();
        r2.x = 50;       r2.y = 100;
        r2.width = 20;   r2.height = 80;
        System.out.println("r1: "+ r1.width +"x"+ r1.height);
        int area1 = r1.width * r1.height;
        System.out.println("area = "+ area1);
        System.out.println("r2: "+ r2.width +"x"+ r2.height);
        int area2 = r2.width * r2.height;
        System.out.println("area = "+ area2);
        // grow both rectangles
        r1.width += 50;  r1.height += 10;
        r2.width += 5;   r2.height += 30;
        System.out.println("r1: "+ r1.width +"x"+ r1.height);
        System.out.println("r2: "+ r2.width +"x"+ r2.height);
    }
}
```

Revised Client Program

```java
public class RectangleClient {
    public static void main(String[] args) {
        Rectangle r1 = new Rectangle();
        r1.x = 10;       r1.y = 20;
        r1.width = 100;  r1.height = 50;
        Rectangle r2 = new Rectangle();
        r2.x = 50;       r2.y = 100;
        r2.width = 20;   r2.height = 80;
        System.out.println("r1: "+ r1.width +"x"+ r1.height);
        System.out.println("area = "+ r1.area());
        System.out.println("r2: "+ r2.width +"x"+ r2.height);
        System.out.println("area = "+ r2.area());
        // grow both rectangles
        r1.grow(50, 10);
        r2.grow(5, 30);
        System.out.println("r1: "+ r1.width +"x"+ r1.height);
        System.out.println("r2: "+ r2.width +"x"+ r2.height);
    }
}
```
Practice Defining Instance Methods

• Add a mutator method that moves the rectangle to the right by a specified amount.
  ```java
  public ______ moveRight(___________________) {
  }
  ```

• Add an accessor method that determines if the rectangle is a square (true or false).
  ```java
  public __________ isSquare(____________) {
  }
  ```

Defining a Constructor

• Our current client program has to use several lines to initialize each Rectangle object:
  ```java
  Rectangle r1 = new Rectangle();
  r1.x = 10;       r1.y = 20;
  r1.width = 100;  r1.height = 50;
  ```

• We'd like to be able to do something like this instead:
  ```java
  Rectangle r1 = new Rectangle(10, 20, 100, 50);
  ```

• To do so, we need to define a constructor, a special method that initializes the state of an object when it is created.
Defining a Constructor (cont.)

- Here it is:
  ```java
  public Rectangle(int initialX, int initialY, int initialWidth, int initialHeight) {
      this.x = initialX;
      this.y = initialY;
      this.width = initialWidth;
      this.height = initialHeight;
  }
  ```

- General syntax for a constructor:
  ```java
  public <class name>(<parameter list>) {
  
  body of the constructor
  
  }
  ```

- Note that a constructor has no return type.

Third Version of our Rectangle Class

```java
public class Rectangle {
    int x;
    int y;
    int width;
    int height;

    public Rectangle(int initialX, int initialY, int initialWidth, int initialHeight) {
        this.x = initialX;
        this.y = initialY;
        this.width = initialWidth;
        this.height = initialHeight;
    }

    public void grow(int dWidth, int dHeight) {
        this.width += dWidth;
        this.height += dHeight;
    }

    public int area() {
        return this.width * this.height;
    }
}
```
Revised Client Program

```java
public class RectangleClient {
    public static void main(String[] args) {
        Rectangle r1 = new Rectangle(10, 20, 100, 50);
        Rectangle r2 = new Rectangle(50, 100, 20, 80);
        System.out.println("r1: "+ r1.width +" x"+ r1.height);
        System.out.println("area = " + r1.area());
        System.out.println("r2: "+ r2.width +" x"+ r2.height);
        System.out.println("area = " + r2.area());
        // grow both rectangles
        r1.grow(50, 10);
        r2.grow(5, 30);
        System.out.println("r1: "+ r1.width +" x"+ r1.height);
        System.out.println("area = " + r1.area());
    }
}
```

A Closer Look at Creating an Object

- What happens when the following line is executed?
  ```java
  Rectangle r1 = new Rectangle(10, 20, 100, 50);
  ```

- Several different things actually happen:
  1) a new `Rectangle` object is created
     - initially, all fields have their default values
  2) the constructor is then called to assign values to the fields
  3) a reference to the new object is stored in the variable `r1`

```
  r1  x 0
       y 0
       width 0
       height 0

  r1  x 10
       y 20
       width 100
       height 50

  r1  x 10
       y 20
       width 100
       height 50
```
Limiting Access to Fields

- The current version of our Rectangle class allows clients to directly access a Rectangle object's fields:

  ```java
  r1.width = 100;
r1.height += 20;
  ```

- This means that clients can make inappropriate changes:

  ```java
  r1.width = -100;
  ```

- To prevent this, we can declare the fields to be `private`:

  ```java
  public class Rectangle {
    private int x;
    private int y;
    private int width;
    private int height;
    ...
  }
  ```

- This indicates that these fields can only be accessed or modified by methods that are part of the Rectangle class.

Limiting Access to Fields (cont.)

- Now that the fields are private, our client program won't compile:

  ```java
  public class RectangleClient {
    public static void main(String[] args) {
      Rectangle r1 = new Rectangle(10, 20, 100, 50);
      Rectangle r2 = new Rectangle(50, 100, 20, 80);
      System.out.println("r1: "+r1.width+" x"+r1.height);
      System.out.println("area = "+ r1.area());
      System.out.println("r2: "+r2.width+" x"+r2.height);
      System.out.println("area = "+ r2.area());
      // grow both rectangles
      r1.grow(50, 10);
      r2.grow(5, 30);
      System.out.println("r1: "+r1.width+" x"+r1.height);
      System.out.println("r2: "+r2.width+" x"+r2.height);
    }
  }
  ```
Adding Accessor Methods for the Fields

```java
public class Rectangle {
    private int x;
    private int y;
    private int width;
    private int height;
    ...
    public int getX() {
        return this.x;
    }
    public int getY() {
        return this.y;
    }
    public int getWidth() {
        return this.width;
    }
    public int getHeight() {
        return this.height;
    }
}
```

- These methods are `public`, which indicates that they can be used by code that is outside the Rectangle class.

Revised Client Program

```java
public class RectangleClient {
    public static void main(String[] args) {
        Rectangle r1 = new Rectangle(10, 20, 100, 50);
        Rectangle r2 = new Rectangle(50, 100, 20, 80);
        System.out.println("r1: "+r1.getWidth()+"x"+
                           r1.getHeight());
        System.out.println("area = " + r1.area());
        System.out.println("r2: "+r2.getWidth()+"x"+
                           r2.getHeight());
        System.out.println("area = " + r2.area());

        // grow both rectangles
        r1.grow(50, 10);
        r2.grow(5, 30);
        System.out.println("r1: "+r1.getWidth()+"x"+
                           r1.getHeight());
        System.out.println("r2: "+r2.getWidth()+"x"+
                           r2.getHeight());
    }
}
```
Access Modifiers

- public and private are known as *access modifiers*.
  - they specify where a class, field, or method can be used

- A class is usually declared to be public:
  ```java
  public class Rectangle {
  
  • indicates that objects of the class can be used anywhere, including in other classes
  
  • Fields are usually declared to be private.
  
  • Methods are usually declared to be public.
  
  • We occasionally define private methods.
    • serve as *helper methods* for the public methods
    • cannot be invoked by code that is outside the class
  ```

Allowing Appropriate Changes

- To allow for appropriate changes to an object, we add whatever mutator methods make sense.

- These methods can prevent inappropriate changes:
  ```java
  public void setLocation(int newX, int newY) {
    if (newX < 0 || newY < 0) {
      throw new IllegalArgumentException();
    }
    this.x = newX;
    this.y = newY;
  }
  ```
Allowing Appropriate Changes (cont.)

• Here are two other mutator methods:

```java
public void setWidth(int newWidth) {
    if (newWidth <= 0) {
        throw new IllegalArgumentException();
    }
    this.width = newWidth;
}

public void setHeight(int newHeight) {
    if (newHeight <= 0) {
        throw new IllegalArgumentException();
    }
    this.height = newHeight;
}
```

Instance Methods Calling Other Instance Methods

• Here’s another mutator method that we already had:

```java
public void grow(int dWidth, int dHeight) {
    this.width += dWidth;
    this.height += dHeight;
}
```

• However, it doesn't prevent inappropriate changes.

• Rather than adding error-checking to it, we can have it call the new mutator methods:

```java
public void grow(int dWidth, int dHeight) {
    this.setWidth(this.width + dWidth);
    this.setHeight(this.height + dHeight);
}
```
Revised Constructor

- To prevent invalid values in the fields of a Rectangle object, we also need to modify our constructor.

- Here again, we take advantage of the error-checking code that's already present in the mutator methods:

  ```java
  public Rectangle(int initialX, int initialY, int initialWidth, int initialHeight)
  {
    this.setLocation(initialX, initialY);
    this.setWidth(initialWidth);
    this.setHeight(initialHeight);
  }
  ```

- setLocation, setWidth, and setHeight operate on the newly created Rectangle object

Encapsulation

- **Encapsulation** is one of the key principles of object-oriented programming.
  - another name for it is *information hiding*

- It refers to the practice of “hiding” the implementation of a class from users of the class.
  - prevent *direct* access to the internals of an object
    - making the fields private
  - provide *limited, indirect* access through a set of methods
    - making them public

- In addition to preventing inappropriate changes, encapsulation allows us to change the implementation of a class without breaking the client code that uses it.
Abstraction

- *Abstraction* involves focusing on the essential properties of something, rather than its inner or low-level details.
  - an important concept in computer science

- Encapsulation leads to abstraction.
  - example: rather than treating a Rectangle as four ints, we treat it as an object that's capable of growing itself, changing its location, etc.

Practice Defining Instance Methods

- Add a mutator method that scales the dimensions of a Rectangle object by a specified factor.
  - make the factor a double, to allow for fractional values
  - take advantage of existing mutator methods
  - use a type cast to turn the result back into an integer

- Add an accessor method that gets the perimeter of a Rectangle object.
Testing for Equivalent Objects

• Let's say that we have two different Rectangle objects, both of which represent equivalent rectangles:

\[
\text{Rectangle rect1 = new Rectangle(10, 100, 20, 55);}
\]
\[
\text{Rectangle rect2 = new Rectangle(10, 100, 20, 55);}
\]

• What is the value of the following condition?

\[
\text{rect1 == rect2}
\]

Testing for Equivalent Objects (cont.)

• The condition

\[
\text{rect1 == rect2}
\]

compares the references stored in rect1 and rect2.

• It doesn't compare the objects themselves.
Testing for Equivalent Objects (cont.)

- Recall: to test for equivalent objects, we need to use the `equals` method:
  
  ```java
  rect1.equals(rect2)
  ```

- Java's built-in classes have `equals` methods that:
  - return `true` if the two objects are equivalent to each other
  - return `false` otherwise

Default `equals()` Method

- If we don't write an `equals()` method for a class, objects of that class get a default version of this method.

- The default `equals()` just tests if the memory addresses of the two objects are the same.
  - the same as what `==` does!

- To ensure that we're able to test for equivalent objects, we need to write our own `equals()` method.
equals() Method for Our Rectangle Class

```java
public boolean equals(Rectangle other) {
    if (other == null) {
        return false;
    } else if (this.x != other.x) {
        return false;
    } else if (this.y != other.y) {
        return false;
    } else if (this.width != other.width) {
        return false;
    } else if (this.height != other.height) {
        return false;
    } else {
        return true;
    }
}
```

• **Note:** The method is able to access the fields in other directly (without using accessor methods).

• Instance methods can access the private fields of any object from the same class as the method.

equals() Method for Our Rectangle Class (cont.)

• Here’s an alternative version:

```java
public boolean equals(Rectangle other) {
    return (other != null
            && this.x == other.x
            && this.y == other.y
            && this.width == other.width
            && this.height == other.height);
}
```
Converting an Object to a String

- The `toString()` method allows objects to be displayed in a human-readable format.
  - It returns a string representation of the object.

- This method is called implicitly when you attempt to print an object or when you perform string concatenation:
  ```java
  Rectangle r1 = new Rectangle(10, 20, 100, 80);
  System.out.println(r1);
  // the second line above is equivalent to:
  System.out.println(r1.toString());
  ```

- If we don't write a `toString()` method for a class, objects of that class get a default version of this method.
  - Here again, it usually makes sense to write our own version.

`toString()` Method for Our `Rectangle` Class

```java
public String toString() {
    return this.width + " x " + this.height;
}
```

- Note: the method does not do any printing.

- It returns a string that can then be printed.
Revised Client Program

```java
public class RectangleClient {
    public static void main(String[] args) {
        Rectangle r1 = new Rectangle(10, 20, 100, 50);
        Rectangle r2 = new Rectangle(50, 100, 20, 80);
        System.out.println("r1: "+ r1);
        System.out.println("area = "+ r1.area());
        System.out.println("r2: "+ r2);
        System.out.println("area = "+ r2.area());
        // grow both rectangles
        r1.grow(50, 10);
        r2.grow(5, 30);
        System.out.println("r1: "+ r1);
        System.out.println("r2: "+ r2);
    }
}
```

Conventions for Accessors and Mutators

- **Accessors:**
  - usually have no parameters
  - all of the necessary info. is inside the called object
  - have a non-void return type
  - often have a name that begins with "get" or "is"
    - examples: getWidth(), isSquare()
    - but not always: area(), perimeter()

- **Mutators:**
  - usually have one or more parameter
  - usually have a void return type
  - often have a name that begins with "set"
    - examples: setLocation(), setWidth()
    - but not always: grow(), scale()
The Implicit Parameter and Method Frames

- When we call an instance method, the implicit parameter is included in its method frame.
  - example: `r1.grow(50, 10)`

```
  r1------------------------------
    grow
       this
    dwidth 50
    dHeight 10

  x 10
  y 20
  width 100
  height 50
```

- The method uses `this` to access the fields in the called object.
  - even if the code doesn't explicitly use it

```
  width += dWidth;
  height += dHeight;
```

```
  this.width += dWidth;
  this.height += dHeight;
```

Example: Method Frames for Instance Methods

```java
public class RectangleClient {
  public static void main(String[] args) {
    Rectangle r1 = new Rectangle(10, 20, 100, 50);
    Rectangle r2 = new Rectangle(50, 100, 20, 80);
    ...
    r1.grow(50, 10);
    r2.grow(5, 30);
    ...
  }
}
```

- After the objects are created:

```
  main
  r1
    r2

  x 10
  y 20
  width 100
  height 50

  x 50
  y 100
  width 20
  height 80
```
Example: Method Frames for Instance Methods

```java
public class RectangleClient {
    public static void main(String[] args) {
        Rectangle r1 = new Rectangle(10, 20, 100, 50);
        Rectangle r2 = new Rectangle(50, 100, 20, 80);
        r1.grow(50, 10);
        r2.grow(5, 30);
        ...
    }
}
```

- During the method call `r1.grow(50, 10)`:

  ![Diagram showing the method frame during the call](image)

- After the method call `r1.grow(50, 10)`:

  ![Diagram showing the updated method frame](image)
Example: Method Frames for Instance Methods

```java
public class RectangleClient {
    public static void main(String[] args) {
        Rectangle r1 = new Rectangle(10, 20, 100, 50);
        Rectangle r2 = new Rectangle(50, 100, 20, 80);
        ...
        r1.grow(50, 10);
        r2.grow(5, 30);
        ...
    }
}
```

• During the method call `r2.grow(5, 30)`: 

![Method Frame Diagram](image1)

• After the method call `r2.grow(5, 30)`: 

![Method Frame Diagram](image2)
Why Mutators Don't Need to Return Anything

- A mutator operates directly on the called object, so any changes it makes will be there after the method returns.
- example: the call `r2.grow(5, 30)` from the last slide
- during this call, `grow` gets a copy of the reference in `r2`, so it changes the object to which `r2` refers

Variable Scope: Static vs. Non-Static Methods

```java
public class Foo {
    private int x;
    
    public static int bar(int b, int c, Foo f) {
        c = c + this.x;         // would not compile
        return 3*b + f.x;       // would compile
    }
    
    public int boo(int d, Foo f) {
        d = d + this.x + f.x;   // would compile
        return 2 * d;
    }
}
```

- Static methods (like `bar` above) do NOT have a called object, so they can't access its fields.
- Instance/non-static methods (like `boo` above) do have a called object, so they can access its fields.
- Any method of a class can access fields in an object of that class that is passed in as a parameter (like the parameter `f` above).
A Common Use of the Implicit Parameter

• Here's our setLocation method:
  ```java
  public void setLocation(int newX, int newY) {
    if (newX < 0 || newY < 0) {
      throw new IllegalArgumentException();
    }
    this.x = newX;
    this.y = newY;
  }
  ```

• Here's an equivalent version:
  ```java
  public void setLocation(int x, int y) {
    if (x < 0 || y < 0) {
      throw new IllegalArgumentException();
    }
    this.x = x;
    this.y = y;
  }
  ```

• When the parameters have the same names as the fields, we must use this to access the fields.

Defining a Second Constructor

• Here's our Rectangle constructor:
  ```java
  public Rectangle(int initialX, int initialY, int initialWidth, int initialHeight) {
    this.setLocation(initialX, initialY);
    this.setWidth(initialWidth);
    this.setHeight(initialHeight);
  }
  ```

• It requires four parameters:
  ```java
  Rectangle r1 = new Rectangle(10, 20, 100, 50);
  ```

• A class can have an arbitrary number of constructors, provided that each of them has a distinct parameter list.
Defining a Second Constructor (cont.)

• Here’s a constructor that only takes values for width and height:
  ```java
  public Rectangle(int width, int height) {
      this.setWidth(width);
      this.setHeight(height);
      this.x = 0;
      this.y = 0;
  }
  ```
  • it puts the rectangle at the location (0, 0)

• Equivalently, we can call the original constructor, and let it perform the actual assignments:
  ```java
  public Rectangle(int width, int height) {
      this(0, 0, width, height);  // call other constr.
  }
  ```
  • we use the keyword this instead of Rectangle
  • this is the way that one constructor calls another

Practice Exercise: Writing Client Code

• Write a static method called processRectangle() that:
  • takes a Rectangle object (call it r) and an integer (call it delta) as parameters
  • prints the existing dimensions and area of the Rectangle (hint: take advantage of the toString() method)
  • increases both of the Rectangle’s dimensions by delta
  • prints the new dimensions and area
Collections of Data

- There are many situations in which we need a program to maintain a collection of data.

- Examples include:
  - all of the grades on a given assignment/exam
  - a simple database of song info (e.g., in a music player)

Using an Array for a Collection

- We've used an array to maintain a collection of primitive data values.

- It's also possible to have an array of objects:
A Class for a Collection

- Rather than just using an array, it's often helpful to create a blueprint class for the collection.

- Example: a Gradeset class for a collection of grades from a single assignment or exam
  - possible field definitions:
    ```java
    public class GradeSet {
        private String name;
        private int possiblePoints;
        private double[] grades;
        private int gradeCount;
    }
    ```
  - The array of values is "inside" the collection object, along with other relevant information associated with the collection.
  - In addition, we would add methods for maintaining and processing the collection.

A Blueprint Class for Grade Objects

- Rather than just representing the grades as ints or doubles, we'll use a separate blueprint class for a single grade:
  ```java
  public class Grade {
      private double rawScore;
      private int latePenalty;  // as a percent
  }
  ```
  - This allows us to store both the raw score and the late penalty (if any).
  - Constructors and methods include:
    ```java
    Grade(double raw, int late)
    Grade(double raw)
    getRawScore()
    getLatePenalty()
    setRawScore(double newScore)
    setLatePenalty(int newPenalty)
    getAdjustedScore()   // with late penalty
    ```
Revised GradeSet Class

```java
public class GradeSet {
    private String name;
    private int possiblePoints;
    private Grade[] grades;
    private int gradeCount;
}
```

• Here's what one of these objects would look like in memory:

```
q2
```

```
<table>
<thead>
<tr>
<th>name</th>
<th>&quot;quiz 2&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>possiblePoints</td>
<td>30</td>
</tr>
<tr>
<td>grades</td>
<td>null null null</td>
</tr>
<tr>
<td>gradeCount</td>
<td>1</td>
</tr>
<tr>
<td>rawScore</td>
<td>26</td>
</tr>
<tr>
<td>latePenalty</td>
<td>0</td>
</tr>
</tbody>
</table>
```

GradeSet Constructor/Methods

• Constructor:
  GradeSet(String name, int possPts, int numGrades)

• Accessor methods:
  String getName()
  int getPossiblePoints()
  int getGradeCount()
  Grade getGrade(int i) // get grade at position i
double averageGrade(boolean includePenalty)

• Mutator methods:
  void setName(String name)
  void setPossiblePoints(int possPoints)
  void addGrade(Grade g)
  Grade removeGrade(int i) // remove grade at posn i

• Let's review the code for these, and write some of them together.
GradeSet Constructor/Methods
GradeSet: Adding a Grade

```
GradeSet ps4 = new GradeSet("PS 4", 100, 4);
ps4.addGrade(new Grade(95, 0));
ps4.addGrade(new Grade(80, 10));
```

GradeSet: Adding a Grade

```
GradeSet ps4 = new GradeSet("PS 4", 100, 4);
ps4.addGrade(new Grade(95, 0));
ps4.addGrade(new Grade(80, 10));
```
GradeSet: Adding a Grade

GradeSet ps4 = new GradeSet("PS 4", 100, 4);
ps4.addGrade(new Grade(95, 0));
ps4.addGrade(new Grade(80, 10));
Inheritance and Polymorphism

Computer Science S-111
Harvard University
David G. Sullivan, Ph.D.

A Class for Modeling an Automobile

public class Automobile {
    private String make;
    private String model;
    private int year;
    private int mileage;
    private String plateNumber;
    private int numSeats;
    private boolean isSUV;
    public Automobile(String make, String model, int year, int numSeats, boolean isSUV) {
        this.make = make;
        this.model = model;
        if (year < 1900) {
            throw new IllegalArgumentException();
        }
        this.year = year;
        this.numSeats = numSeats;
        this.isSUV = isSUV;
        this.mileage = 0;
        this.plateNumber = "unknown";
    }
    public Automobile(String make, String model, int year) {
        this(make, model, year, 5, false);
    }
    // continued...
A Class for Modeling an Automobile (cont.)

```java
public String getMake() {
    return this.make;
}
public String getModel() {
    return this.model;
}
public int getYear() {
    return this.year;
}
public int getMileage() {
    return this.mileage;
}
public String getPlateNumber() {
    return this.plateNumber;
}
public int getNumSeats() {
    return this.numSeats;
}
public boolean isSUV() {
    return this.isSUV;
} // continued…
```

public void setMileage(int newMileage) {
    if (newMileage < this.mileage) {
        throw new IllegalArgumentException();
    }
    this.mileage = newMileage;
}
public void setPlateNumber(String plate) {
    this.plateNumber = plate;
}
public String toString() {
    String str = this.make + " " + this.model;
    str += "( " + this.numSeats + " seats)";
    return str;
}
```

• There are no mutators for the other fields. Why not?
Modeling a Related Class

- What if we now want to write a class to represent a taxi?

- The Taxi class will have the same fields and methods as the Automobile class.

- It will also have its own fields and methods:
  
  ```
  taxiID  getID, setID
  fareTotal getFareTotal, addFare
  numFares getNumFares, getAverageFare
  resetFareInfo
  ```

- We may also want the Taxi versions of some of the Automobile methods to behave differently. Examples:
  
  - we may want the toString method to include values from different fields
  - we may want the getNumSeats method to return only the number of seats available for passengers

Inheritance

- To avoid redefining all of the Automobile fields and methods, we specify that the Taxi class extends the Automobile class:
  
  ```
  public class Taxi extends Automobile {
  ```

- The Taxi class will inherit the fields and methods of the Automobile class.
  
  - it doesn't have to redefine them
A Class for Modeling a Taxi

```java
public class Taxi extends Automobile {
    // We don't need to include the fields
    // from Automobile!
    private String taxiID;
    private double fareTotal;
    private int numFares;
    // constructor goes here...
    // we don't need to include the methods
    // from Automobile!
    public String getID() {
        return this.taxiID;
    }
    public void addFare(double fare) {
        if (fare < 0) {
            throw new IllegalArgumentException();
        }
        this.fareTotal += fare;
        this.numFares++;
    }
    ...
}
```

Using Inherited Methods

- Because Taxi extends Automobile, we can invoke a method defined in the Automobile class on a Taxi object.
  - example:
    ```java
    Taxi t = new Taxi(...);
    t.setMileage(25000);
    ```

- This works even though there is no setMileage method defined in the Taxi class!
  - Taxi inherits it from Automobile
Overriding an Inherited Method

• A subclass can override an inherited method, replacing it with its own version.

• To override a method, the new method must have the same:
  • return type
  • name
  • number and types of parameters

• Example: our Taxi class can define its own toString method:
  ```java
  public String toString() {
    return "Taxi (id = " + this.taxiID + ");
  }
  ```
  • it overrides the toString method inherited from Automobile

Rethinking Our Design

• What if we also want to be able to capture information about other types of vehicles?
  • motorcycles
  • trucks

• The classes for these other vehicles should not inherit from Automobile. Why not?

• Solution: define a Vehicle class
  • fields and methods common to all vehicles are defined there
  • leave automobile-specific state and behavior in Automobile
    • everything else is inherited from Vehicle
  • define Motorcycle and Truck classes that also inherit from Vehicle
A Class for Modeling a Vehicle

```java
public class Vehicle {
    private String make;
    private String model;
    private int year;
    private int mileage;
    private String plateNumber;
    private int numWheels;  // this was not in Automobile
    public Vehicle(String make, String model, int year,
                    int numWheels) {
        this.make = make;
        this.model = model;
        if (year < 1900) {
            throw new IllegalArgumentException();
        }
        this.year = year;
        this.numWheels = numWheels;
        this.mileage = 0;
        this.plateNumber = "unknown";
    }
    public String getMake() {
        return this.make;
    }
    // etc.
```

Revised Automobile Class

```java
public class Automobile extends Vehicle {
    // make, model, etc. are now inherited from Vehicle
    // The following are specific to automobiles,
    // so we leave them here.
    private int numSeats;
    private boolean isSUV;
    // constructor goes here...
    // getMake(), etc. are now inherited from Vehicle
    // The following are specific to automobiles,
    // so we leave them here.
    public int getNumSeats() {
        return this.numSeats;
    }
    public boolean isSUV() {
        return this.isSUV;
    }
    // ...
}
```
Inheritance Hierarchy

• Inheritance leads classes to be organized in a hierarchy:
  
  ![Inheritance Hierarchy Diagram]

  - A class in Java inherits \textit{directly} from at most one class.
  - However, a class can inherit \textit{indirectly} from a class higher up in the hierarchy.
    - example: Taxi inherits indirectly from Vehicle

Terminology

• When class C extends class D (directly or indirectly):
  • class D is known as a \textit{superclass} or \textit{base class} of C
    - super – comes \textit{above} it in the hierarchy
  • class C is known as a \textit{subclass} or \textit{derived class} of D
    - sub – comes \textit{below} it in the hierarchy

• Examples:
  • Automobile is a superclass of Taxi and Limousine
  • Taxi is a subclass of Automobile and Vehicle
Deciding Where to Define a Method

• Assume we only care about the number of axles in truck vehicles.

• Thus, we define the getNumAxles method in the Truck class, rather than in the Vehicle class.
  
  ```java
  public int getNumAxles() {
    return this.getNumWheels() / 2;
  }
  ```

  • it will be inherited by subclasses of Truck
  • it won't be available to non-truck subclasses of Vehicle

• We override this method in the TractorTrailer class, because tractor trailers have four wheels on all but the front axle:
  
  ```java
  public int getNumAxles() {
    int numBackWheels = this.getNumWheels() - 2;
    return 1 + numBackWheels/4;
  }
  ```


What is Accessible From a Superclass?

• A subclass has direct access to the public fields and methods of a superclass.

• A subclass does not have direct access to the private fields and methods of a superclass.

• Example: we can think of an Automobile object as follows:

  - private fields inherited from Vehicle. They cannot be accessed directly by methods in Automobile.
  - fields defined in Automobile. They can be accessed directly by methods in Automobile.
What is Accessible From a Superclass? (cont.)

- Example: now that make and model are defined in Vehicle, we're no longer able to access them directly in the Automobile version of toString:

```java
public String toString() {
    String str = this.make + " " + this.model;
    str += " ( " + this.numSeats + " seats)";
    return str;
}
```

- Instead, we need to make method calls to access the inherited fields:

```java
public String toString() {
    String str = this.getMake() + " " + this.getModel();
    str += " ( " + this.numSeats + " seats)";
    return str;
}
```

What is Accessible From a Superclass? (cont.)

- Faulty approach: redefine the inherited fields in the subclass

```java
public class Vehicle {
    private String make;
    private String model;
    ...
}

class Automobile extends Vehicle {
    private String make; // NOT a good idea!
    private String model;
    ...
}
```

- You should NOT do this!
Writing a Constructor for a Subclass

• Another example of illegally accessing inherited private fields:

```java
public Automobile(String make, String model, int year,
    int numSeats, boolean isSUV) {
    this.make = make;
    this.model = model;
    ...
}
```

• To initialize inherited fields, a constructor should invoke a constructor from the superclass.

```java
public Automobile(String make, String model, int year,
    int numSeats, boolean isSUV) {
    super(make, model, year, 4);   // 4 is for numWheels
    this.numSeats = numSeats;
    this.isSUV = isSUV;
}
```

• use the keyword super followed by appropriate parameters for the superclass constructor

• must be done as the very first line of the constructor

Writing a Constructor for a Subclass (cont.)

• If a subclass constructor doesn't explicitly invoke a superclass constructor, the compiler tries to insert a call to the superclass constructor with no parameters.

• If there isn't such a constructor, we get a compile-time error.
  • example: this constructor won't compile:

```java
public Taxi(String make, String model, int year, String ID) {
    this.taxiID = ID;
}
```

  • the compiler attempts to insert the following call:
    `super();`
  • there isn't an Automobile constructor with no parameters
The Object Class

- If a class doesn't explicitly extend another class, it implicitly extends a special class called Object.

- Thus, the object class is at the top of the class hierarchy.
  - all classes are subclasses of this class
  - the default toString and equals methods are defined in this class

Inheritance in the Java API

```
java.awt

Class Rectangle

java.lang.object
  java.awt.geom.RectangularShape
    java.awt.geom.Rectangle2D
      java.awt.Rectangle

All Implemented Interfaces:
  Shape, Serializable, Cloneable

Direct Known Subclasses:
  DefaultCaret
```
More Examples of Method Overriding

- **Vehicle** inherits the fields and methods of **Object**.

- The inherited **toString** method isn't very helpful.

- We define a **Vehicle** version that overrides the inherited one:

  ```java
  public String toString() {    // Vehicle version
    String str = this.make + " " + this.model;
    return str;
  }
  ```

- When **toString** is invoked on a **Vehicle** object, the **Vehicle** version is executed:

  ```java
  Vehicle v = new Vehicle("Radio Flyer", "Classic Tricycle", 2002, 3);
  System.out.println(v);
  ```

  **outputs:** Radio Flyer Classic Tricycle

More Examples of Method Overriding (cont.)

- The **Automobile** class inherits the **Vehicle** version of **toString**.

- If we didn't define a **toString()** method in **Automobile**, the inherited version would be used.

- The **Automobile** version overrides the **Vehicle** version so that the number of seats can be included in the string:

  ```java
  public String toString() {
    String str = this.getMake() + " " +
    this.getModel();
    str += " ( " + this.numSeats + " seats)";
    return str;
  }
  ```
Invoking an Overridden Method

- When a subclass overrides an inherited method, we can invoke the inherited version by using the keyword `super`.

- Example: the `Automobile` version of `toString()` begins with the same fields as the `Vehicle` version:
  ```java
  public String toString() {
    String str = this.getMake() + " " +
      this.getModel();
    str += " ( " + this.numSeats + " seats)";
    return str;
  }
  ```
  - instead of calling the accessor methods, we can do this:
    ```java
    public String toString() {
      String str = super.toString();
      str += " ( " + this.numSeats + " seats)";
      return str;
    }
  ```

Another Example of Inheritance

- A square is a special type of rectangle.
  - but the width and height must be the same

- Assume that we also want `Square` objects to have a field for the unit of measurement (e.g., "cm").

- `Square` objects should mostly behave like `Rectangle` objects:
  ```java
  Rectangle r = new Rectangle(20, 30);
  int area1 = r.area();
  Square sq = new Square(40, "cm");
  int area2 = sq.area();
  ```
  - But there may be differences as well:
    ```java
    System.out.println(r);  // output: 20 x 30
    System.out.println(sq);  // output: square with 40-cm sides
    ```
Another Example of Inheritance (cont.)

```java
public class Rectangle {
    private int width;
    private int height;
    ...
    public Rectangle(int initWidth, int initHeight) {
        ...
    }
    public int getWidth() {
        ...
    }
    ...
    // other methods
}
```

```java
public class Square extends Rectangle {
    private String unit; // inherits other fields
    public Square(int side, String unit) {
        super(side, side);
        this.unit = unit;
    }
    public String toString() {  // overrides
        String s = "square with ";
        s += this.getWidth() + "-";
        s += this.unit + " sides";
        return s;
    }
    // inherits other methods
}
```

Another Example of Inheritance (cont.)

```java
public class Rectangle {
    private int width;
    private int height;
    ...
    public Rectangle(int initWidth, int initHeight) {
        ...
    }
    public int getWidth() {
        ...
    }
    ...
    // other methods
}
```

```java
public class Square extends Rectangle {
    private String unit; // inherits other fields
    public Square(int side, String unit) {
        super(side, side);
        this.unit = unit;
    }
    public String toString() {  // overrides
        String s = "square with ";
        s += this.getWidth() + "-";
        s += this.unit + " sides";
        return s;
    }
    // inherits other methods
}
```
Another Example of Method Overriding

• The Rectangle class has the following mutator method:
  ```java
  public void setWidth(int w) {
      if (w <= 0) {
          throw new IllegalArgumentException();
      }
      this.width = w;
  }
  ```

• The Square class inherits it. Why should we override it?

Which of these works?

A. // Square version, which overrides // the version inherited from Rectangle
   public void setWidth(int w) {
       this.width = w;
       this.height = w;
   }

B. // Square version, which overrides // the version inherited from Rectangle
   public void setWidth(int w) {
       this.setWidth(w);
       this.setHeight(w);
   }

C. either version would work

D. neither version would work
Accessing Methods from the Superclass

• The solution: use super to access the inherited version of the method – the one we are overriding:

```java
// Square version
public void setWidth(int w) {
    super.setWidth(w);  // call the Rectangle version
    super.setHeight(w);
}
```

• Only use super if you want to call a method from the superclass that has been overridden.

• If the method has not been overridden, use this as usual.

Accessing Methods from the Superclass

• We need to override all of the inherited mutators:

```java
// Square versions
public void setWidth(int w) {
    super.setWidth(w);
    super.setHeight(w);
}
public void setHeight(int h) {
    super.setWidth(h);
    super.setHeight(h);
}
public void grow(int dw, int dh) {
    if (dw != dh) {
        throw new IllegalArgumentException();
    }
    super.setWidth(this.getWidth() + dw);
    super.setHeight(this.getHeight() + dh);
}
```

getWidth() and getHeight() are not overridden, so we use this.
is-a Relationships

- We use inheritance to capture is-a relationships.
  - an automobile is a vehicle
  - a taxi is an automobile
  - a tractor trailer is a truck

```
Object
    Vehicle
      Motorcycle
      Automobile
      Truck
        Limousine
        Taxi
        MovingVan
        TractorTrailer
```

has-a Relationships

- Another type of relationship is a has-a relationship.
  - one type of object "owns" another type of object
  - example: a driver has a vehicle

- Inheritance should not be used to capture has-a relationships.
  - it does not make sense to make the Driver class a subclass of Vehicle

- Instead, we give the "owner" object a field that refers to the "owned" object:

  ```java
  public class Driver {
      String name;
      String ID;
      Vehicle v;
      ...
  }
  ```
Polymorphism

• We've been using reference variables like this:

```java
Automobile a = new Automobile("Ford", "Model T", ...);
```
• variable a is declared to be of type Automobile
• it holds a reference to an Automobile object

• In addition, a reference variable of type T can hold a reference to an object from a subclass of T:

```java
Automobile a = new Taxi("Ford", "Tempo", ...);
```
• this works because Taxi is a subclass of Automobile
• a taxi is an automobile!

• The name for this feature of Java is **polymorphism**.
  • from the Greek for “many forms”
  • the same code can be used with objects of different types!

Polymorphism and Collections of Objects

• Polymorphism is useful when we have a collection of objects of different but related types.

• Example:
  • let's say that a company has a collection of vehicles of different types
  • we can store all of them in an array of type Vehicle:

```java
Vehicle[] fleet = new Vehicle[5];
fleet[0] = new Automobile("Honda", "Civic", ...);
fleet[1] = new Motorcycle("Harley", ...);
fleet[2] = new TractorTrailer("Mack", ...);
fleet[4] = new Truck("Dodge", ...);
```
Processing a Collection of Objects

- We can determine the average age of the vehicles in the company's fleet by doing the following:

```java
int totalAge = 0;
for (int i = 0; i < fleet.length; i++) {
    int age = CURRENT_YEAR - fleet[i].getYear();
    totalAge += age;
}
double averageAge = (double)totalAge / fleet.length;
```

- We can invoke `getYear()` on each object in the array, regardless of its type.
  - their classes are all subclasses of `Vehicle`
  - thus, they must all have a `getYear()` method

Practice with Polymorphism

- Which of these assignments would be allowed?

  ```java
  Vehicle v1 = new Motorcycle(...);
  TractorTrailer t1 = new Truck(...);
  Truck t2 = new MovingVan(...);
  Taxi t3 = new Automobile(...);
  Vehicle v2 = new TractorTrailer(...);
  MovingVan m1 = new TractorTrailer(...);
  ```
Declared Type vs. Actual Type

- An object's declared type may not match its actual type:
  - declared type: type specified when declaring a variable
  - actual type: type specified when creating an object

- Recall this client code:
  ```java
  Vehicle[] fleet = new Vehicle[5];
  fleet[0] = new Automobile("Honda", "Civic", 2005);
  fleet[1] = new Motorcycle("Harley", ...);
  fleet[2] = new TractorTrailer("Mack", ...);
  ```

- Here are the types:

<table>
<thead>
<tr>
<th>object</th>
<th>declared type</th>
<th>actual type</th>
</tr>
</thead>
<tbody>
<tr>
<td>fleet[0]</td>
<td>Vehicle</td>
<td>Automobile</td>
</tr>
<tr>
<td>fleet[1]</td>
<td>Vehicle</td>
<td>Motorcycle</td>
</tr>
<tr>
<td>fleet[2]</td>
<td>Vehicle</td>
<td>TractorTrailer</td>
</tr>
</tbody>
</table>

- The compiler uses the "declared" type of an object to determine if a method call is valid.
- starts at the declared type, and goes up the inheritance hierarchy as needed looking for a version of the method
- if it can't find a version, the method call will not compile

Example: the following would not work:
```java
Vehicle[] fleet = new Vehicle[5];
...
fleet[2] = new TractorTrailer("Mack", ...);
...
System.out.println(fleet[2].getNumAxles());
```

- the declared type of fleet[2] is Vehicle
- there's no getNumAxles() method defined in or inherited by Vehicle
Determining if a Method Call is Valid (cont.)

- In such cases, we can use casting to create a reference with the necessary declared type:
  
  ```java
  Vehicle[] fleet = new Vehicle[5];
  ...
  fleet[2] = new TractorTrailer("Mack", ...);
  ...
  TractorTrailer t = (TractorTrailer) fleet[2];
  ```

- The following will work:

  ```java
  System.out.println(t.getNumAxles());
  ```

  - the declared type of `t` is `TractorTrailer`
  - there is a `getNumAxles()` method defined in `TractorTrailer`, so the compiler is happy

Determining Which Method to Execute

- Truck also has a `getNumAxles` method, so this would be another way to handle the previous problem:

  ```java
  Vehicle[] fleet = new Vehicle[5];
  ...
  fleet[2] = new TractorTrailer("Mack", ...);
  ...
  Truck t2 = (Truck) fleet[2];
  System.out.println(t2.getNumAxles());
  ```

- The object represented by `t2` has:
  - a declared type of __________
  - an actual type of __________

- Both `Truck` and `TractorTrailer` have a `getNumAxles`. Which version will be executed?

- More generally, how does the interpreter decide which version of a method should be used?
Dynamic Binding

• At runtime, the Java interpreter selects the version of a method that is appropriate to the actual type of the object.
  • starts at the actual type, and goes up the inheritance hierarchy as needed until it finds a version of the method
  • known as dynamic binding

• Given the code from the previous slide
  ```java
  Vehicle[] fleet = new Vehicle[5]
  fleet[2] = new TractorTrailer("Mack", ...);
  Truck t2 = (Truck) fleet[2];
  System.out.println(t2.getNumAxles());
  ```
  the TractorTrailer version of getNumAxles would be run
  • TractorTrailer is the actual type of t2, and that class has its own version of getNumAxles

Dynamic Binding (cont.)

• Another example:
  ```java
  public static void printFleet(Vehicle[] fleet) {
    for (int i = 0; i < fleet.length; i++) {
      System.out.println(fleet[i]);
    }
  }
  ```
  • the toString() method is implicitly invoked on each element of the array when we go to print it.
  • the appropriate version is selected by dynamic binding
  • note: the selection must happen at runtime, because the actual types of the objects may not be known when the code is compiled
Dynamic Binding (cont.)

- Recall our initialization of the array:
  ```java
  Vehicle[] fleet = new Vehicle[5];
  fleet[0] = new Automobile("Honda", "Civic", ...);
  fleet[1] = new Motorcycle("Harley", ...);
  fleet[2] = new TractorTrailer("Mack", ...);
  ...
  ```

- `System.out.println(fleet[0]);` will invoke the `Automobile` version of the `toString()` method.

- `Motorcycle` does not define its own `toString()` method, so `System.out.println(fleet[1]);` will invoke the `Vehicle` version of `toString()`, which is inherited by `Motorcycle`.

- `TractorTrailer` does not define its own `toString()` but `Truck` does, so `System.out.println(fleet[2]);` will invoke the `Truck` version of `toString()`, which is inherited by `TractorTrailer`.

Dynamic Binding (cont.)

- Dynamic binding also applies to method calls on the called object that occur within other methods.

- Example: the `Truck` class has the following `toString` method:
  ```java
  public String toString() {
      String str = this.getMake() + " " +
                   this.getModel();
      str = str + ", capacity = " + this.capacity;
      str = str + ", " + this.getNumAxles() + " axles";
      return str;
  }
  ```

- The `TractorTrailer` class inherits it and does not override it.

- When `toString` is called on a `TractorTrailer` object:
  - this `Truck` version of `toString()` will run
  - the `TractorTrailer` version of `getNumAxles()` will run when the code above is executed
The Power of Polymorphism

- Recall our `printFleet` method:
  ```java
  public static void printFleet(Vehicle[] fleet) {
      for (int i = 0; i < fleet.length; i++) {
          System.out.println(fleet[i]);
      }
  }
  ```
  - polymorphism allows this method to use a single `println` statement to print the appropriate info. for any kind of vehicle.

- Without polymorphism, we would need a large if-else-if:
  ```java
  if (fleet[i] is an Automobile) {
      print the appropriate info for Automobiles
  } else if (fleet[i] is a Truck) {
      print the appropriate info for Trucks
  } else if ...
  ```

- Polymorphism allows us to easily write code that works for more than one type of object.

Polymorphism and the Object Class

- The `Object` class is a superclass of every other class.
- Thus, we can use an `Object` variable to store a reference to any object.
  ```java
  Object o1 = "Hello World";
  Object o2 = new Temperature(20, 'C');
  Object o3 = new Taxi("Ford", "Tempo", 2000, "T253");
  ```
Summary and Extra Practice

- To determine if a method call is valid:
  - start at the *declared* type
  - go up the hierarchy as needed to see if you can find the specified method in the declared type or a superclass
  - if you don't find it, the method call is *not* valid

- Given the following:
  TractorTrailer t1 = new TractorTrailer(...);
  Vehicle v = new Truck(...);
  MovingVan m = new MovingVan(...);
  Truck t2 = new TractorTrailer(...);

- Which of the following are valid?
  v.getNumAxles()
  m.getNumAxles()
  t1.getMake()
  t1.isSleeper()
  t2.isSleeper

Summary and Extra Practice (cont.)

- To determine which version of a method will run (dynamic binding):
  - start at the *actual* type
  - go up the hierarchy as needed until you find the method
  - the first version you encounter is the one that will run

- Given the following:
  TractorTrailer t1 = new TractorTrailer(...);
  Vehicle v = new Truck(...);
  MovingVan m = new MovingVan(...);
  Truck t2 = new TractorTrailer(...);

- Which version of the method will run?
  m.getNumAxles()
  t1.getNumAxles()
  t2.getNumAxles()
  v.getMake()
  t2.getMake()
More Practice

```java
public class E extends G {
    public void method2() {
        System.out.print("E 2 ");
        this.method1();
    }
    public void method3() {
        System.out.print("E 3 ");
        this.method1();
    }
}
public class F extends G {
    public void method2() {
        System.out.print("F 2 ");
    }
}
public class G {
    public void method1() {
        System.out.print("G 1 ");
    }
    public void method2() {
        System.out.print("G 2 ");
    }
}
public class H extends E {
    public void method1() {
        System.out.print("H 1 ");
    }
}
```

More Practice (cont.)

- Which of these would compile and which would not?
  
  ```java
  E e1 = new E();
  E e2 = new H();
  E e3 = new G();
  E e4 = new F();
  G g1 = new H();
  G g2 = new F();
  H h1 = new H();
  ```

- To answer these questions, draw the inheritance hierarchy:

Here are the classes again...

```java
public class E extends G {
    public void method2() {
        System.out.print("E 2 ");
        this.method1();
    }
    public void method3() {
        System.out.print("E 3 ");
        this.method1();
    }
}
public class F extends G {
    public void method2() {
        System.out.print("F 2 ");
    }
}
public class G {
    public void method1() {
        System.out.print("G 1 ");
    }
    public void method2() {
        System.out.print("G 2 ");
    }
}
public class H extends E {
    public void method1() {
        System.out.print("H 1 ");
    }
}
```

More Practice (cont.)

```java
E e1 = new E();
G g1 = new H();
G g2 = new F();

• Which of the following would compile and which would not? For the ones that would compile, what is the output?

e1.method1();
e1.method2();
e1.method3();
g1.method1();
g1.method2();
g1.method3();
g2.method1();
g2.method2();
g2.method3();
```
Abstract Data Types
and Data Structures

Computer Science S-111
Harvard University
David G. Sullivan, Ph.D.

Congrats on completing the first half!

• In the second half, we will study fundamental data structures.
  • ways of imposing order on a collection of information
  • sequences: lists, stacks, and queues
  • trees
  • hash tables
  • graphs

• We will also:
  • study algorithms related to these data structures
  • learn how to compare data structures & algorithms

• Goals:
  • learn to think more intelligently about programming problems
  • acquire a set of useful tools and techniques
Sample Problem I: Finding Shortest Paths

- Given a set of routes between pairs of cities, determine the shortest path from city A to city B.

Sample Problem II: A Data "Dictionary"

- Given a large collection of data, how can we arrange it so that we can efficiently:
  - add a new item
  - search for an existing item

- Some data structures provide better performance than others for this application.

- More generally, we'll learn how to characterize the efficiency of different data structures and their associated algorithms.
Example of Comparing Algorithms

• Consider the problem of finding a phone number in a phonebook.

• Let’s informally compare the time efficiency of two algorithms for this problem.

Algorithm 1 for Finding a Phone Number

```java
findNumber(person) {
    for (p = number of first page; p <= number of the last page; p++) {
        if person is found on page p {
            return the person's phone number
        }
    }
    return NOT_FOUND
}
```

• If there were 1,000 pages in the phonebook, how many pages would this look at in the worst case?

• What if there were 1,000,000 pages?
Algorithm 2 for Finding a Phone Number

findNumber(person) {
    min = the number of the first page
    max = the number of the last page
    while (min <= max) {
        mid = (min + max) / 2 // page number of the middle page
        if person is found on page mid {
            return the person's number
        } else if the person's name comes earlier in the book {
            max = mid – 1
        } else {
            min = mid + 1
        }
    }
    return NOT_FOUND
}

• If there were 1,000 pages in the phonebook, how many pages would this look at in the worst case?
• What if there were 1,000,000 pages?

Searching a Collection of Data

• The phonebook problem is one example of a common task: searching for an item in a collection of data.
  • another example: searching for a record in a database

• Algorithm 1 is known as sequential search.
  • also called linear search

• Algorithm 2 is known as binary search.
  • only works if the items in the data collection are sorted
Abstract Data Types

- An abstract data type (ADT) is a model of a data structure that specifies:
  - the characteristics of the collection of data
  - the operations that can be performed on the collection

- It’s abstract because it doesn’t specify how the ADT will be implemented.
  - does not commit to any low-level details

- A given ADT can have multiple implementations.

A Simple ADT: A Bag

- A bag is just a container for a group of data items.
  - analogy: a bag of candy

- The positions of the data items don’t matter (unlike a list).
  - \{3, 2, 10, 6\} is equivalent to \{2, 3, 6, 10\}

- The items do not need to be unique (unlike a set).
  - \{7, 2, 10, 7, 5\} isn’t a set, but it is a bag
A Simple ADT: A Bag (cont.)

- The operations we want a Bag to support:
  - add(item): add item to the Bag
  - remove(item): remove one occurrence of item (if any) from the Bag
  - contains(item): check if item is in the Bag
  - numItems(): get the number of items in the Bag
  - grab(): get an item at random, without removing it
    - reflects the fact that the items don't have a position (and thus we can't say "get the 5th item in the Bag")
  - toArray(): get an array containing the current contents of the bag

- We want the bag to be able to store objects of any type.

Specifying an ADT Using an Interface

- In Java, we can use an interface to specify an ADT:

```java
public interface Bag {
    boolean add(Object item);
    boolean remove(Object item);
    boolean contains(Object item);
    int numItems();
    Object grab();
    Object[] toArray();
}
```

- An interface specifies a set of methods.
  - includes only the method headers
  - does not typically include the full method definitions

- Like a class, it must go in a file with an appropriate name.
  - in this case: Bag.java
Implementing an ADT Using a Class

• To implement an ADT, we define a class:

  ```java
  public class ArrayBag implements Bag {
      ...
      public boolean add(Object item) {
          ...
      }
  }
  ```

• When a class header includes an `implements` clause, the class must define all of the methods in the interface.
  • if the class doesn't define them, it won't compile

All Interface Methods Are Public

• Methods specified in an interface **must** be public, so we don’t use the keyword `public` in the definition:

  ```java
  public interface Bag {
      boolean add(Object item);
      boolean remove(Object item);
      boolean contains(Object item);
      int numItems();
      Object grab();
      Object[] toArray();
  }
  ```

• However, when we actually implement the methods in a class, we **do** need to use `public`:

  ```java
  public class ArrayBag implements Bag {
      ...
      public boolean add(Object item) {
          ...
      }
  }
  ```
One Possible Bag Implementation

• One way to store the items in the bag is to use an array:
  ```java
  public class ArrayBag {
      private ______________[] items;
      ...
  }
  ```
• What type should the array be?
• This allows us to store any type of object in the `items` array, thanks to the power of polymorphism:
  ```java
  ArrayBag bag = new ArrayBag();
  bag.add("hello");
  bag.add(new Rectangle(20, 30));
  ```
• How could we keep track of how many items are in a bag?

Another Example of Polymorphism

• An interface name can be used as the type of a variable:
  ```java
  Bag b;
  ```
• Variables with an interface type can refer to objects of any class that implements the interface:
  ```java
  Bag b = new ArrayBag();
  ```
• Using the interface as the type allows us to write code that works with any implementation of an ADT:
  ```java
  public void processBag(Bag b) {
      for (int i = 0; i < b.numItems(); i++) {
          ...
      }
  }
  ```
  • the param can be an instance of any Bag implementation
  • we must use method calls to access the object's internals, because the fields are not part of the interface
Memory Management: Looking Under the Hood

- To understand how data structures are implemented, you need to understand how memory is managed.

- There are three main types of memory allocation in Java.

- They correspond to three different regions of memory.

Memory Management, Type I: Static Storage

- Static storage is used for class variables, which are declared outside any method using the keyword static:

  ```java
  public class MyMethods {
      public static int numCompares;
      public static final double PI = 3.14159;
  }
  ```

- There is only one copy of each class variable.
  - shared by all objects of the class
  - Java's version of a global variable

- The Java runtime allocates memory for class variables when the class is first encountered.
  - this memory stays fixed for the duration of the program
Memory Management, Type II: Stack Storage

- Method parameters and local variables are stored in a region of memory known as the stack.

- For each method call, a new stack frame is added to the top of the stack.

```
public class Foo {
    public static int x(int i) {
        int j = i - 2;
        if (i >= 6) {
            return i;
        }
        return x(i + j);
    }
    public static void main(String[] args) {
        System.out.println(x(5));
    }
}
```

- When a method completes, its stack frame is removed.

Memory Management, Type III: Heap Storage

- Objects are stored in a memory region known as the heap.

- Memory on the heap is allocated using the new operator:

```
int[] values = new int[3];
ArrayBag b = new ArrayBag();
```

- new returns the memory address of the start of the object on the heap.
  - a reference!

- An object stays on the heap until there are no remaining references to it.

- Unused objects are automatically reclaimed by a process known as garbage collection.
  - makes their memory available for other objects
Two Constructors for the ArrayBag Class

```java
public class ArrayBag implements Bag {
    private Object[] items;
    private int numItems;
    public static final int DEFAULT_MAX_SIZE = 50;

    public ArrayBag() {
        this.items = new Object[DEFAULT_MAX_SIZE];
        this.numItems = 0;
    }
    public ArrayBag(int maxSize) {
        if (maxSize <= 0) {
            throw new IllegalArgumentException(
                "maxSize must be > 0");
        }
        this.items = new Object[maxSize];
        this.numItems = 0;
    }
    ...
}
```

• As we’ve seen before, we can have multiple constructors.
  • the parameters must differ in some way

• The first one is useful for small bags.
  • creates an array with room for 50 items.

• The second one allows the client to specify the max # of items.

• If the user inputs an invalid maxSize, we throw an exception.
Example: Creating Two ArrayBag Objects

```java
// client
public static void main(String[] args) {
    ArrayBag b1 = new ArrayBag(2);
    ArrayBag b2 = new ArrayBag(4);
    ...
}
```

```java
// constructor
public ArrayBag(int maxSize) {
    ...  // error-checking
    this.items = new Object[maxSize];
    this.numItems = 0;
}
```

- After the objects have been created, here's what we have:
Adding Items

- We fill the array from left to right. Here's an empty bag:

  ![Diagram showing an empty bag with items and numItems set to 0]

- After adding the first item:

  ![Diagram showing an item added to the bag, increasing numItems to 1]

- After adding the second item:

  ![Diagram showing two items added to the bag, increasing numItems to 2]

Adding Items (cont.)

- After adding the third item:

  ![Diagram showing three items added to the bag, increasing numItems to 3]

- After adding the fourth item:

  ![Diagram showing four items added to the bag, increasing numItems to 4]

- At this point, the ArrayBag is full!
  - It's non-trivial to "grow" an array, so we don't!
  - Additional items cannot be added until one is removed
A Method for Adding an Item to a Bag

```java
public class ArrayBag implements Bag {
    private Object[] items;
    private int numItems;
    ...

    public boolean add(Object item) {
        if (item == null) {
            throw new IllegalArgumentException("no nulls");
        } else if (this.numItems == this.items.length) {
            return false;   // no more room!
        } else {
            this.items[this.numItems] = item;
            this.numItems++;
            return true;    // success!
        }
    }
    ...
}
```

- takes an object of any type!
- returns a boolean to indicate if the operation succeeded

Initially, this.numItems is 0, so the first item goes in position 0.
- We increase this.numItems because we now have 1 more item.
- and so the next item added will go in the correct position!
Example: Adding an Item

```java
public static void main(String[] args) {
    String message = "hello, world";
    ArrayBag b = new ArrayBag(4);
    b.add(message);
    ...
}
```

```java
public boolean add(Object item) {
    if (this.numItems == this.items.length) {
        return false;
    } else {
        this.items[this.numItems] = item;
        this.numItems++;
        return true;
    }
}
```

- **add**'s stack frame includes:
  - **item**, which stores...
  - **this**, which stores...

Example: Adding an Item (cont.)

```java
public static void main(String[] args) {
    String message = "hello, world";
    ArrayBag b = new ArrayBag(4);
    b.add(message);
    ...
}
```

```java
public boolean add(Object item) {
    if (this.numItems == this.items.length) {
        return false;
    } else {
        this.items[this.numItems] = item;
        this.numItems++;
        return true;
    }
}
```

- **add**'s stack frame includes:
  - **item**, which stores...
  - **this**, which stores...
Example: Adding an Item (cont.)

public boolean add(Object item) {
    if (this.numItems == this.items.length) {
        this.items = Arrays.copyOf(this.items, this.items.length + 1);
    }
    this.items[this.numItems] = item;
    this.numItems++;
    return true;
}

• The method modifies the items array and numItems.
  • note that the array holds a copy of the reference to the item, not a copy of the item itself.

Example: Adding an Item (cont.)

After the method call returns, add's stack frame is removed from the stack.
Extra Practice: Determining if a Bag Contains an Item

- Let’s write the `ArrayBag contains()` method together.
  - should return `true` if an object equal to `item` is found, and `false` otherwise.

```
items numItems null null null null ...
"hello, world" "oh my!" "what's in the bag?"
```

```java
____________ contains(____________ item) {

}
```
Would this work instead?

```
items : [null, null, null, null, ...]
numItems : 3

"hello, world" "oh my!" "what's in the bag?"
```

- Let's write the ArrayBag `contains()` method together.
  - should return `true` if an object equal to `item` is found, and `false` otherwise.

```java
public boolean contains(Object item) {
    for (int i = 0; i < this.items.length; i++) {
        if (this.items[i].equals(item)) { // not ==
            return true;
        }
    }
    return false;
}
```

Another Incorrect `contains()` Method

```java
public boolean contains(Object item) {
    for (int i = 0; i < this.numItems; i++) {
        if (this.items[i].equals(item)) {
            return true;
        } else {
            return false;
        }
    }
    return false;
}
```

- What's the problem with this?
A Method That Takes a Bag as a Parameter

```java
public boolean containsAll(Bag otherBag) {
    if (otherBag == null || otherBag.numItems() == 0) {
        return false;
    }
    Object[] otherItems = otherBag.toArray();
    for (int i = 0; i < otherItems.length; i++) {
        if (!this.contains(otherItems[i])) {
            return false;
        }
    }
    return true;
}
```

• We use `Bag` instead of `ArrayBag` as the type of the parameter.
  • allows this method to be part of the `Bag` interface
  • allows us to pass in any object that implements `Bag`

• We must use methods in the interface to manipulate `otherBag`.
  • we can't use the fields, because they're not in the interface

A Type Mismatch

• Here are the headers of two `ArrayBag` methods:
  ```java
  public boolean add(Object item)
  public Object grab()
  ```

• Polymorphism allows us to pass `String` objects into `add()`:
  ```java
  ArrayBag stringBag = new ArrayBag();
  stringBag.add("hello");
  stringBag.add("world");
  ```

• However, this will not work:
  ```java
  String str = stringBag.grab(); // compiler error
  ```
  • the return type of `grab()` is `Object`
  • `Object` isn't a subclass of `String`, so polymorphism doesn't help!

• Instead, we need to use a type cast:
  ```java
  String str = (String)stringBag.grab();
  ```
  • this cast doesn't actually change the value being assigned
  • it just reassures the compiler that the assignment is okay
Unit 6, Part 2

Recursion Revisited;
Recursive Backtracking

Computer Science S-111
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Review: Recursive Problem-Solving
• When we use recursion, we reduce a problem to
a simpler problem of the same kind.
• We keep doing this until we reach a problem that is
simple enough to be solved directly.
• This simplest problem is known as the base case.
public static void printSeries(int n1, int n2) {
// base case
if (n1 == n2) {
System.out.println(n2);
} else {
System.out.print(n1 + ", ");
printSeries(n1 + 1, n2);
}
}

• The base case stops the recursion, because it doesn't
make another call to the method.

CSCI S-111

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Review: Recursive Problem-Solving (cont.)

- If the base case hasn't been reached, we execute the recursive case.

```java
public static void printSeries(int n1, int n2) {
    if (n1 == n2) { // base case
        System.out.println(n2);
    } else { // recursive case
        System.out.print(n1 +", ");
        printSeries(n1 + 1, n2);
    }
}
```

- The recursive case:
  - reduces the overall problem to one or more simpler problems of the same kind
  - makes recursive calls to solve the simpler problems

Raising a Number to a Power

- We want to write a recursive method to compute

\[ x^n = x \times x \times \ldots \times x \]

where \( x \) and \( n \) are both integers and \( n \geq 0 \).

- Examples:
  - \( 2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024 \)
  - \( 10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100000 \)

- Computing a power recursively:

\[
\begin{align*}
2^{10} &= 2 \times 2^9 \\
&= 2 \times (2 \times 2^8) \\
&= \ldots
\end{align*}
\]

- Recursive definition:

\[
\begin{align*}
x^n &= x \times x^{n-1} \text{ when } n > 0 \\
x^0 &= 1
\end{align*}
\]
Power Method: First Try

```java
public static int power1(int x, int n) {
    if (n < 0) {
        throw new IllegalArgumentException();
    } else if (n == 0) {
        return 1;
    } else {
        int pow_rest = power1(x, n-1);
        return x * pow_rest;
    }
}
```

Example: `power1(5,3)`

```
x 5 n 0
  x 5 n 1
    x 5 n 2
      x 5 n 3 return 5
    x 5 n 2
  x 5 n 1
    x 5 n 2
      x 5 n 3 return 5
    x 5 n 2
  x 5 n 0
```

Power Method: Second Try

- There's a better way to break these problems into subproblems. For example: \(2^{10} = (2\cdot2\cdot2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2)\)
  \[= (2^5) \cdot (2^5) = (2^5)^2\]
- A more efficient recursive definition of \(x^n\) (when \(n > 0\)):
  \(x^n = (x^{n/2})^2\) when \(n\) is even
  \(x^n = x \cdot (x^{n/2})^2\) when \(n\) is odd (using integer division for \(n/2\))

```java
public static int power2(int x, int n) {
    // code to handle n < 0 goes here...
}
```
Analyzing power2

- How many method calls would it take to compute $2^{1000}$?

```java
power2(2, 1000)
power2(2, 500)
power2(2, 250)
power2(2, 125)
power2(2, 62)
power2(2, 31)
power2(2, 15)
power2(2, 7)
power2(2, 3)
power2(2, 1)
power2(2, 0)
```

- Much more efficient than power1() for large n.
- It can be shown that it takes approx. $\log_2 n$ method calls.

An Inefficient Version of power2

- What’s wrong with the following version of power2()?

```java
public static int power2(int x, int n) {
    // code to handle n < 0 goes here...
    if (n == 0) {
        return 1;
    } else {
        // int pow_rest = power2(x, n/2);
        if (n % 2 == 0) {
            return power2(x, n/2)*power2(x, n/2);
        } else {
            return x * power2(x, n/2) * power2(x, n/2);
        }
    }
}
```
Review: Processing a String Recursively

- A string is a recursive data structure. It is either:
  - empty ("")
  - a single character, followed by a string

- Thus, we can easily use recursion to process a string.
  - process one or two of the characters ourselves
  - make a recursive call to process the rest of the string

- Example: print a string vertically, one character per line:
  ```java
  public static void printVertical(String str) {
      if (str == null || str.equals("") {
          return;
      }
      System.out.println(str.charAt(0));  // first char
      printVertical(str.substring(1));   // rest of string
  }
  ```

Removing Vowels From a String

- `removeVowels(s)` - removes the vowels from the string `s`, returning its "vowel-less" version!
  - `removeVowels("recursive")` should return "rcrsv"
  - `removeVowels("vowel")` should return "vw1"

- Can we take the usual approach to recursive string processing?
  - base case: empty string
  - delegate `s.substring(1)` to the recursive call
  - we're responsible for handling `s.charAt(0)`
Applying the String-Processing Template

public static String removeVowels(String s) {
    if (s.equals("")) { // base case
        return __________;
    } else { // recursive case
        String rem_rest = ________________;
        // do our one step!
    }
}

Consider Concrete Cases

removeVowels("after")   # first char is a vowel
  • what is its solution?
  • what is the next smaller subproblem?
  • what is the solution to that subproblem?
  • how can we use the solution to the subproblem?
    What is our one step?

removeVowels("recurse") # first char is not a vowel
  • what is its solution?
  • what is the next smaller subproblem?
  • what is the solution to that subproblem?
  • how can we use the solution to the subproblem?
    What is our one step?
removeVowels()  
public static String removeVowels(String s) {
    if (s.equals("")) { // base case
        return "";
    } else { // recursive case
        String rem_rest = removeVowels(s.substring(1));
        if ("aeiou".indexOf(s.charAt(0)) != -1) {
            ____________________________
        } else {
            ____________________________
        }
    }
}

The n-Queens Problem

- **Goal:** to place n queens on an n x n chessboard so that no two queens occupy:
  - the same row
  - the same column
  - the same diagonal.

- Sample solution for n = 8:  

```
  q  q  q  q  q  q  q  q
```

- This problem can be solved using a technique called *recursive backtracking.*
Recursive Strategy for n-Queens

- **findSolution(row)** – to place a queen in the specified row:
  - try one column at a time, looking for a "safe" one
  - if we find one: – place the queen there
    - make a recursive call to go to the next row
  - if we can’t find one: – backtrack by returning from the call
    - try to find another safe column in the previous row

- Example:
  - row 0:
    - col 0: safe
  - row 1:
    - col 0: same col
    - col 1: same diag
    - col 2: safe

4-Queens Example (cont.)

- row 2:
  - col 0: same col
  - col 1: same diag
  - col 2: same col/diag
  - col 3: same diag

- We’ve run out of columns in row 2!
- *Backtrack* to row 1 by returning from the recursive call.
  - pick up where we left off
  - we had already tried columns 0-2, so now we try column 3:

- Continue the recursion as before.
4-Queens Example (cont.)

- row 2:

- row 3:

- Backtrack to row 2:

- Backtrack to row 1. No columns left, so backtrack to row 0!

4-Queens Example (cont.)

- row 0:

- row 1:

- row 2:

- row 3:

A solution!
A Blueprint Class for an N-Queens Solver

public class NQueens {
    private boolean[][] board;  // state of the chessboard
    // other fields go here...

    public NQueens(int n) {
        this.board = new boolean[n][n];
        // initialize other fields here...
    }
    ...

    private void placeQueen(int row, int col) {
        this.board[row][col] = true;
        // modify other fields here...
    }

    // Here's what the object looks like initially:
    NQueens object
    board [false false false false][false false false false][false false false false][false false false false]
    // other fields

    // Here's what it looks like after placing some queens:
    NQueens object
    board [false false false false][false false false true][false true false false][false false false false]
    // other fields
A Blueprint Class for an N-Queens Solver

```java
public class NQueens {
    private boolean[][] board; // state of the chessboard
    // other fields go here...

    public NQueens(int n) {
        this.board = new boolean[n][n];
        // initialize other fields here...
    }

    private void placeQueen(int row, int col) {
        this.board[row][col] = true;
        // modify other fields here...
    }

    private void removeQueen(int row, int col) {
        this.board[row][col] = false;
        // modify other fields here...
    }

    private boolean isSafe(int row, int col) {
        // returns true if [row][col] is "safe", else false
    }

    private boolean findSolution(int row) {
        // see next slide!
    }
}
```

Recursive-Backtracking Method

```java
private boolean findSolution(int row) {
    if (row == this.board.length) {
        this.displayBoard();
        return true;
    }
    for (int col = 0; col < this.board.length; col++) {
        if (this.isSafe(row, col)) {
            this.placeQueen(row, col);
            if (this.findSolution(row + 1)) {
                return true;
            }
            this.removeQueen(row, col);
        }
    }
    return false;
}
```

- takes the index of a row (initially 0)
- uses a loop to consider all possible columns in that row
- makes a recursive call to move onto the next row
- returns true if a solution has been found; false otherwise
Tracing `findSolution()`

```java
private boolean findSolution(int row) {
    if (row == this.board.length) {
        // code to process a solution goes here...
    }
    for (int col = 0; col < this.board.length; col++) {
        if (this.isSafe(row, col)) {
            this.placeQueen(row, col);
            if (this.findSolution(row + 1)) {
                return true;
            }
            this.removeQueen(row, col);
        }
    }
    return false;
}
```

Once we place a queen in the last row...

```java
private boolean findSolution(int row) {
    if (row == this.board.length) {
        this.displayBoard();
        return true;
    }
    for (int col = 0; col < this.board.length; col++) {
        if (this.isSafe(row, col)) {
            this.placeQueen(row, col);
            if (this.findSolution(row + 1)) {
                return true;
            }
            this.removeQueen(row, col);
        }
    }
    return false;
}
```
...we make one more recursive call...

```java
private boolean findSolution(int row) {
    if (row == this.board.length) {
        this.displayBoard();
        return true;
    }
    for (int col = 0; col < this.board.length; col++) {
        if (this.isSafe(row, col)) {
            this.placeQueen(row, col);
            if (this.findSolution(row + 1)) {
                return true;
            }
            this.removeQueen(row, col);
        }
    }
    return false;
}
```

...and hit the base case!

```java
private boolean findSolution(int row) {
    if (row == this.board.length) {
        this.displayBoard();
        return true;
    }
    for (int col = 0; col < this.board.length; col++) {
        if (this.isSafe(row, col)) {
            this.placeQueen(row, col);
            if (this.findSolution(row + 1)) {
                return true;
            }
            this.removeQueen(row, col);
        }
    }
    return false;
}
```
true is sent back...

private boolean findSolution(int row) {
    if (row == this.board.length) {
        this.displayBoard();
        return true;
    }
    for (int col = 0; col < this.board.length; col++) {
        if (this.isSafe(row, col)) {
            this.placeQueen(row, col);
            if (this.findSolution(row + 1)) { // if (true)
                return true;
            }
            this.removeQueen(row, col);
        }
    }
    return false;
}

...and all the earlier calls also return true!
Using a "Wrapper" Method

- The key recursive method is private:
  
  ```java
  private boolean findSolution(int row) {
    ...
  }
  ```

- We use a separate, public "wrapper" method to start the recursion:
  
  ```java
  public boolean findSolution() {
    return findSolution(0);
  }
  ```

  - an example of overloading – two methods with the same name, but different parameters
  - this method takes no parameters
  - it makes the initial call to the recursive method and returns whatever that call returns
  - it allows us to ensure that the correct initial value is passed into the recursive method

Recursive Backtracking in General

- Useful for constraint satisfaction problems
  - involve assigning values to variables according to a set of constraints
  - n-Queens: variables = Queen’s position in each row constraints = no two queens in same row/col/diag
  - many others: factory scheduling, room scheduling, etc.

- Backtracking greatly reduces the number of possible solutions that we consider.
  - ex: 
    
    ![Diagram](image)
    
    • there are 16 possible solutions that begin with queens in these two positions
    • backtracking doesn't consider any of them!

- Recursion makes it easy to handle an arbitrary problem size.
  - stores the state of each variable in a separate stack frame
Template for Recursive Backtracking

// n is the number of the variable that the current
// call of the method is responsible for
boolean findSolution(int n, possibly other params) {
    if (found a solution) {
        this.displaySolution();
        return true;
    }
    // loop over possible values for the nth variable
    for (val = first to last) {
        if (this.isValid(val, n)) {
            this.applyValue(val, n);
            if (this.findSolution(n+1, other params)) {
                return true;
            }
            this.removeValue(val, n);
        }
    }
    return false;     // backtrack!
}

Note: n++ will not work here!

Template for Finding Multiple Solutions

(up to some target number of solutions)
boolean findSolutions(int n, possibly other params) {
    if (found a solution) {
        this.displaySolution();
        this.solutionsFound++;
        return (this.solutionsFound >= this.target);
    }
    // loop over possible values for the nth variable
    for (val = first to last) {
        if (this.isValid(val, n)) {
            this.applyValue(val, n);
            if (this.findSolutions(n + 1, other params)) {
                return true;
            }
            this.removeValue(val, n);
        }
    }
    return false;
}
Data Structures for n-Queens

- Three key operations:
  - `isSafe(row, col)`: check to see if a position is safe
  - `placeQueen(row, col)`
  - `removeQueen(row, col)`

- In theory, our 2-D array of booleans would be sufficient:
  ```java
  public class NQueens {
    private boolean[][] board;
  }
  ```
  It's easy to place or remove a queen:
  ```java
  private void placeQueen(int row, int col) {
    this.board[row][col] = true;
  }
  private void removeQueen(int row, int col) {
    this.board[row][col] = false;
  }
  ...
  ```
  - Problem: `isSafe()` takes a lot of steps. What matters more?

Additional Data Structures for n-Queens

- To facilitate `isSafe()`, add three arrays of booleans:
  ```java
  private boolean[] colEmpty;
  private boolean[] upDiagEmpty;
  private boolean[] downDiagEmpty;
  ```
  An entry in one of these arrays is:
  - true if there are no queens in the column or diagonal
  - false otherwise

- Numbering diagonals to get the indices into the arrays:
  ```java
  upDiag = row + col
  downDiag = (boardSize - 1) + row - col
  ```
Using the Additional Arrays

- Placing and removing a queen now involve updating four arrays instead of just one. For example:

```java
private void placeQueen(int row, int col) {
    this.board[row][col] = true;
    this.colEmpty[col] = false;
    this.upDiagEmpty[row + col] = false;
    this.downDiagEmpty[(this.board.length - 1) + row - col] = false;
}
```

- However, checking if a square is safe is now more efficient:

```java
private boolean isSafe(int row, int col) {
    return (this.colEmpty[col] && this.upDiagEmpty[row + col] && this.downDiagEmpty[(this.board.length - 1) + row - col]);
}
```

Recursive Backtracking II: Map Coloring

- We want to color a map using **only four colors**.

- Bordering states or countries **cannot** have the same color.
  - example:

```
not allowed!
```
Applying the Template to Map Coloring

```java
boolean findSolution(n, perhaps other params) {
    if (found a solution) {
        this.displaySolution();
        return true;
    }
    for (val = first to last) {
        if (this.isValid(val, n)) {
            this.applyValue(val, n);
            if (this.findSolution(n + 1, other params)) {
                return true;
            }
            this.removeValue(val, n);
        }
    }
    return false;
}
```

<table>
<thead>
<tr>
<th>template element</th>
<th>meaning in map coloring</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>found a solution</td>
</tr>
<tr>
<td>val</td>
<td>val</td>
</tr>
<tr>
<td>isValid(val, n)</td>
<td>isValid(val, n)</td>
</tr>
<tr>
<td>applyValue(val, n)</td>
<td>applyValue(val, n)</td>
</tr>
<tr>
<td>removeValue(val, n)</td>
<td>removeValue(val, n)</td>
</tr>
</tbody>
</table>

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

We color Colorado through Utah without a problem.
- Colorado:
- Idaho:
- Kansas:
- Montana:
- Nebraska:
- North Dakota:
- South Dakota:
- Utah:

No color works for Wyoming, so we backtrack...
Map Coloring Example (cont.)

Now we can complete the coloring:
Unit 7

Sorting and Algorithm Analysis

Computer Science S-111
Harvard University
David G. Sullivan, Ph.D.

Sorting an Array of Integers

• Ground rules:
  • sort the values in increasing order
  • sort “in place,” using only a small amount of additional storage

• Terminology:
  • position: one of the memory locations in the array
  • element: one of the data items stored in the array
  • element i: the element at position i

• Goal: minimize the number of comparisons \( C \) and the number of moves \( M \) needed to sort the array.
  • move = copying an element from one position to another
  example: \( arr[3] = arr[5]; \)
Defining a Class for our Sort Methods

```java
public class Sort {
    public static void bubbleSort(int[] arr) {
        ...
    }
    public static void insertionSort(int[] arr) {
        ...
    }
    ...
}
```

- Our `Sort` class is simply a collection of methods like Java's built-in `Math` class.
- Because we never create `Sort` objects, all of the methods in the class must be `static`.
  - outside the class, we invoke them using the class name: e.g., `Sort.bubbleSort(arr)`

---

Defining a Swap Method

- It would be helpful to have a method that swaps two elements of the array.
- Why won't the following work?
  ```java
  public static void swap(int a, int b) {
      int temp = a;
      a = b;
      b = temp;
  }
  ```
An Incorrect Swap Method

public static void swap(int a, int b) {
    int temp = a;
    a = b;
    b = temp;
}

• Trace through the following lines to see the problem:

int[] arr = {15, 7, ...};
swap(arr[0], arr[1]);

A Correct Swap Method

• This method works:

public static void swap(int[] arr, int a, int b) {
    int temp = arr[a];
    arr[a] = arr[b];
    arr[b] = temp;
}

• Trace through the following with a memory diagram to convince yourself that it works:

int[] arr = {15, 7, ...};
swap(arr, 0, 1);
Selection Sort

• Basic idea:
  • consider the positions in the array from left to right
  • for each position, find the element that belongs there and put it in place by swapping it with the element that’s currently there

• Example:

```
0 1 2 3 4
15 6 2 12 4
0 1 2 3 4
2 6 15 12 4
0 1 2 3 4
2 4 15 12 6
0 1 2 3 4
```

Why don’t we need to consider position 4?

Selecting an Element

• When we consider position \( i \), the elements in positions 0 through \( i - 1 \) are already in their final positions.

```
example for \( i = 3 \):
2 4 7 21 25 10 17
```

• To select an element for position \( i \):
  • consider elements \( i, i+1, i+2, \ldots, \text{arr.length} - 1 \), and keep track of \( \text{indexMin} \), the index of the smallest element seen thus far
  
```
indexMin: 3, 5
2 4 7 21 25 10 17
```

• when we finish this pass, \( \text{indexMin} \) is the index of the element that belongs in position \( i \).
• swap \( \text{arr}[i] \) and \( \text{arr}[\text{indexMin}] \):

```
2 4 7 10 25 21 17
```
Implementation of Selection Sort

• Use a helper method to find the index of the smallest element:

```java
private static int indexSmallest(int[] arr, int start) {
    int indexMin = start;
    for (int i = start + 1; i < arr.length; i++) {
        if (arr[i] < arr[indexMin]) {
            indexMin = i;
        }
    }
    return indexMin;
}
```

• The actual sort method is very simple:

```java
public static void selectionSort(int[] arr) {
    for (int i = 0; i < arr.length - 1; i++) {
        int j = indexSmallest(arr, i);
        swap(arr, i, j);
    }
}
```

Time Analysis

• Some algorithms are much more efficient than others.

• The time efficiency or time complexity of an algorithm is some measure of the number of operations that it performs.
  - for sorting, we'll focus on comparisons and moves

• We want to characterize how the number of operations depends on the size, n, of the input to the algorithm.
  - for sorting, n is the length of the array
  - how does the number of operations grow as n grows?

• We'll express the number of operations as functions of n
  - \( C(n) \) = number of comparisons for an array of length n
  - \( M(n) \) = number of moves for an array of length n
Counting Comparisons by Selection Sort

```java
class SelectionSort {
    private static int indexSmallest(int[] arr, int start) {
        int indexMin = start;
        for (int i = start + 1; i < arr.length; i++) {
            if (arr[i] < arr[indexMin]) {
                indexMin = i;
            }
        }
        return indexMin;
    }

    public static void selectionSort(int[] arr) {
        for (int i = 0; i < arr.length - 1; i++) {
            int j = indexSmallest(arr, i);
            swap(arr, i, j);
        }
    }
}
```

- To sort \( n \) elements, selection sort performs \( n - 1 \) passes:
  - On 1st pass, it performs \( 1 \) comparison to find \( \text{indexSmallest} \)
  - On 2nd pass, it performs \( 2 \) comparisons...
  - On the \( (n-1) \)st pass, it performs \( 1 \) comparison

Adding them up:

\[
C(n) = 1 + 2 + \ldots + (n - 2) + (n - 1)
\]

Counting Comparisons by Selection Sort (cont.)

- The resulting formula for \( C(n) \) is the sum of an arithmetic sequence:
  \[
  C(n) = 1 + 2 + \ldots + (n - 2) + (n - 1) = \sum_{i=1}^{n-1} i
  \]
- Formula for the sum of this type of arithmetic sequence:
  \[
  \sum_{i=1}^{m} i = \frac{m(m + 1)}{2}
  \]
- Thus, we can simplify our expression for \( C(n) \) as follows:
  \[
  C(n) = \sum_{i=1}^{n-1} i
  = \frac{(n - 1)((n - 1) + 1)}{2}
  = \frac{(n - 1)n}{2}
  \]
  \[
  C(n) = \frac{n^2}{2} - \frac{n}{2}
  \]
Focusing on the Largest Term

- When $n$ is large, mathematical expressions of $n$ are dominated by their “largest” term — i.e., the term that grows fastest as a function of $n$.

- example:

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n^2/2$</th>
<th>$n/2$</th>
<th>$n^2/2 - n/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>50</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>100</td>
<td>5000</td>
<td>50</td>
<td>4950</td>
</tr>
<tr>
<td>10000</td>
<td>50,000,000</td>
<td>5000</td>
<td>49,995,000</td>
</tr>
</tbody>
</table>

- In characterizing the time complexity of an algorithm, we’ll focus on the largest term in its operation-count expression.
  - for selection sort, $C(n) = n^2/2 - n/2 \approx n^2/2$

- In addition, we’ll typically ignore the coefficient of the largest term (e.g., $n^2/2 \rightarrow n^2$).

Big-O Notation

- We specify the largest term using big-O notation.
  - e.g., we say that $C(n) = n^2/2 - n/2$ is $O(n^2)$

- Common classes of algorithms:

<table>
<thead>
<tr>
<th>name</th>
<th>example expressions</th>
<th>big-O notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant time</td>
<td>1, 7, 10</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>logarithmic time</td>
<td>$3\log_{10}n, \log_2 n + 5$</td>
<td>$O(\log n)$</td>
</tr>
<tr>
<td>linear time</td>
<td>$5n, 10n - 2\log_2 n$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>nlogn time</td>
<td>$4n\log_2 n, n\log_2 n + n$</td>
<td>$O(n \log n)$</td>
</tr>
<tr>
<td>quadratic time</td>
<td>$2n^2 + 3n, n^2 - 1$</td>
<td>$O(n^2)$</td>
</tr>
<tr>
<td>exponential time</td>
<td>$2^n, 5e^n + 2n^2$</td>
<td>$O(c^n)$</td>
</tr>
</tbody>
</table>

- For large inputs, efficiency matters more than CPU speed.
  - e.g., an $O(\log n)$ algorithm on a slow machine will outperform an $O(n)$ algorithm on a fast machine
Ordering of Functions

• We can see below that: $n^2$ grows faster than $n \log_2 n$
  $n \log_2 n$ grows faster than $n$
  $n$ grows faster than $\log_2 n$

Ordering of Functions (cont.)

• Zooming in, we see that: $n^2 \geq n$ for all $n \geq 1$
  $n \log_2 n \geq n$ for all $n \geq 2$
  $n > \log_2 n$ for all $n \geq 1$
Big-O Time Analysis of Selection Sort

- **Comparisons**: we showed that \( C(n) = \frac{n^2}{2} - \frac{n}{2} \)
  - selection sort performs \( O(n^2) \) comparisons

- **Moves**: after each of the \( n-1 \) passes, the algorithm does one swap.
  - \( n-1 \) swaps, 3 moves per swap
  - \( M(n) = 3(n-1) = 3n-3 \)
  - selection sort performs \( O(n) \) moves.

- **Running time (i.e., total operations)**: ?

Mathematical Definition of Big-O Notation

- \( f(n) = O(g(n)) \) if there exist positive constants \( c \) and \( n_0 \) such that \( f(n) \leq cg(n) \) for all \( n \geq n_0 \)

- Example: \( f(n) = \frac{n^2}{2} - \frac{n}{2} \) is \( O(n^2) \), because
  - \( \frac{n^2}{2} - \frac{n}{2} \leq n^2 \) for all \( n \geq 0 \)
  - \( c = 1 \) \( n_0 = 0 \)

- Big-O notation specifies an upper bound on a function \( f(n) \) as \( n \) grows large.
Big-O Notation and Tight Bounds

- Strictly speaking, big-O notation provides an upper bound, not a tight bound (upper and lower).

- Example:
  - $3n - 3$ is $O(n^2)$ because $3n - 3 \leq n^2$ for all $n \geq 1$
  - $3n - 3$ is also $O(2^n)$ because $3n - 3 \leq 2^n$ for all $n \geq 1$

- However, it is common to use big-O notation to characterize a function as closely as possible – as if it specified a tight bound.
  - for our example, we would say that $3n - 3$ is $O(n)$
  - this is how you should use big-O in this class!

Insertion Sort

- Basic idea:
  - going from left to right, “insert” each element into its proper place with respect to the elements to its left
  - “slide over” other elements to make room

- Example:

```
0 1 2 3 4
  15 4 2 12 6
  4 15 2 12 6
  2 4 15 12 6
  2 4 12 15 6
  2 4 6 12 15
```
Comparing Selection and Insertion Strategies

- In selection sort, we start with the positions in the array and select the correct elements to fill them.
- In insertion sort, we start with the elements and determine where to insert them in the array.
- Here’s an example that illustrates the difference:

```
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>12</td>
<td>15</td>
<td>9</td>
<td>25</td>
<td>2</td>
<td>17</td>
</tr>
</tbody>
</table>
```

- Sorting by selection:
  - consider position 0: find the element (2) that belongs there
  - consider position 1: find the element (9) that belongs there
  - ...
- Sorting by insertion:
  - consider the 12: determine where to insert it
  - consider the 15: determine where to insert it
  - ...

Inserting an Element

- When we consider element \( i \), elements 0 through \( i-1 \) are already sorted with respect to each other.

```
example for \( i = 3 \):

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 & 4 \\
6 & 14 & 19 & 9 & \ldots
\end{array}
\]
```

- To insert element \( i \):
  - make a copy of element \( i \), storing it in the variable \( \text{toInsert} \):

```
\[
\begin{array}{cccc}
\text{toInsert} & 9 & 6 & 14 & 19 & 9
\end{array}
\]
```
  - consider elements \( i-1, i-2, \ldots \)
    - if an element \( > \) \( \text{toInsert} \), slide it over to the right
    - stop at the first element \( \leq \) \( \text{toInsert} \)

```
\[
\begin{array}{cccc}
\text{toInsert} & 9 & 6 & 14 & 19
\end{array}
\]
```

- copy \( \text{toInsert} \) into the resulting “hole”:

```
\[
\begin{array}{cccc}
6 & 9 & 14 & 19
\end{array}
\]"
Insertion Sort Example (done together)

**description of steps**
12 5 2 13 18 4

Implementation of Insertion Sort

```java
public class Sort {
    ...
    public static void insertionSort(int[] arr) {
        for (int i = 1; i < arr.length; i++) {
            if (arr[i] < arr[i-1]) {
                int toInsert = arr[i];
                int j = i;
                do {
                    arr[j] = arr[j-1];
                    j = j - 1;
                } while (j > 0 && toInsert < arr[j-1]);
                arr[j] = toInsert;
            }
        }
    }
}
```
Time Analysis of Insertion Sort

- The number of operations depends on the contents of the array.
- **best case:** array is sorted
  - each element is only compared to the element to its left
  - we never execute the do-while loop!
  - \( C(n) = \underline{________}, \ M(n) = \underline{________}, \) running time = ______
- **worst case:** array is in reverse order
  - each element is compared to all of the elements to its left:
    - \( arr[1] \) is compared to 1 element (\( arr[0] \))
    - \( arr[2] \) is compared to 2 elements (\( arr[0] \) and \( arr[1] \))
    - \( \ldots \)
    - \( arr[n-1] \) is compared to \( n-1 \) elements
  - \( C(n) = 1 + 2 + \ldots + (n - 1) = \underline{________} \)
  - similarly, \( M(n) = \underline{________}, \) running time = ______
- **average case:** elements are randomly arranged
  - on average, each element is compared to half of the elements to its left
  - still get \( C(n) = M(n) = \underline{________}, \) running time = ______

Shell Sort

- Developed by Donald Shell
- Improves on insertion sort
  - takes advantage of the fact that it's fast for almost-sorted arrays
  - eliminates a key disadvantage: an element may need to move many times to get to where it belongs.
- Example: if the largest element starts out at the beginning of the array, it moves one place to the right on every insertion!

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>…</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>999</td>
<td>42</td>
<td>56</td>
<td>30</td>
<td>18</td>
<td>23</td>
<td>…</td>
<td>11</td>
</tr>
</tbody>
</table>

- Shell sort uses larger moves that allow elements to quickly get close to where they belong in the sorted array.
**Sorting Subarrays**

- Basic idea:
  - use insertion sort on subarrays that contain elements separated by some increment $\text{incr}$
    - increments allow the data items to make larger "jumps"
  - repeat using a decreasing sequence of increments
- Example for an initial increment of 3:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>36</td>
<td>18</td>
<td>10</td>
<td>27</td>
<td>3</td>
<td>20</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

  - three subarrays:
    1) elements 0, 3, 6
    2) elements 1, 4, 7
    3) elements 2 and 5
- Sort the subarrays using insertion sort to get the following:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
<td>3</td>
<td>10</td>
<td>27</td>
<td>8</td>
<td>20</td>
<td>36</td>
<td>18</td>
</tr>
</tbody>
</table>

- Next, we complete the process using an increment of 1.

**Shell Sort: A Single Pass**

- We don’t actually consider the subarrays one at a time.
- For each element from position $\text{incr}$ to the end of the array, we insert the element into its proper place with respect to the elements from its subarray that come before it.

- The same example ($\text{incr} = 3$):

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>36</td>
<td>18</td>
<td>10</td>
<td>27</td>
<td>3</td>
<td>20</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>27</td>
<td>18</td>
<td>10</td>
<td>36</td>
<td>3</td>
<td>20</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
<td>3</td>
<td>10</td>
<td>36</td>
<td>18</td>
<td>20</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>3</td>
<td>10</td>
<td>36</td>
<td>18</td>
<td>20</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>3</td>
<td>10</td>
<td>27</td>
<td>18</td>
<td>20</td>
<td>36</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>3</td>
<td>10</td>
<td>27</td>
<td>8</td>
<td>20</td>
<td>36</td>
<td>18</td>
</tr>
</tbody>
</table>
Inserting an Element in a Subarray

- When we consider element \( i \), the other elements in its subarray are already sorted with respect to each other.

  **example for \( i = 6 \):**
  
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>3</td>
<td>10</td>
<td>36</td>
<td>18</td>
<td>20</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

  the other element's in 9's subarray (the 27 and 36) are already sorted with respect to each other

- To insert element \( i \):
  - make a copy of element \( i \), storing it in the variable `toInsert`:
  
  ```
  toInsert: 9
  0 1 2 3 4 5 6 7
  | 27 | 3 | 10 | 36 | 18 | 20 | 9 | 8 |
  ```

  - consider elements \( i-incr \), \( i-(2\times incr) \), \( i-(3\times incr) \), ...
    - if an element > `toInsert`, slide it right *within the subarray*
    - stop at the first element <= `toInsert`
  
  ```
  toInsert: 9
  0 1 2 3 4 5 6 7
  | 3 | 10 | 27 | 18 | 20 | 36 | 8 |
  ```

  - copy `toInsert` into the “hole”:
  
  ```
  0 1 2 3 4
  | 9 | 3 | 10 | 27 | 18 | ...
  ```

The Sequence of Increments

- Different sequences of decreasing increments can be used.

- Our version uses values that are one less than a power of two.
  - \( 2^k - 1 \) for some \( k \)
  - ... 63, 31, 15, 7, 3, 1
  - can get to the next lower increment using integer division:
    
    ```
    incr = incr/2;
    ```

- Should avoid numbers that are multiples of each other.
  - otherwise, elements that are sorted with respect to each other in one pass are grouped together again in subsequent passes
    - repeat comparisons unnecessarily
    - get fewer of the large jumps that speed up later passes
  
  - example of a bad sequence: 64, 32, 16, 8, 4, 2, 1
    - what happens if the largest values are all in odd positions?
Implementation of Shell Sort

public static void shellSort(int[] arr) {
    int incr = 1;
    while (2 * incr <= arr.length) {
        incr = 2 * incr;
    }
    incr = incr - 1;
    while (incr >= 1) {
        for (int i = incr; i < arr.length; i++) {
            if (arr[i] < arr[i-incr]) {
                int toInsert = arr[i];
                int j = i;
                do {
                    arr[j] = arr[j-incr];
                    j = j - incr;
                } while (j > incr-1 &&
                        toInsert < arr[j-incr]);
                arr[j] = toInsert;
            }
        }
        incr = incr/2;
    }
}

(If you replace incr with 1 in the for-loop, you get the code for insertion sort.)

Time Analysis of Shell Sort

• Difficult to analyze precisely
  • typically use experiments to measure its efficiency

• With a bad interval sequence, it’s $O(n^2)$ in the worst case.

• With a good interval sequence, it’s better than $O(n^2)$.
  • at least $O(n^{1.5})$ in the average and worst case
  • some experiments have shown average-case running times of $O(n^{1.25})$ or even $O(n^{7/6})$

• Significantly better than insertion or selection for large n:

<table>
<thead>
<tr>
<th>n</th>
<th>$n^2$</th>
<th>$n^{1.5}$</th>
<th>$n^{1.25}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
<td>31.6</td>
<td>17.8</td>
</tr>
<tr>
<td>100</td>
<td>10,000</td>
<td>1000</td>
<td>316</td>
</tr>
<tr>
<td>10,000</td>
<td>100,000,000</td>
<td>1,000,000</td>
<td>100,000</td>
</tr>
<tr>
<td>$10^6$</td>
<td>$10^{12}$</td>
<td>$10^9$</td>
<td>$3.16 \times 10^7$</td>
</tr>
</tbody>
</table>

• We’ve wrapped insertion sort in another loop and increased its efficiency! The key is in the larger jumps that Shell sort allows.
Practicing Time Analysis

• Consider the following static method:
  public static int mystery(int n) {
    int x = 0;
    for (int i = 0; i < n; i++) {
      x += i; // statement 1
      for (int j = 0; j < i; j++) {
        x += j;
      }
    }
    return x;
  }
  
• What is the big-O expression for the number of times that statement 1 is executed as a function of the input n?

What about now?

• Consider the following static method:
  public static int mystery(int n) {
    int x = 0;
    for (int i = 0; i < 3*n + 4; i++) {
      x += i; // statement 1
      for (int j = 0; j < i; j++) {
        x += j;
      }
    }
    return x;
  }
  
• What is the big-O expression for the number of times that statement 1 is executed as a function of the input n?
Practicing Time Analysis

• Consider the following static method:
  ```java
  public static int mystery(int n) {
      int x = 0;
      for (int i = 0; i < n; i++) {
          x += i;         // statement 1
          for (int j = 0; j < i; j++) {
              x += j;     // statement 2
          }
      }
      return x;
  }
  ```

• What is the big-O expression for the number of times that statement 2 is executed as a function of the input value of i number of times statement 2 is executed

---

Bubble Sort

• Perform a sequence of passes from left to right
  • each pass swaps adjacent elements if they are out of order
  • larger elements “bubble up” to the end of the array

• At the end of the kth pass:
  • the k rightmost elements are in their final positions
  • we don’t need to consider them in subsequent passes.

• Example:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>24</td>
<td>37</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

  after the first pass:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>28</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>37</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

  after the second:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>15</td>
<td>5</td>
<td>28</td>
</tr>
<tr>
<td>37</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

  after the third:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>5</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>37</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

  after the fourth:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>37</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Implementation of Bubble Sort

```java
class Sort {
    ...
    public static void bubbleSort(int[] arr) {
        for (int i = arr.length - 1; i > 0; i--) {
            for (int j = 0; j < i; j++) {
                if (arr[j] > arr[j+1]) {
                    swap(arr, j, j+1);
                }
            }
        }
    }
}
```

- Nested loops:
  - the inner loop performs a single pass
  - the outer loop governs:
    - the number of passes (arr.length - 1)
    - the ending point of each pass (the current value of i)

Time Analysis of Bubble Sort

- **Comparisons** (n = length of array):
  - they are performed in the inner loop
  - *how many repetitions does each execution of the inner loop perform?*

<table>
<thead>
<tr>
<th>value of i</th>
<th>number of comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td>n – 1</td>
<td>n – 1</td>
</tr>
<tr>
<td>n – 2</td>
<td>n – 2</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
1 + 2 + ... + n - 1 = \]

```java
class Sort {
    ...
    public static void bubbleSort(int[] arr) {
        for (int i = arr.length - 1; i > 0; i--) {
            for (int j = 0; j < i; j++) {
                if (arr[j] > arr[j+1]) {
                    swap(arr, j, j+1);
                }
            }
        }
    }
}
```
Time Analysis of Bubble Sort

- **Comparisons**: the kth pass performs n - k comparisons, so we get \( C(n) = \sum_{i=1}^{n-k} i = \frac{n^2}{2} - \frac{n}{2} = O(n^2) \)

- **Moves**: depends on the contents of the array
  - in the worst case:
    - \( M(n) = \)
  - in the best case:

- **Running time**:
  - \( C(n) \) is always \( O(n^2) \), \( M(n) \) is never worse than \( O(n^2) \)
  - therefore, the largest term of \( C(n) + M(n) \) is \( O(n^2) \)

- Bubble sort is a quadratic-time or \( O(n^2) \) algorithm.
  - can’t do much worse than bubble!

Quicksort

- Like bubble sort, quicksort uses an approach based on swapping out-of-order elements, but it’s more efficient.

- A recursive, divide-and-conquer algorithm:
  - **divide**: rearrange the elements so that we end up with two subarrays that meet the following criterion:
    
    *each element in left array <= each element in right array*

    example:

    \[
    \begin{array}{cccccc}
    12 & 8 & 14 & 4 & 6 & 13 \\
    \end{array}
    \]

    \[
    \begin{array}{cccccc}
    6 & 8 & 4 & 14 & 12 & 13 \\
    \end{array}
    \]

  - **conquer**: apply quicksort recursively to the subarrays, stopping when a subarray has a single element
  - **combine**: nothing needs to be done, because of the way we formed the subarrays
Partitioning an Array Using a Pivot

• The process that quicksort uses to rearrange the elements is known as partitioning the array.

• It uses one of the values in the array as a pivot, rearranging the elements to produce two subarrays:
  • left subarray: all values <= pivot
  • right subarray: all values >= pivot

\[
\begin{array}{cccccccc}
7 & 15 & 4 & 9 & 6 & 18 & 9 & 12 \\
\end{array}
\]

\[\text{partition using a pivot of 9}\]

\[
\begin{array}{cccccccc}
7 & 9 & 4 & 6 & 9 & 18 & 15 & 12 \\
\end{array}
\]

all values <= 9 all values >= 9
equivalent to the criterion on the previous page.

• The subarrays will not always have the same length.

• This approach to partitioning is one of several variants.

Possible Pivot Values

• First element or last element
  • risky, can lead to terrible worst-case behavior
  • especially poor if the array is almost sorted

\[
\begin{array}{cccccc}
4 & 8 & 14 & 12 & 6 & 18 \\
\end{array}
\]

\[
\begin{array}{cccccc}
4 & 8 & 14 & 12 & 6 & 18 \\
\end{array}
\]

pivot = 18

• Middle element (what we will use)

• Randomly chosen element

• Median of three elements
  • left, center, and right elements
  • three randomly selected elements
  • taking the median of three decreases the probability of getting a poor pivot
Partitioning an Array: An Example

- Maintain indices $i$ and $j$, starting them “outside” the array:
  
  $i = \text{first} - 1$
  $j = \text{last} + 1$

- Find “out of place” elements:
  - increment $i$ until $arr[i] \geq pivot$
  - decrement $j$ until $arr[j] \leq pivot$

- Swap $arr[i]$ and $arr[j]$:

Partitioning Example (cont.)

- Find:

- Swap:

- Find:

and now the indices have crossed, so we return $j$.

- Subarrays: $\text{left} = \text{from first to j}$, $\text{right} = \text{from j+1 to last}$
Partitioning Example 2

- Start (pivot = 13):
  
  \[
  \begin{array}{cccccccc}
  24 & 5 & 2 & 13 & 18 & 4 & 20 & 19 \\
  \end{array}
  \]

- Find:
  
  \[
  \begin{array}{cccccccc}
  24 & 5 & 2 & 13 & 18 & 4 & 20 & 19 \\
  \end{array}
  \]

- Swap:
  
  \[
  \begin{array}{cccccccc}
  4 & 5 & 2 & 13 & 18 & 24 & 20 & 19 \\
  \end{array}
  \]

- Find:
  
  \[
  \begin{array}{cccccccc}
  4 & 5 & 2 & 13 & 18 & 24 & 20 & 19 \\
  \end{array}
  \]

  and now the indices are equal, so we return \( j \).

- Subarrays:
  
  \[
  \begin{array}{cccccccc}
  4 & 5 & 2 & 13 & 18 & 24 & 20 & 19 \\
  \end{array}
  \]

Partitioning Example 3 (done together)

- Start (pivot = 5):
  
  \[
  \begin{array}{cccccccc}
  4 & 14 & 7 & 5 & 2 & 19 & 26 & 6 \\
  \end{array}
  \]

- Find:
  
  \[
  \begin{array}{cccccccc}
  4 & 14 & 7 & 5 & 2 & 19 & 26 & 6 \\
  \end{array}
  \]
Partitioning Example 4

- Start (pivot = 15):

```
8 10 7 15 20 9 6 18
```

- Find:

```
8 10 7 15 20 9 6 18
```

partition() Helper Method

```
private static int partition(int[] arr, int first, int last) {
    int pivot = arr[(first + last)/2];
    int i = first - 1;   // index going left to right
    int j = last + 1;   // index going right to left
    while (true) {
        do {
            i++;
        } while (arr[i] < pivot);
        do {
            j--;
        } while (arr[j] > pivot);
        if (i < j) {
            swap(arr, i, j);
        } else {
            return j;   // arr[j] = end of left array
        }
    }
}
```
Implementation of Quicksort

```java
public static void quickSort(int[] arr) { // "wrapper" method
    qSort(arr, 0, arr.length - 1);
}

private static void qSort(int[] arr, int first, int last) {
    int split = partition(arr, first, last);
    if (first < split) { // if left subarray has 2+ values
        qSort(arr, first, split); // sort it recursively!
    }
    if (last > split + 1) { // if right has 2+ values
        qSort(arr, split + 1, last); // sort it!
    }
} // note: base case is when neither call is made,
   // because both subarrays have only one element!
```

A Quick Review of Logarithms

- \( \log_b n \) = the exponent to which \( b \) must be raised to get \( n \)
- \( \log_b n = p \) if \( b^p = n \)
- examples: \( \log_2 8 = 3 \) because \( 2^3 = 8 \)
  \( \log_{10} 10000 = 4 \) because \( 10^4 = 10000 \)

- Another way of looking at logs:
  - let's say that you repeatedly divide \( n \) by \( b \) (using integer division)
  - \( \log_b n \) is an upper bound on the number of divisions needed to reach 1
- example: \( \log_2 18 \) is approx. 4.17
  \[
  \begin{align*}
  18/2 &= 9 \\
  9/2 &= 4 \\
  4/2 &= 2 \\
  2/2 &= 1
  \end{align*}
  \]
A Quick Review of Logs (cont.)

• \(O(\log n)\) algorithm – one in which the number of operations is proportional to \(\log_b n\) for any base \(b\)

• \(\log_b n\) grows much more slowly than \(n\)

\[
\begin{array}{|c|c|}
\hline
n & \log_2 n \\
\hline
2 & 1 \\
1024 (1K) & 10 \\
1024*1024 (1M) & 20 \\
1024*1024*1024 (1G) & 30 \\
\hline
\end{array}
\]

• Thus, for large values of \(n\):
  • a \(O(\log n)\) algorithm is much faster than a \(O(n)\) algorithm
    • \(\log n \ll n\)
  • a \(O(n \log n)\) algorithm is much faster than a \(O(n^2)\) algorithm
    • \(n \times \log n \ll \log n\) \(n\times n\)

\[n \log n \ll n^2\]

• It's also faster than a \(O(n^{1.5})\) algorithm like Shell sort

Time Analysis of Quicksort

• Partitioning an array requires approx. \(n\) comparisons.
  • most elements are compared with the pivot once; a few twice
  • best case: partitioning always divides the array in half
    • repeated recursive calls give:

\[
\begin{array}{c}
\text{comparisons} \\
\hline
n \\
2^{n/2} = n \\
4^{n/4} = n \\
\ldots \ldots \ldots \\
1 \ 1 \ 1 \ 1 \ 1 \ ... \ 1 \ 1 \ 1 \\
0 \\
\hline
\end{array}
\]

• at each "row" except the bottom, we perform \(n\) comparisons
• there are ________ rows that include comparisons
• \(C(n) = ?\)
• Similarly, \(M(n)\) and running time are both ________
Time Analysis of Quicksort (cont.)

- **worst case**: pivot is always the smallest or largest element
  - one subarray has 1 element, the other has \( n - 1 \)
  - repeated recursive calls give:

\[
\begin{align*}
C(n) &= \sum_{i=2}^{n} i = O(n^2). \\
M(n) &\text{ and run time are also } O(n^2).
\end{align*}
\]

- **average case** is harder to analyze
  - \( C(n) > n \log_2 n \), but it’s still \( O(n \log n) \)

Mergesort

- The algorithms we’ve seen so far have sorted the array in place.
  - use only a small amount of additional memory

- Mergesort requires an additional temporary array of the same size as the original one.
  - it needs \( O(n) \) additional space, where \( n \) is the array size

- It is based on the process of **merging** two sorted arrays.
  - example:
Merging Sorted Arrays

• To merge sorted arrays A and B into an array C, we maintain three indices, which start out on the first elements of the arrays:

\[
\begin{array}{c}
A \quad 2 \quad 8 \quad 14 \quad 24 \\
B \quad 5 \quad 7 \quad 9 \quad 11 \\
\end{array}
\]

\[
\begin{array}{c}
j \quad i \\
\end{array}
\]

k

• We repeatedly do the following:
  • compare A[i] and B[j]
  • copy the smaller of the two to C[k]
  • increment the index of the array whose element was copied
  • increment k

\[
\begin{array}{c}
A \quad 2 \quad 8 \quad 14 \quad 24 \\
B \quad 5 \quad 7 \quad 9 \quad 11 \\
\end{array}
\]

\[
\begin{array}{c}
j \quad i \\
k \\
\end{array}
\]

Merging Sorted Arrays (cont.)

• Starting point:

\[
\begin{array}{c}
A \quad 2 \quad 8 \quad 14 \quad 24 \\
B \quad 5 \quad 7 \quad 9 \quad 11 \\
\end{array}
\]

\[
\begin{array}{c}
j \quad i \\
k \\
\end{array}
\]

• After the first copy:

\[
\begin{array}{c}
A \quad 2 \quad 8 \quad 14 \quad 24 \\
B \quad 5 \quad 7 \quad 9 \quad 11 \\
\end{array}
\]

\[
\begin{array}{c}
j \quad i \\
k \\
\end{array}
\]

• After the second copy:

\[
\begin{array}{c}
A \quad 2 \quad 8 \quad 14 \quad 24 \\
B \quad 5 \quad 7 \quad 9 \quad 11 \\
\end{array}
\]

\[
\begin{array}{c}
j \quad i \\
k \\
\end{array}
\]
Merging Sorted Arrays (cont.)

• After the third copy:

\[
\begin{array}{cccc}
A & 2 & 8 & 14 & 24 \\
B & 5 & 7 & 9 & 11 \\
\end{array}
\]

\[
\begin{array}{cccc}
C & 2 & 5 & 7 & \\
& & & \\
& & & \\
& & & \\
\end{array}
\]

• After the fourth copy:

\[
\begin{array}{cccc}
A & 2 & 8 & 14 & 24 \\
B & 5 & 7 & 9 & 11 \\
\end{array}
\]

\[
\begin{array}{cccc}
C & 2 & 5 & 7 & 8 \\
& & & \\
& & & \\
& & & \\
\end{array}
\]

• After the fifth copy:

\[
\begin{array}{cccc}
A & 2 & 8 & 14 & 24 \\
B & 5 & 7 & 9 & 11 \\
\end{array}
\]

\[
\begin{array}{cccc}
C & 2 & 5 & 7 & 8 & 9 \\
& & & \\
& & & \\
& & & \\
\end{array}
\]

• After the sixth copy:

\[
\begin{array}{cccc}
A & 2 & 8 & 14 & 24 \\
B & 5 & 7 & 9 & 11 \\
\end{array}
\]

\[
\begin{array}{cccc}
C & 2 & 5 & 7 & 8 & 9 & 11 \\
& & & & \\
& & & & \\
& & & & \\
\end{array}
\]

• There’s nothing left in B, so we simply copy the remaining elements from A:

\[
\begin{array}{cccc}
A & 2 & 8 & 14 & 24 \\
B & 5 & 7 & 9 & 11 \\
\end{array}
\]

\[
\begin{array}{cccc}
C & 2 & 5 & 7 & 8 & 9 & 11 & 14 & 24 \\
& & & & & & & & \\
& & & & & & & & \\
& & & & & & & & \\
\end{array}
\]
Divide and Conquer

- Like quicksort, mergesort is a divide-and-conquer algorithm.
- **divide**: split the array in half, forming two subarrays
- **conquer**: apply mergesort recursively to the subarrays, stopping when a subarray has a single element
- **combine**: merge the sorted subarrays

```
12 8 14 4 6 33 2 27
```

Tracing the Calls to Mergesort

the initial call is made to sort the entire array:
```
12 8 14 4 6 33 2 27
```

split into two 4-element subarrays, and make a recursive call to sort the left subarray:
```
12 8 14 4 6 33 2 27
```

```
12 8 14 4
```

split into two 2-element subarrays, and make a recursive call to sort the left subarray:
```
12 8 14 4 6 33 2 27
```

```
12 8 14 4
```

```
12 8
```
Tracing the Calls to Mergesort

split into two 1-element subarrays, and make a recursive call to sort the left subarray:

split into two 1-element subarrays, and make a recursive call to sort its right subarray:

base case, so return to the call for the subarray \{12, 8\}:

base case, so return to the call for the subarray \{12, 8\}:
Tracing the Calls to Mergesort

merge the sorted halves of \{12, 8\}:

```
12 8 14 4 6 33 2 27
```

```
12 8 14 4
```

```
12 8 ➞ 8 12
```

dead end of the method, so return to the call for the 4-element subarray, which now has a sorted left subarray:

```
12 8 14 4 6 33 2 27
```

```
8 12 14 4
```

Tracing the Calls to Mergesort

make a recursive call to sort the right subarray of the 4-element subarray

```
12 8 14 4 6 33 2 27
```

```
8 12 14 4
```

```
14 4
```

dead split it into two 1-element subarrays, and make a recursive call to sort the left subarray:

```
12 8 14 4 6 33 2 27
```

```
8 12 14 4
```

```
14 4
```

```
14
```

base case...
Tracing the Calls to Mergesort

return to the call for the subarray \{14, 4\}:

\[
\begin{array}{cccccccccc}
12 & 8 & 14 & 4 & 6 & 33 & 2 & 27 \\
8 & 12 & 14 & 4 \\
8 & 12 & 14 & 4 \\
14 & 4 \\
\end{array}
\]

make a recursive call to sort its right subarray:

\[
\begin{array}{cccccccccc}
12 & 8 & 14 & 4 & 6 & 33 & 2 & 27 \\
8 & 12 & 14 & 4 \\
8 & 12 & 14 & 4 \\
14 & 4 \\
\end{array}
\]

merge the sorted halves of \{14, 4\}:

\[
\begin{array}{cccccccccc}
12 & 8 & 14 & 4 & 6 & 33 & 2 & 27 \\
8 & 12 & 14 & 4 \\
8 & 12 & 14 & 4 \\
14 & 4 \\
\end{array}
\]

base case...
Tracing the Calls to Mergesort

end of the method, so return to the call for the 4-element subarray, which now has two sorted 2-element subarrays:

```
  12  8  14  4  6  33  2  27
  8  12  4  14
```

merge the 2-element subarrays:

```
  12  8  14  4  6  33  2  27
  8  12  4  14
  →  4  8  12  14
```

end of the method, so return to the call for the original array, which now has a sorted left subarray:

```
  4  8  12  14  6  33  2  27
```

perform a similar set of recursive calls to sort the right subarray. here's the result:

```
  4  8  12  14  2  6  27  33
```

finally, merge the sorted 4-element subarrays to get a fully sorted 8-element array:

```
  4  8  12  14  2  6  27  33
  2  4  6  8  12  14  27  33
```
Implementing Mergesort

- In theory, we could create new arrays for each new pair of subarrays, and merge them back into the array that was split.

- Instead, we'll create a temp. array of the same size as the original.
  - pass it to each call of the recursive mergesort method
  - use it when merging subarrays of the original array:

```
arr 8 12 4 14 6 33 2 27
```

```
temp 4 8 12 14
```

- after each merge, copy the result back into the original array:

```
arr 4 8 12 14 6 33 2 27
```

```
temp 4 8 12 14
```

A Method for Merging Subarrays

```java
private static void merge(int[] arr, int[] temp, int leftStart, int leftEnd, int rightStart, int rightEnd) {
    int i = leftStart;    // index into left subarray
    int j = rightStart;   // index into right subarray
    int k = leftStart;    // index into temp
    while (i <= leftEnd && j <= rightEnd) {
        if (arr[i] < arr[j]) {
            temp[k] = arr[i];
            i++; k++;
        } else {
            temp[k] = arr[j];
            j++; k++;
        }
    }
    while (i <= leftEnd) {
        temp[k] = arr[i];
        i++; k++;
    }
    while (j <= rightEnd) {
        temp[k] = arr[j];
        j++; k++;
    }
    for (i = leftStart; i <= rightEnd; i++) {
        arr[i] = temp[i];
    }
}
```
A Method for Merging Subarrays

```java
private static void merge(int[] arr, int[] temp, int leftStart, int leftEnd, int rightStart, int rightEnd) {
    int i = leftStart; // index into left subarray
    int j = rightStart; // index into right subarray
    int k = leftStart; // index into temp
    while (i <= leftEnd && j <= rightEnd) { // both subarrays still have values
        if (arr[i] < arr[j]) {
            temp[k] = arr[i];
            i++;
            k++;
        } else {
            temp[k] = arr[j];
            j++;
            k++;
        }
    }
}
```

Methods for Mergesort

- Here's the key recursive method:

```java
private static void mSort(int[] arr, int[] temp, int start, int end){
    if (start >= end) {  // base case: subarray of length 0 or 1
        return;
    } else {
        int middle = (start + end)/2;
        mSort(arr, temp, start, middle);
        mSort(arr, temp, middle + 1, end);
        merge(arr, temp, start, middle, middle + 1, end);
    }
}
```
Methods for Mergesort

- Here's the key recursive method:

```java
private static void mSort(int[] arr, int[] temp, int start, int end){
    if (start >= end) {  // base case: subarray of length 0 or 1
        return;
    } else {
        int middle = (start + end)/2;
        mSort(arr, temp, start, middle);
        mSort(arr, temp, middle + 1, end);
        merge(arr, temp, start, middle, middle + 1, end);
    }
}
```

- We use a "wrapper" method to create the temp array, and to make the initial call to the recursive method:

```java
public static void mergeSort(int[] arr) {
    int[] temp = new int[arr.length];
    mSort(arr, temp, 0, arr.length - 1);
}
```

Time Analysis of Mergesort

- Merging two halves of an array of size n requires 2n moves. Why?

- Mergesort repeatedly divides the array in half, so we have the following call tree (showing the sizes of the arrays):

```
   n
  / \
 n/2 n/2
 / \ / \n n/4 n/4 n/4 n/4
 ... ...
 1 1 1 1 1 1 1 ...
```

   - at all but the last level of the call tree, there are 2n moves
   - how many levels are there?
   - M(n) = ?
   - C(n) = ?
Summary: Sorting Algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Best Case</th>
<th>Avg Case</th>
<th>Worst Case</th>
<th>Extra Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection sort</td>
<td>O(n²)</td>
<td>O(n²)</td>
<td>O(n²)</td>
<td>O(1)</td>
</tr>
<tr>
<td>Insertion sort</td>
<td>O(n)</td>
<td>O(n²)</td>
<td>O(n²)</td>
<td>O(1)</td>
</tr>
<tr>
<td>Shell sort</td>
<td>O(n log n)</td>
<td>O(n^{1.5})</td>
<td>O(n^{1.5})</td>
<td>O(1)</td>
</tr>
<tr>
<td>Bubble sort</td>
<td>O(n²)</td>
<td>O(n²)</td>
<td>O(n²)</td>
<td>O(1)</td>
</tr>
<tr>
<td>Quicksort</td>
<td>O(n log n)</td>
<td>O(n log n)</td>
<td>O(n²)</td>
<td>best/avg: O(log n) worst: O(n)</td>
</tr>
<tr>
<td>Mergesort</td>
<td>O(n log n)</td>
<td>O(n log n)</td>
<td>O(n²)</td>
<td>O(n)</td>
</tr>
</tbody>
</table>

- Insertion sort is best for nearly sorted arrays.
- Mergesort has the best worst-case complexity, but requires O(n) extra memory – and moves to and from the temp. array.
- Quicksort is comparable to mergesort in the best/average case.
  - efficiency is also O(n log n), but less memory and fewer moves
  - its extra memory is from...
  - with a reasonable pivot choice, its worst case is seldom seen

Comparison-Based vs. Distributive Sorting

- All of the sorting algorithms we've considered have been comparison-based:
  - treat the keys as wholes (comparing them)
  - don’t “take them apart” in any way
  - all that matters is the relative order of the keys, not their actual values

- No comparison-based sorting algorithm can do better than O(n log₂ n) on an array of length n.
  - O(n log₂ n) is a lower bound for such algorithms.

- Distributive sorting algorithms do more than compare keys; they perform calculations on the values of individual keys.

- Moving beyond comparisons allows us to overcome the lower bound.
  - tradeoff: use more memory.
Distributive Sorting Example: Radix Sort

• Relies on the representation of the data as a sequence of $m$ quantities with $k$ possible values.

• Examples:
  - integer in range 0 ... 999 3 10
  - string of 15 upper-case letters 15 26
  - 32-bit integer 32 2 (in binary)
    4 256 (as bytes)

• Strategy: Distribute according to the last element in the sequence, then concatenate the results:
  
  33 41 12 24 31 14 13 42 34
  
  get: 41 31 | 12 42 | 33 13 | 24 14 34

• Repeat, moving back one digit each time:
  
  get: |

Analysis of Radix Sort

• Recall that we treat the values as a sequence of $m$ quantities with $k$ possible values.

• Number of operations is $O(n^m)$ for an array with $n$ elements
  - better than $O(n \log n)$ when $m < \log n$

• Memory usage increases as $k$ increases.
  - $k$ tends to increase as $m$ decreases
  - tradeoff: increased speed requires increased memory usage
Big-O Notation Revisited

- We’ve seen that we can group functions into classes by focusing on the fastest-growing term in the expression for the number of operations that they perform.
- e.g., an algorithm that performs \( \frac{n^2}{2} - \frac{n}{2} \) operations is a \( O(n^2) \)-time or quadratic-time algorithm

Common classes of algorithms:

<table>
<thead>
<tr>
<th>name</th>
<th>example expressions</th>
<th>big-O notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant time</td>
<td>1, 7, 10</td>
<td>( O(1) )</td>
</tr>
<tr>
<td>logarithmic time</td>
<td>( 3\log_{10}n, \log_2n + 5 )</td>
<td>( O(\log n) )</td>
</tr>
<tr>
<td>linear time</td>
<td>( 5n, 10n - 21\log_2n )</td>
<td>( O(n) )</td>
</tr>
<tr>
<td>nlogn time</td>
<td>( 4n\log_2n, n\log_2n + n )</td>
<td>( O(n\log n) )</td>
</tr>
<tr>
<td>quadratic time</td>
<td>( 2n^2 + 3n, n^2 - 1 )</td>
<td>( O(n^2) )</td>
</tr>
<tr>
<td>cubic time</td>
<td>( n^2 + 3n^3, 5n^3 - 5 )</td>
<td>( O(n^3) )</td>
</tr>
<tr>
<td>exponential time</td>
<td>( 2^n, 5e^n + 2n^2 )</td>
<td>( O(c^n) )</td>
</tr>
<tr>
<td>factorial time</td>
<td>( 3n!, 5n + n! )</td>
<td>( O(n!) )</td>
</tr>
</tbody>
</table>

How Does the Number of Operations Scale?

- Let's say that we have a problem size of 1000, and we measure the number of operations performed by a given algorithm.

- If we double the problem size to 2000, how would the number of operations performed by an algorithm increase if it is:
  - \( O(n) \)-time
  - \( O(n^2) \)-time
  - \( O(n^3) \)-time
  - \( O(\log_2n) \)-time
  - \( O(2^n) \)-time
How Does the Actual Running Time Scale?

- How much time is required to solve a problem of size n?
  - assume that each operation requires 1 μsec (1 x 10^{-6} sec)

<table>
<thead>
<tr>
<th>time function</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>.00001 s</td>
<td>.00002 s</td>
<td>.00003 s</td>
<td>.00004 s</td>
<td>.00005 s</td>
<td>.00006 s</td>
</tr>
<tr>
<td>n^2</td>
<td>.0001 s</td>
<td>.0004 s</td>
<td>.0009 s</td>
<td>.0016 s</td>
<td>.0025 s</td>
<td>.0036 s</td>
</tr>
<tr>
<td>n^5</td>
<td>.1 s</td>
<td>3.2 s</td>
<td>24.3 s</td>
<td>1.7 min</td>
<td>5.2 min</td>
<td>13.0 min</td>
</tr>
<tr>
<td>2^n</td>
<td>.001 s</td>
<td>1.0 s</td>
<td>17.9 min</td>
<td>12.7 days</td>
<td>35.7 yrs</td>
<td>36,600 yrs</td>
</tr>
</tbody>
</table>

- sample computations:
  - when n = 10, an n^2 algorithm performs 10^2 operations.
    \[ 10^2 \times (1 \times 10^{-6} \text{ sec}) = .0001 \text{ sec} \]
  - when n = 30, a 2^n algorithm performs 2^{30} operations.
    \[ 2^{30} \times (1 \times 10^{-6} \text{ sec}) = 1073 \text{ sec} = 17.9 \text{ min} \]

What's the Largest Problem That Can Be Solved?

- What's the largest problem size n that can be solved in a given time T? (again assume 1 μsec per operation)

<table>
<thead>
<tr>
<th>time function</th>
<th>1 min</th>
<th>1 hour</th>
<th>1 week</th>
<th>1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>60,000,000</td>
<td>3.6 x 10^9</td>
<td>6.0 x 10^{11}</td>
<td>3.1 x 10^{13}</td>
</tr>
<tr>
<td>n^2</td>
<td>7745</td>
<td>60,000</td>
<td>777,688</td>
<td>5,615,692</td>
</tr>
<tr>
<td>n^5</td>
<td>35</td>
<td>81</td>
<td>227</td>
<td>500</td>
</tr>
<tr>
<td>2^n</td>
<td>25</td>
<td>31</td>
<td>39</td>
<td>44</td>
</tr>
</tbody>
</table>

- sample computations:
  - 1 hour = 3600 sec
    that's enough time for \[ 3600/(1 \times 10^{-6}) = 3.6 \times 10^6 \text{ operations} \]
  - n^2 algorithm:
    \[ n^2 = 3.6 \times 10^9 \Rightarrow n = (3.6 \times 10^9)^{1/2} = 60,000 \]
  - 2^n algorithm:
    \[ 2^n = 3.6 \times 10^9 \Rightarrow n = \log_2(3.6 \times 10^9) \approx 31 \]
Linked Lists

Computer Science S-111
Harvard University
David G. Sullivan, Ph.D.

Representing a Sequence of Data

• Sequence – an ordered collection of items (position matters)
  • we will look at several types: lists, stacks, and queues
• Most common representation = an array
• Advantages of using an array:
  • easy and efficient access to any item in the sequence
  • items[i] gives you the item at position i in O(1) time
  • known as random access
  • very compact (but can waste space if positions are empty)
• Disadvantages of using an array:
  • have to specify an initial array size and resize it as needed
  • inserting/deleting items can require shifting other items
  • ex: insert 63 between 52 and 72

items:  31  52  72  ...

Alternative Representation: A Linked List

- A linked list stores a sequence of items in separate nodes.
- Each node is an object that contains:
  - a single item
  - a "link" (i.e., a reference) to the node containing the next item
- The last node in the linked list has a link value of null.
- The linked list as a whole is represented by a variable that holds a reference to the first node.
  - e.g., items in the example above

Arrays vs. Linked Lists in Memory

- In an array, the elements occupy consecutive memory locations:
- In a linked list, the nodes are distinct objects.
  - do not have to be next to each other in memory
  - that's why we need the links to get from one node to the next!
Linked Lists in Memory

Here's how the above linked list might actually look in memory:

```
<table>
<thead>
<tr>
<th>Address</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0x200</td>
<td>31</td>
</tr>
<tr>
<td>0x204</td>
<td></td>
</tr>
<tr>
<td>0x208</td>
<td>72</td>
</tr>
<tr>
<td>0x212</td>
<td>null</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>0x520</td>
<td>31</td>
</tr>
<tr>
<td>0x524</td>
<td>0x812</td>
</tr>
<tr>
<td>0x528</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>0x812</td>
<td>52</td>
</tr>
<tr>
<td>0x816</td>
<td>0x208</td>
</tr>
</tbody>
</table>
```

- the variable `items`
- the last node
- the first node
- the second node

Features of Linked Lists

- They can grow without limit (provided there is enough memory).
- Easy to insert/delete an item – no need to "shift over" other items.
  - for example, to insert 63 between 52 and 72:

```
before:
items: 31 ———> 52 ———> 72

after:
items: 31 ———> 52 ———> 63 ———> 72
```

- Disadvantages:
  - they don't provide random access
  - need to "walk down" the list to access an item
  - the links take up additional memory
A String as a Linked List of Characters

- Each node represents one character.
- Java class for this type of node:
  ```java
  public class StringNode {
      private char ch;
      private StringNode next;
      public StringNode(char c, StringNode n) {
          this.ch = c;
          this.next = n;
      }
  }
  ```
- The string as a whole is represented by a variable that holds a reference to the node for the first character (e.g., `str1` above).

A String as a Linked List (cont.)

- An empty string will be represented by a null value.
  ```java
  StringNode str2 = null;
  ```
- We will use `static` methods that take the string as a parameter.
  - e.g., we'll write `length(str1)` instead of `str1.length()`
  - outside the class, call the methods using the class name:
    ```java
    StringNode.length(str1)
    ```
- This approach allows the methods to handle empty strings.
  - if `str1 == null`:
    - `length(str1)` will work
    - `str1.length()` will throw a `NullPointerException`
Review of Variables

- A variable or variable expression represents both:
  - a "box" or location in memory (the *address* of the variable)
  - the contents of that "box" (the *value* of the variable)

- Practice:

  ```
  StringNode str;  // points to the first node
  StringNode temp;  // points to the second node
  ```

- Expressions

<table>
<thead>
<tr>
<th>expression</th>
<th>address</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>str</td>
<td>0x200</td>
<td>0x520 (ref to the 'd' node)</td>
</tr>
<tr>
<td>str.ch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>str.next</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

  Assumptions:
  - *ch* field has the same memory address as the node itself.
  - *next* field comes 2 bytes after the start of the node.

More Complicated Expressions

- Example: `temp.next.ch`

- Start with the beginning of the expression: `temp.next`
  - It represents the *next* field of the node to which `temp` refers.
    - address =
    - value =

- Next, consider `temp.next.ch`
  - It represents the *ch* field of the node to which `temp.next` refers.
    - address =
    - value =
What are the address and value of `str.next.next`?

- `str.next` is...
- thus, `str.next.next` is...

What expression using `τ` would give us `'e'`?

- `s'
- `0xc000` → `0x0be00` → `0x03004` → `0x0bb00` → `0xa004`
What expression using \( t \) would give us 'e'?

Working backwards…
- I know that I need the ch field in the 'e' node
- Where do I have a reference to the 'e' node?
- What expression can I use for the box containing that reference?

Review of Assignment Statements
- An assignment of the form
  \[ \text{var1} = \text{var2}; \]
  - takes the value of \( \text{var2} \)
  - copies it into the box at the address of \( \text{var1} \)

- Example involving integers:
  \[
  \begin{align*}
  \text{int } x &= 5; \\
  \text{int } y &= x;
  \end{align*}
  \]

- Example involving references:
  \[
  \begin{align*}
  \text{int[]} \ a1 &= \{3, 4, 5\}; \\
  \text{int[]} \ a2 &= a1;
  \end{align*}
  \]
1) \texttt{str.next = temp.next;}

2) \texttt{temp.next = temp.next.next;}

**Writing an Appropriate Assignment**

- If \texttt{temp} didn't already refer to the 'o' node, what assignment would be needed to make it refer to that node?

  - start by asking: where do I currently have a reference to the 'o' node?
  - then ask: what expression can I use for that box?
  - then write the assignment:
A Linked List Is a Recursive Data Structure!

- Recursive definition: a linked list is either
  a) empty or
  b) a single node, followed by a linked list

- Viewing linked lists in this way allows us to write recursive methods that operate on linked lists.

Recursively Finding the Length of a String

- For a Java String object:

```java
public static int length(String str) {
    if (str.equals("")) {
        return 0;
    } else {
        int lenRest = length(str.substring(1));
        return 1 + lenRest;
    }
}
```

- For a linked-list string:

```java
public static int length(StringNode str) {
    if (str == null) {
        return 0;
    } else {
        int lenRest = length(str.next);
        return 1 + lenRest;
    }
}
```
An Alternative Version of the Method

- Original version:
  ```java
  public static int length(StringNode str) {
    if (str == null) {
      return 0;
    } else {
      int lenRest = length(str.next);
      return 1 + lenRest;
    }
  }
  ```

- Version without a variable for the result of the recursive call:
  ```java
  public static int length(StringNode str) {
    if (str == null) {
      return 0;
    } else {
      return 1 + length(str.next);
    }
  }
  ```

Tracing `length()`

```java
public static int length(StringNode str) {
  if (str == null) {
    return 0;
  } else {
    return 1 + length(str.next);
  }
}
```

- Example: `StringNode.length(str1)`

```
str: null
    return 0;
  
str: 0x404
    "t"
      str: 0x720
        "at"
          str: 0x128
            "cat"
              str: 0x404
                "t"
                  str: 0x720
                    "at"
                      str: 0x128
                        "cat"
                          str: 0x404
                            "t"
                              str: 0x720
                                "at"
                                  str: 0x128
                                    "cat"
                                      str: 0x404
                                        "t"
                                          str: 0x720
                                            "at"
                                              str: 0x128
                                                "cat"
                                                  str: 0x404
                                                    "t"
                                                      str: 0x720
                                                        "at"
                                                          str: 0x128
                                                            "cat"
                                                              str: 0x404
                                                                "t"
                                                                  str: 0x720
                                                                    "at"
                                                                      str: 0x128
                                                                        "cat"
                                                                          str: 0x404
                                                                            "t"
                                                                              str: 0x720
                                                                                "at"
                                                                                  str: 0x128
                                                                                    "cat"
                                                                                      str: 0x404
                                                                                        "t"
                                                                                          str: 0x720
                                                                                           "at"
                                                                                                str: 0x128
                                                                                                    "cat"
                                                                                                        return 1+2
```
Using Iteration to Traverse a Linked List

• Many tasks require us to traverse or "walk down" a linked list.
• We just saw a method that used recursion to do this.
• It can also be done using iteration (for loops, while loops, etc.).
• We make use of a variable (call it trav) that keeps track of where we are in the linked list.

```
StringNode trav = str;    // start with first node
while (trav != null) {
    // process the current node here
    trav = trav.next;     // move trav to next node
}
```

• Template for traversing an entire linked list:

Example of Iterative Traversal

• toUpperCase(str): converting str to all upper-case letters

```
public static void toUpperCase(StringNode str) {
    StringNode trav = str;
    while (trav != null) {
        trav.ch = Character.toUpperCase(trav.ch);
        trav = trav.next;
    }
}
```
Calling `StringNode.toUpperCase(str)` adds a stack frame to the stack:

```
StringNode trav = str;
```

We enter the while loop:
```
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

Tracing `toUpperCase()`: Part II
Tracing toUpperCase(): Part II

From the previous page:
```java
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

Results of the first pass through the loop:

Tracing toUpperCase(): Part III

```java
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```
Tracing `toUpperCase()`: Part III

```java
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

Results of the second pass through the loop:

```
str
str
str
'F'
'I'
'n'
'e'
null
str
str
str
'F'
'I'
'N'
'e'
null
```
Tracing toUpperCase(): Part III

while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}

results of the second pass through the loop:

results of the third pass:

Tracing toUpperCase(): Part IV

while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}

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Tracing `toUpperCase()`: Part IV

```java
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

Results of the fourth pass through the loop:

- `trav` is `null`.
- The `str` is updated to `'FINENull'`.

And now `trav == null`, so we end the loop and return:

- The `str` is updated to `'FINENull'`.
Getting the Node at Position $i$ in a Linked List

- `getNode(str, i)` – should return a reference to the $i$th node in the linked list to which `str` refers

  ![Linked List Diagram]

  - `str` — ['f', 'i', 'n'] — `null`

- Examples:
  - `getNode(str, 0)` should return a ref. to the 'f' node
  - `getNode(str, 3)` should return a ref. to the 'e' node
  - `getNode(str.next, 2)` should return a ref. to...

- More generally, when $0 < i < \text{length of list}$,
  `getNode(str, i)` is equivalent to `getNode(str.next, i-1)`

---

Getting the Node at Position $i$ in a Linked List

- Recursive approach to `getNode(str, i)`:
  - if $i = 0$, return `str` (base case)
  - else call `getNode(str.next, i-1)` and return what it returns!

- Here's the method:

  ```java
  private static StringNode getNode(StringNode str, int i) {
      if (i < 0 || str == null) { // base case 1: no node i
          return null;
      } else if (i == 0) { // base case 2: just found
          return str;
      } else {
          return getNode(str.next, i-1);
      }
  }
  ```
Deleting the Item at Position i

- **Special case:** \( i == 0 \) (deleting the first item)
- **Update our reference to the first node by doing:**
  
  \[
  \text{str} = \text{str}.next;
  \]

Deleting the Item at Position i (cont.)

- **General case:** \( i > 0 \)

  1. Obtain a reference to the *previous* node:
     
    \[
    \text{StringNode prevNode} = \text{getNode}(i - 1);
    \]

    (example for \( i == 1 \))

    \[
    \text{str} \quad \rightarrow \quad \text{'j'} \quad \rightarrow \quad \text{'a'} \quad \rightarrow \quad \text{'v'} \quad \rightarrow \quad \text{'a'} \quad \rightarrow \quad \text{null}
    \]

    \[
    \text{prevNode} \quad \rightarrow \quad \text{null}
    \]
Deleting the Item at Position i (cont.)

- **General case:** $i > 0$
  2. Update the references to remove the node

  *(example for $i = 1$)*

  **before:**
  
  ![Diagram of before state]

  **after:**
  
  ![Diagram of after state]

Inserting an Item at Position $i$

- **Special case:** $i == 0$ (insertion at the front of the list)

- **Step 1:** *Create* the new node. *Fill in the blanks!*

  **before:**
  
  ![Diagram of before state]

  **after:**
  
  ![Diagram of after state]

  ```
  StringNode newNode = new StringNode(_______, _______);
  ```
Inserting an Item at Position i (cont.)

- **Special case:** \( i == 0 \) (continued)
- **Step 2:** Insert the new node. *Write the assignment!*

**before (result of previous slide):**

\[
\begin{array}{c}
\text{ch} \quad | \quad \text{prevNode} \quad | \quad \text{newNode} \quad | \quad \text{str} \\
\text{'f'} \quad | \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ quad...
Returning a Reference to the First Node

- Both deleteChar() and insertChar() return a reference to the first node in the linked list. For example:

```java
public static StringNode deleteChar(StringNode str, int i) {
    if (i == 0) { // special case
        str = str.next;
    } else { // general case
        StringNode prevNode = getNode(str, i-1);
        if (prevNode != null && prevNode.next != null) {
            prevNode.next = prevNode.next.next;
        }
    }
    return str;
}
```

- Clients should call them as part of an assignment:
  ```java
  s1 = StringNode.deleteChar(s1, 0);
  s2 = StringNode.insertChar(s2, 0, 'h');
  ```

- If the first node changes, the client's variable will be updated to point to the new first node.

Creating a Copy of a Linked List

- copy(str) – create a copy of str and return a reference to it

  - Recursive approach:
    - base case: if str is empty, return null
    - else: copy the first character
      - make a recursive call to copy the rest

```java
public static StringNode copy(StringNode str) {
    if (str == null) { // base case
        return null;
    }

    // copy the first node (the one to which str refers)
    StringNode copyFirst = new StringNode(str.ch, null);

    // make a recursive call to copy the rest, and
    // store the result in copyFirst's next field
    copyFirst.next = copy(str.next);
    return copyFirst;
}
```
Tracing copy(): the initial call

• From a client: StringNode s2 = StringNode.copy(s1);

```java
public static StringNode copy(StringNode str) {
    if (str == null) {
        return null;
    }
    StringNode copyFirst = new StringNode(str.ch, null);
    copyFirst.next = copy(str.next);
    return copyFirst;
}
```

---

Tracing copy(): the initial call

• From a client: StringNode s2 = StringNode.copy(s1);

```java
public static StringNode copy(StringNode str) {
    if (str == null) {
        return null;
    }
    StringNode copyFirst = new StringNode(str.ch, null);
    copyFirst.next = copy(str.next);
    return copyFirst;
}
```
Tracing copy(): the recursive calls

- From a client: `StringNode s2 = StringNode.copy(s1);

```java
public static StringNode copy(StringNode str) {
    if (str == null) {
        return null;
    }
    StringNode copyFirst = new StringNode(str.ch, null);
    copyFirst.next = copy(str.next);
    return copyFirst;
}
```

Tracing copy(): the recursive calls
Tracing `copy()`: the recursive calls

```java
public static StringNode copy(...) {
    if (str == null) {
        return null;
    }

    StringNode copyFirst = new ...;
    copyFirst.next = copy(str.next);
    return copyFirst;
}
```

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Tracing copy(): the base case

```java
public static StringNode copy(...) {
    if (str == null) {
        return null;
    }
    StringNode copyFirst = new ...
    copyFirst.next = copy(str.next);
    return copyFirst;
}
```

Tracing copy(): returning from the base case

```java
public static StringNode copy(...) {
    if (str == null) {
        return null;
    }
    StringNode copyFirst = new ...
    copyFirst.next = copy(str.next);
    return copyFirst;
}
```
Tracing copy(): returning from the base case

```java
public static StringNode copy(...) {
    if (str == null) {
        return null;
    }

    StringNode copyFirst = new ...
    copyFirst.next = copy(str.next);
    return copyFirst;
}
```

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From a client: \begin{verbatim} StringNode s2 = StringNode.copy(s1); \end{verbatim}

Tracing \texttt{copy()}: returning from the base case

- \texttt{s2} now holds a reference to a linked list that is a copy of the linked list to which \texttt{s1} holds a reference.

Tracing \texttt{copy()}: Final Result

- \texttt{s2} now holds a reference to a linked list that is a copy of the linked list to which \texttt{s1} holds a reference.
Using a "Trailing Reference" During Traversal

• When traversing a linked list, one trav may not be enough.

• Ex: insert \( ch = 'n' \) at the right place in this sorted linked list:

\[
\text{str} \quad \text{'a'} \quad \text{'c'} \quad \text{'p'} \quad \text{'z'} \quad \text{null}
\]

• Traverse the list to find the right position:

```java
StringNode trav = str;
while (trav != null && trav.ch < ch) {
    trav = trav.next;
}
```

• When we exit the loop, where will trav point? Can we insert \( 'n' \)?

• The following changed version doesn't work either. Why not?

```java
while (trav != null && trav.next.ch < ch) {
    trav = trav.next;
}
```

Using a "Trailing Reference" (cont.)

• To get around the problem seen on the previous page, we traverse the list using two different references:
  • trav, which we use as before
  • trail, which stays one node behind trav

\[
\text{str} \quad \text{'a'} \quad \text{'c'} \quad \text{'p'} \quad \text{'z'} \quad \text{null}
\]

```java
StringNode trav = str;
StringNode trail = null;
while (trav != null && trav.ch < ch) {
    trail = trav;
    trav = trav.next;
}
// if trail == null, insert at the front of the list
// else insert after the node to which trail refers
```
Using a "Trailing Reference" (cont.)

- To get around the problem seen on the previous page, we traverse the list using two different references:
  - `trav`, which we use as before
  - `trail`, which stays one node behind `trav` 

```java
StringNode trav = str;
StringNode trail = null;
while (trav != null && trav.ch < ch) {
    trail = trav;
    trav = trav.next;
}
// if trail == null, insert at the front of the list
// else insert after the node to which trail refers
```

Using a "Trailing Reference" (cont.)

- To get around the problem seen on the previous page, we traverse the list using two different references:
  - `trav`, which we use as before
  - `trail`, which stays one node behind `trav` 

```java
StringNode trav = str;
StringNode trail = null;
while (trav != null && trav.ch < ch) {
    trail = trav;
    trav = trav.next;
}
// if trail == null, insert at the front of the list
// else insert after the node to which trail refers
```
Doubly Linked Lists

- In a doubly linked list, every node stores two references:
  - `next`, which works the same as before
  - `prev`, which holds a reference to the previous node
    - in the first node, `prev` has a value of null

- The `prev` references allow us to "back up" as needed.
  - remove the need for a trailing reference during traversal!

- Insertion and deletion must update both types of references.

Find the address and value of `s.next.next.ch`

<table>
<thead>
<tr>
<th>address</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 0xbe00</td>
<td>'r'</td>
</tr>
<tr>
<td>B. 0x3004</td>
<td>'e'</td>
</tr>
<tr>
<td>C. 0xbb00</td>
<td>'a'</td>
</tr>
<tr>
<td>D. none of these</td>
<td></td>
</tr>
</tbody>
</table>
Find the address and value of `s.next.next.ch`

- `s.next` is the next field in the node to which `s` refers
  - it holds a reference to the 'r' node
- thus, `s.next.next` is the next field in the 'r' node
  - it holds a reference to the 'e' node
- thus, `s.next.next.ch` is the ch field in the 'e' node
  - it holds the 'e'!

<table>
<thead>
<tr>
<th>Address</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0xbe00</td>
<td>'r'</td>
</tr>
<tr>
<td>0x3004</td>
<td>'e'</td>
</tr>
<tr>
<td>0xbb00</td>
<td>'a'</td>
</tr>
<tr>
<td>0xa004</td>
<td>null</td>
</tr>
</tbody>
</table>

- A. 0xbe00 'r'
- B. 0x3004 'e'
- C. 0xbb00 'a'
- D. none of these
Lists, Stacks, and Queues

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Representing a Sequence: Arrays vs. Linked Lists

- Sequence – an ordered collection of items (position matters)
  - we will look at several types: lists, stacks, and queues
- Can represent any sequence using an array or a linked list

<table>
<thead>
<tr>
<th></th>
<th>array</th>
<th>linked list</th>
</tr>
</thead>
<tbody>
<tr>
<td>representation in memory</td>
<td>elements occupy consecutive memory locations</td>
<td>nodes can be at arbitrary locations in memory; the links connect the nodes together</td>
</tr>
<tr>
<td>advantages</td>
<td>• provide random access (access to any item in constant time)</td>
<td>• can grow to an arbitrary length</td>
</tr>
<tr>
<td></td>
<td>• no extra memory needed for links</td>
<td>• allocate nodes as needed</td>
</tr>
<tr>
<td></td>
<td>• inserting or deleting does not require shifting items</td>
<td>• inserting or deleting does not require shifting items</td>
</tr>
<tr>
<td>disadvantages</td>
<td>• have to preallocate the memory needed for the maximum sequence size</td>
<td>• no random access (may need to traverse the list)</td>
</tr>
<tr>
<td></td>
<td>• inserting or deleting can require shifting items</td>
<td>• need extra memory for links</td>
</tr>
</tbody>
</table>
The List ADT

- A list is a sequence in which items can be accessed, inserted, and removed at any position in the sequence.

- The operations supported by our List ADT:
  - `getItem(i)`: get the item at position i
  - `addItem(item, i)`: add the specified item at position i
  - `removeItem(i)`: remove the item at position i
  - `length()`: get the number of items in the list
  - `isFull()`: test if the list already has the maximum number of items

- Note that we don’t specify how the list will be implemented.

Our List Interface

```java
public interface List {
    Object getItem(int i);
    boolean addItem(Object item, int i);
    Object removeItem(int i);
    int length();
    boolean isFull();
}
```

- Recall that all methods in an interface must be public, so we don’t need the keyword public in the headers.

- We use the object type to allow for items of any type.

- `addItem()` returns false if the list is full, and true otherwise.
Implementing a List Using an Array

```java
public class ArrayList implements List {
    private Object[] items;
    private int length;

    public ArrayList(int maxSize) {
        // code to check for invalid maxSize goes here...
        this.items = new Object[maxSize];
        this.length = 0;
    }

    public int length() {
        return this.length;
    }

    public boolean isFull() {
        return (this.length == this.items.length);
    }
}
```

![Diagram of ArrayList object](image)

Recall: The Implicit Parameter

- All non-static methods have an implicit parameter `(this)` that refers to the called object.
- In most cases, we're allowed to omit it!
  - we'll do so in the remaining notes
Omitting The Implicit Parameter

```java
public class ArrayList implements List {
    private Object[] items;
    private int length;

    public ArrayList(int maxSize) {
        items = new Object[maxSize];
        length = 0;
    }

    public int length() {
        return length;
    }

    public boolean isFull() {
        return (length == items.length);
    }
}
```

- In a non-static method, if we use a variable that
  - isn't declared in the method
  - has the name of one of the fields
  Java assumes that we're using the field.

Adding an Item to an ArrayList

- Adding at position i (shifting items i, i+1, ... to the right by one):
  ```java
  public boolean addItem(Object item, int i) {
      if (item == null || i < 0 || i > length) {
          throw new IllegalArgumentException();
      } else if (isFull()) {
          return false;
      }
      // make room for the new item
      for (int j = length - 1; j >= i; j--) {
          items[j + 1] = items[j];
      }
      items[i] = item;
      length++;
      return true;
  }
  ```

```
example for i = 3:

<table>
<thead>
<tr>
<th>items</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
```
Adding an Item to an ArrayList

- Adding at position i (shifting items i, i+1, ... to the right by one):

  public boolean addItem(Object item, int i) {
    if (item == null || i < 0 || i > length) {
      throw new IllegalArgumentException();
    } else if (isFull()) {
      return false;
    }
    // make room for the new item
    for (int j = length - 1; j >= i; j--) {
      items[j + 1] = items[j];
    }
    items[i] = item;
    length++;
    return true;
  }

example for i = 3:

<table>
<thead>
<tr>
<th>items</th>
<th>length</th>
</tr>
</thead>
<tbody>
<tr>
<td>null</td>
<td>7</td>
</tr>
</tbody>
</table>

Removing an Item from an ArrayList

- Removing item i (shifting items i+1, i+2, ... to the left by one):

  public Object removeItem(int i) {
    if (i < 0 || i >= length) {
      throw new IndexOutOfBoundsException();
    }
    Object removed = items[i];
    // shift items after items[i] to the left
    for (int j = i; j < length - 1; j++) {
      ______________________;
    }
    items[length - 1] = null;
    length--;
    return removed;
  }

example for i = 1:

<table>
<thead>
<tr>
<th>items</th>
<th>length</th>
</tr>
</thead>
<tbody>
<tr>
<td>null</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>removed</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Kylie&quot;</td>
</tr>
</tbody>
</table>
Getting an Item from an ArrayList

- Getting item i (without removing it):
  ```java
  public Object getItem(int i) {
      if (i < 0 || i >= length) {
          throw new IndexOutOfBoundsException();
      }
      return items[i];
  }
  ```

toString() Method for the ArrayList Class

```java
public String toString() {
    String str = "{";
    if (length > 0) {
        for (int i = 0; i < length - 1; i++) {
            str = str + items[i] + "", ";
        }
        str = str + items[length - 1];
    }
    str = str + "}";
    return str;
}
```

- Produces a string of the following form:
  ```
  {items[0], items[1], ... }
  ```

- Why is the last item added outside the loop?
- Why do we need the if statement?
Implementing a List Using a Linked List

```java
public class LLList implements List {
    private Node head;
    private int length;
    ...
}
```

- Differences from the linked lists we used for strings:
  - we "embed" the linked list inside another class
    - users of our LLList class won't actually touch the nodes
  - we use non-static methods instead of static ones
    - `myList.length()` instead of `length(myList)`
  - we use a special *dummy head node* as the first node

Using a Dummy Head Node

- The dummy head node is always at the front of the linked list.
  - like the other nodes in the linked list, it's of type `Node`
  - it does *not* store an item
  - it does *not* count towards the length of the list

- Using it allows us to avoid special cases when adding and removing nodes from the linked list.

- An empty LLList still has a dummy head node:
An Inner Class for the Nodes

public class LLList implements List {
    private class Node {
        private Object item;
        private Node next;
        private Node(Object i, Node n) {
            item = i;
            next = n;
        }
    }
    private Node head;
    private int length;
    public LLList() {
        head = new Node(null, null);
        length = 0;
    }
    public boolean isFull() {
        return false;
    }
    ...
}

- We make Node an *inner class*, defining it within LLList.
  - allows the LLList methods to directly access Node's private fields, while restricting access from outside LLList
  - the compiler creates this class file: LLList$Node.class

- For simplicity, our diagrams may show the items inside the nodes.

Other Details of Our LLList Class

public class LLList implements List {
    private class Node {
        // see previous slide
    }
    private Node head;
    private int length;
    public LLList() {
        head = new Node(null, null);
        length = 0;
    }
    public boolean isFull() {
        return false;
    }
    ...
}

- Unlike ArrayList, there's no need to preallocate space for the items. The constructor simply creates the dummy head node.
- The linked list can grow indefinitely, so the list is never full!
Getting a Node

- Private helper method for getting node i
- to get the dummy head node, use i = -1

```java
private Node getNode(int i) {
    // private method, so we assume i is valid!
    Node trav = null;
    int travIndex = -1;
    while (trav != null) {
        travIndex++;
        trav = trav.next;
    }
    return trav;
}
```

**Example for i = 1:**

```
head | length: 3
item 3
```

Getting an Item

```java
public Object getItem(int i) {
    if (i < 0 || i >= length) {
        throw new IndexOutOfBoundsException();
    }
    Node n = getNode(i);
    return _____;
}
```

**Example for i = 1:**

```
head | length: 3
item 3
```

```
n
```

```
head | length: 3
item 3
```

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head | length: 3
item 3
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head | length: 3
item 3
```

```
head | length: 3
item 3
```
Adding an Item to an LList

```java
public boolean addItem(Object item, int i) {
    if (item == null || i < 0 || i > length) {
        throw new IllegalArgumentException();
    }
    Node newNode = new Node(item, null);
    Node prevNode = getNode(i - 1);
    newNode.next = prevNode.next;
    prevNode.next = newNode;
    length++;
    return true;
}
```

- This works even when adding at the front of the list (i = 0):

![Diagram of adding an item at the front of the list]

addItem() Without a Dummy Head Node

```java
public boolean addItem(Object item, int i) {
    if (item == null || i < 0 || i > length) {
        throw new IllegalArgumentException();
    }
    Node newNode = new Node(item, null);

    if (i == 0) {
        // case 1: add to front
        newNode.next = head;
        head = newNode;
    } else {
        // case 2: i > 0
        Node prevNode = getNode(i - 1);
        newNode.next = prevNode.next;
        prevNode.next = newNode;
    }
    length++;
    return true;
}
```

*(the gray code shows what we would need to add if we didn't have a dummy head node)*
Removing an Item from an LList

```java
public Object removeItem(int i) {
    if (i < 0 || i >= length) {
        throw new IndexOutOfBoundsException();
    }
    Node prevNode = getNode(i - 1);
    Object removed = prevNode.next.item;
    // what line goes here?
    length--;
    return removed;
}
```

- This works even when removing the first item (i = 0):

```
CSCI S-111
```

```
CSCI S-111
```

```
David G. Sullivan, Ph.D.
```

```
David G. Sullivan, Ph.D.
```

```
434
```

```
434
```

toString() Method for the LList Class

```java
public String toString() {
    String str = "{
    // what should go here?

    str = str + "}";
    return str;
}
```
### Efficiency of the List ADT Implementations

\( n = \text{number of items in the list} \)

<table>
<thead>
<tr>
<th></th>
<th>ArrayList</th>
<th>LList</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>getItem()</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>only one case:</td>
<td></td>
<td>best:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>worst:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>average:</td>
</tr>
<tr>
<td><strong>addItem()</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>best:</strong></td>
<td></td>
<td>best:</td>
</tr>
<tr>
<td><strong>worst:</strong></td>
<td></td>
<td>worst:</td>
</tr>
<tr>
<td><strong>average:</strong></td>
<td></td>
<td>average:</td>
</tr>
</tbody>
</table>

### Efficiency of the List ADT Implementations (cont.)

\( n = \text{number of items in the list} \)

<table>
<thead>
<tr>
<th></th>
<th>ArrayList</th>
<th>LList</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>removeItem()</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>best:</strong></td>
<td></td>
<td>best:</td>
</tr>
<tr>
<td><strong>worst:</strong></td>
<td></td>
<td>worst:</td>
</tr>
<tr>
<td><strong>average:</strong></td>
<td></td>
<td>average:</td>
</tr>
</tbody>
</table>

| space efficiency |           |       |
Counting the Number of Occurrences of an Item

```java
public class MyClass {
    public static int numOccur(List l, Object item) {
        int numOccur = 0;
        for (int i = 0; i < l.length(); i++) {
            Object itemAt = l.getItem(i);
            if (itemAt.equals(item)) {
                numOccur++;
            }
        }
        return numOccur;
    }
    ... 
}
```

- This method works fine if we pass in an ArrayList object.
  - time efficiency (as a function of the length, n) = ?
- However, it's not efficient if we pass in an LList object.
  - each call to getItem() calls getNode()
  - to access item 0, getNode() accesses 2 nodes (dummy + node 0)
  - to access item 1, getNode() accesses 3 nodes
  - to access item i, getNode() accesses i+2 nodes
  - 2 + 3 + ... + (n+1) = ?

Solution: Provide an Iterator

```java
public class MyClass {
    public static int numOccur(List l, Object item) {
        int numOccur = 0;
        ListIterator iter = l.iterator();
        while (iter.hasNext()) {
            Object itemAt = iter.next();
            if (itemAt.equals(item)) {
                numOccur++;
            }
        }
        return numOccur;
    }
    ...
}
```

- We add an iterator() method to the List interface.
  - it returns a separate iterator object that can efficiently iterate over the items in the list
- The iterator has two key methods:
  - hasNext(): tells us if there are items we haven't seen yet
  - next(): returns the next item and advances the iterator
An Interface for List Iterators

• Here again, the interface only includes the method headers:
  ```java
  public interface ListIterator { // in ListIterator.java
    boolean hasNext();
    Object next();
  }
  ```

• We can then implement this interface for each type of list:
  • LLListIterator for an iterator that works with LLLists
  • ArrayListIterator for an iterator for ArrayLists

• We use the interfaces when declaring variables in client code:
  ```java
  public class MyClass {
    public static int numOccur(List l, Object item) {
      int numOccur = 0;
      ListIterator iter = l.iterator();
      ...
    }
  }
  ```

  • doing so allows the code to work for any type of list!

Using an Inner Class for the Iterator

```java
public class LLList {
  private Node head;
  private int length;

  private class LLListIterator implements ListIterator {
    private Node nextNode;  // points to node with the next item
    public LLListIterator() {
      nextNode = head.next;  // skip over dummy head node
    }
    ...
    public ListIterator iterator() {
      return new LLListIterator();
    }
    ...

  }
}
```

• Using an inner class gives the iterator access to the list’s internals.

• The iterator() method is an LLList method.
  • it creates an instance of the inner class and returns it
  • its return type is the interface type
    • so it will work in the context of client code
Full LLListIterator Implementation

```java
private class LLListIterator implements ListIterator {
    private Node nextNode; // points to node with the next item
    public LLListIterator() {
        nextNode = head.next; // skip over the dummy head node
    }
    public boolean hasNext() {
        return (nextNode != null);
    }
    public Object next() {
        // throw an exception if nextNode is null
        Object item = _______________;     // how
        nextNode = _______________;         // are
        return item; // you
    }
}
```

Stack ADT

- A stack is a sequence in which:
  - items can be added and removed only at one end (the top)
  - you can only access the item that is currently at the top
- Operations:
  - push: add an item to the top of the stack
  - pop: remove the item at the top of the stack
  - peek: get the item at the top of the stack, but don’t remove it
  - isEmpty: test if the stack is empty
  - isFull: test if the stack is full
- Example: a stack of integers

```plaintext
<table>
<thead>
<tr>
<th>start:</th>
<th>push 8:</th>
<th>pop:</th>
<th>pop:</th>
<th>push 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>15</td>
<td>15</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>
```
A Stack Interface: First Version

public interface Stack {
    boolean push(Object item);
    Object pop();
    Object peek();
    boolean isEmpty();
    boolean isFull();
}

• push() returns false if the stack is full, and true otherwise.
• pop() and peek() take no arguments, because we know that we always access the item at the top of the stack.
  • return null if the stack is empty.
• The interface provides no way to access/insert/delete an item at an arbitrary position.
  • encapsulation allows us to ensure that our stacks are only manipulated in appropriate ways.

Implementing a Stack Using an Array: First Version

public class ArrayStack implements Stack {
    private Object[] items;
    private int top;    // index of the top item
    public ArrayStack(int maxSize) {
        // code to check for invalid maxSize goes here...
        items = new Object[maxSize];
        top = -1;
    }
    ...

• Example: the stack

• Items are added from left to right (top item = the rightmost one).
  • push() and pop() won't require any shifting!
public class ArrayStack implements Stack {
    private Object[] items;
    private int top;  // index of the top item
    ...
}

- So far, our collections have allowed us to add objects of any type.
  ArrayStack s1 = new ArrayStack(4);
  s1.push(7);  // 7 is turned into an Integer object for 7
  s1.push("hi");
  String item = s1.pop();           // won't compile
  String item = (String)s1.pop();   // need a type cast

- We'd like to be able to limit a given collection to one type.
  ArrayStack<String> s2 = new ArrayStack<String>(10);
  s2.push(7);               // won't compile
  s2.push("hello");
  String item = s2.pop();   // no cast needed!

### Limiting a Stack to Objects of a Given Type

- We can do this by using a generic interface and class.

- Here's a generic version of our stack interface:
  ```java
  public interface Stack<T> {
      boolean push(T item);
      T pop();
      T peek();
      boolean isEmpty();
      boolean isFull();
  }
  ```

- It includes a type variable `T` in its header and body.
  - used as a placeholder for the actual type of the items
A Generic ArrayStack Class

public class ArrayStack<T> implements Stack<T> {
    private T[] items;  // index of the top item
    private int top;
    ...
    public boolean push(T item) {
        ...
    }
}

• Once again, a type variable T is used as a placeholder for the actual type of the items.

• When we create an ArrayStack, we specify the type of items that we intend to store in the stack:
  
  ```java
  ArrayStack<String> s1 = new ArrayStack<String>(10);
  ArrayStack<Integer> s2 = new ArrayStack<Integer>(25);
  ```

• We can still allow for a mixed-type collection:
  
  ```java
  ArrayStack<Object> s3 = new ArrayStack<Object>(20);
  ```

Using a Generic Class

```java
public class ArrayStack<String> {
    private String[] items;
    private int top;
    ...
    public boolean push(String item) {
        ...
    }
}
```

```java
ArrayStack<String> s1 = new ArrayStack<String>(10);

public class ArrayStack<T> {...
    private T[] items;
    private int top;
    ...
    public boolean push(T item) {
        ...
    }

ArrayStack<Integer> s2 = new ArrayStack<Integer>(25);

public class ArrayStack<Integer> {
    private Integer[] items;
    private int top;
    ...
    public boolean push(Integer item) {
        ...
    }
```
ArrayStack Constructor

- Java doesn't allow you to create an object or array using a type variable. Thus, we cannot do this:
  ```java
  public ArrayStack(int maxSize) {
    // code to check for invalid maxSize goes here...
    items = new T[maxSize]; // not allowed
    top = -1;
  }
  }
  ```

- Instead, we do this:
  ```java
  public ArrayStack(int maxSize) {
    // code to check for invalid maxSize goes here...
    items = (T[]) new Object[maxSize];
    top = -1;
  }
  ```

- The cast generates a compile-time warning, but we'll ignore it.

- Java's built-in ArrayList class takes this same approach.

Testing if an ArrayStack is Empty or Full

- Empty stack:
  ```java
  public boolean isEmpty() {
    return (top == -1);
  }
  ```

- Full stack:
  ```java
  public boolean isFull() {
    return (top == items.length - 1);
  }
  ```
Pushing an Item onto an ArrayStack

```java
public boolean push(T item) {
    // code to check for a null item goes here
    if (isFull()) {
        return false;
    }
    top++;
    items[top] = item;
    return true;
}
```

ArrayStack pop() and peek()

```java
public T pop() {
    if (isEmpty()) {
        return null;
    }
   removed = items[top];
    items[top] = null;
    top--;
    return removed;
}
```

- peek just returns items[top] without decrementing top.
Implementing a Generic Stack Using a Linked List

```java
public class LLStack<T> implements Stack<T> {
    private Node top;    // top of the stack
    ...
}

• Example: the stack

```

```
• Things worth noting:
  • our LLStack class needs only a single field:
    a reference to the first node, which holds the top item
  • top item = leftmost item (vs. rightmost item in ArrayStack)
  • we don’t need a dummy node
    • only one case: always insert/delete at the front of the list!
```

Other Details of Our LLStack Class

```java
public class LLStack<T> implements Stack<T> {
    private class Node {
        private T item;
        private Node next;
        ...
    }
    private Node top;
    public LLStack() {
        top = null;
    }
    public boolean isEmpty() {
        return (top == null);
    }
    public boolean isFull() {
        return false;
    }
}
```

• The inner Node class uses the type parameter T for the item.
• We don’t need to preallocate any memory for the items.
• The stack is never full!
public boolean push(T item) {
// code to check for a null item goes here
    Node newNode = new Node(item, top);
    top = newNode;
    return true;
}
LLStack pop() and peek()

```java
class LLStack {
    public T pop() {
        if (isEmpty()) {
            return null;
        }
        T removed = top.item;
        return removed;
    }
    public T peek() {
        if (isEmpty()) {
            return null;
        }
        return top.item;
    }
}
```

Efficiency of the Stack Implementations

<table>
<thead>
<tr>
<th></th>
<th>ArrayStack</th>
<th>LLStack</th>
</tr>
</thead>
<tbody>
<tr>
<td>push()</td>
<td>O(1)</td>
<td>O(1)</td>
</tr>
<tr>
<td>pop()</td>
<td>O(1)</td>
<td>O(1)</td>
</tr>
<tr>
<td>peek()</td>
<td>O(1)</td>
<td>O(1)</td>
</tr>
<tr>
<td>space efficiency</td>
<td>O(m) where m is the anticipated maximum number of items</td>
<td>O(n) where n is the number of items currently on the stack</td>
</tr>
</tbody>
</table>
Applications of Stacks

- Converting a recursive algorithm to an iterative one
  - use a stack to emulate the runtime stack

- Making sure that delimiters (parens, brackets, etc.) are balanced:
  - push open (i.e., left) delimiters onto a stack
  - when you encounter a close (i.e., right) delimiter, pop an item off the stack and see if it matches
  - example:
    $$5 \times [3 + \{(5 + 16 - 2)\}]$$

- Evaluating arithmetic expressions

Queue ADT

- A queue is a sequence in which:
  - items are added at the rear and removed from the front
    - first in, first out (FIFO) (vs. a stack, which is last in, first out)
    - you can only access the item that is currently at the front

- Operations:
  - insert: add an item at the rear of the queue
  - remove: remove the item at the front of the queue
  - peek: get the item at the front of the queue, but don’t remove it
  - isEmpty: test if the queue is empty
  - isFull: test if the queue is full

- Example: a queue of integers

  start: 12 8
  insert 5: 12 8 5
  remove: 8 5
Our Generic Queue Interface

```java
public interface Queue<T> {
    boolean insert(T item);
    T remove();
    T peek();
    boolean isEmpty();
    boolean isFull();
}
```

- `insert()` returns `false` if the queue is full, and `true` otherwise.
- `remove()` and `peek()` take no arguments, because we always access the item at the front of the queue.
  - return `null` if the queue is empty.
- Here again, we will use encapsulation to ensure that the data structure is manipulated only in valid ways.

Implementing a Queue Using an Array

```java
public class ArrayQueue<T> implements Queue<T> {
    private T[] items;
    private int front;
    private int rear;
    private int numItems;
    ...
}
```

- Example:
  - We maintain two indices:
    - `front`: the index of the item at the front of the queue
    - `rear`: the index of the item at the rear of the queue
Avoiding the Need to Shift Items

- Problem: what do we do when we reach the end of the array?
  - example: a queue of integers:

<table>
<thead>
<tr>
<th>front</th>
<th>rear</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>4</td>
</tr>
<tr>
<td>21</td>
<td>17</td>
</tr>
<tr>
<td>89</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

the same queue after removing two items and inserting two:

<table>
<thead>
<tr>
<th>front</th>
<th>rear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
</tr>
<tr>
<td>17</td>
<td>89</td>
</tr>
<tr>
<td>65</td>
<td>43</td>
</tr>
<tr>
<td>81</td>
<td></td>
</tr>
</tbody>
</table>

we have room for more items, but shifting to make room is inefficient

- Solution: maintain a circular queue. When we reach the end of the array, we wrap around to the beginning.

  insert 5: wrap around!

<table>
<thead>
<tr>
<th>rear</th>
<th>front</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>17</td>
<td>89</td>
</tr>
<tr>
<td>65</td>
<td>43</td>
</tr>
<tr>
<td>81</td>
<td></td>
</tr>
</tbody>
</table>

Maintaining a Circular Queue

- We use the mod operator (%) when updating front or rear:
  - front = (front + 1) % items.length;
  - rear = (rear + 1) % items.length;

- Example:
Maintaining a Circular Queue

- We use the mod operator (%) when updating front or rear:
  \[ \text{front} = (\text{front} + 1) \mod \text{items.length}; \]
  \[ \text{rear} = (\text{rear} + 1) \mod \text{items.length}; \]

- Example:
  - \text{q.insert(81):} // rear is not at end of array
    \[ \text{rear} = (\text{rear} + 1) \mod \text{items.length}; \]
    \[ = (4 + 1) \mod 6 \]
    \[ = 5 \mod 6 = 5 \text{ (mod has no effect)} \]

- \text{q.insert(33):} // rear is at end of array
  \[ \text{rear} = (\text{rear} + 1) \mod \text{items.length}; \]
  \[ = (5 + 1) \mod 6 \]
  \[ = 6 \mod 6 = 0 \text{ wrap around!} \]
Inserting an Item in an ArrayQueue

- We increment rear before adding the item:

  ![ArrayQueue diagram](image)

  public boolean insert(T item) {
      // code to check for a null item goes here
      if (isFull()) {
          return false;
      }
      rear = (rear + 1) % items.length;
      items[rear] = item;
      numItems++;
      return true;
  }

ArrayQueue remove()

- Remove the item at the front of the queue:

  ![ArrayQueue diagram](image)

  public T remove() {
      if (isEmpty()) {
          return null;
      }
      T removed = _________________;
      numItems--;
      return removed;
  }
Constructor

```java
public ArrayQueue(int maxSize) {
    // code to check for an invalid maxSize goes here...
    items = (T[])new Object[maxSize];
    front = 0;
    rear = -1;
    numItems = 0;
}
```

- When we insert the first item in a newly created ArrayQueue, we want it to go in position 0. Thus, we need to:
  - start rear at -1, since then it will be incremented to 0 and used to perform the insertion
  - start front at 0, since it is not changed by the insertion

```
<table>
<thead>
<tr>
<th>items</th>
<th>front</th>
<th>rear</th>
<th>numItems</th>
</tr>
</thead>
<tbody>
<tr>
<td>null</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>items</th>
<th>front</th>
<th>rear</th>
<th>numItems</th>
</tr>
</thead>
<tbody>
<tr>
<td>null</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
```

Testing if an ArrayQueue is Empty or Full

- In both empty and full queues, rear is one "behind" front:

```
<table>
<thead>
<tr>
<th>initial configuration:</th>
</tr>
</thead>
<tbody>
<tr>
<td>rear</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>after two insertions and two removals:</td>
</tr>
<tr>
<td>rear</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>after 7 more insertions:</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>
```

- This is why we maintain numItems!

```java
public boolean isEmpty() {
    return (numItems == 0);
}
```

```java
public boolean isFull() {
    return (numItems == items.length);
}
```
Implementing a Queue Using a Linked List

```java
public class LLQueue<T> implements Queue<T> {
    private Node front;    // front of the queue
    private Node rear;     // rear of the queue

    ...
}
```

- Example:

```
In a linked list, we can efficiently:
• remove the item at the front
• add an item to the rear (if we have a ref. to the last node)

Thus, this implementation is simpler than the array-based one!
```

Other Details of Our LLQueue Class

```java
public class LLQueue<T> implements Queue<T> {
    private class Node {
        private T item;
        private Node next;
    }

    private Node front;
    private Node rear;

    public LLQueue() {
        front = null;
        rear = null;
    }

    public boolean isEmpty() {
        return (front == null);
    }

    public boolean isFull() {
        return false;
    }

    ...
}
```
Inserting an Item in an Empty LLQueue

```java
public boolean insert(T item) {
    // code to check for a null item goes here
    Node newNode = new Node(item, null);
    if (isEmpty()) {
        front = newNode;
        rear = newNode;
    } else {
        // we'll add this later!
    }
    return true;
}
```

Inserting an Item in a Non-Empty LLQueue

```java
public boolean insert(T item) {
    // code to check for a null item goes here
    Node newNode = new Node(item, null);
    if (isEmpty()) {
        front = newNode;
        rear = newNode;
    } else {
        // we'll add this later!
    }
    return true;
}
```
Removing from an LLQueue with One Item

public T remove() {
    if (isEmpty()) {
        return null;
    }
    T removed = _________________;
    if (front == rear) {     // removing the only item
        front = null;
        rear = null;
    } else {
        // we'll add this later
    }
    return removed;
}

Removing from an LLQueue with Two or More Items

public T remove() {
    if (isEmpty()) {
        return null;
    }
    T removed = _________________;
    if (front == rear) {     // removing the only item
        front = null;
        rear = null;
    } else {
        // we'll add this later
    }
    return removed;
}
Efficiency of the Queue Implementations

<table>
<thead>
<tr>
<th></th>
<th>ArrayQueue</th>
<th>LLQueue</th>
</tr>
</thead>
<tbody>
<tr>
<td>insert()</td>
<td>O(1)</td>
<td>O(1)</td>
</tr>
<tr>
<td>remove()</td>
<td>O(1)</td>
<td>O(1)</td>
</tr>
<tr>
<td>peek()</td>
<td>O(1)</td>
<td>O(1)</td>
</tr>
<tr>
<td>space efficiency</td>
<td>O(m) where m is the anticipated maximum number of items</td>
<td>O(n) where n is the number of items currently in the queue</td>
</tr>
</tbody>
</table>

Applications of Queues

- first-in first-out (FIFO) inventory control
- OS scheduling: processes, print jobs, packets, etc.
- simulations of banks, supermarkets, airports, etc.
Motivation: Implementing a Dictionary

- A *data dictionary* is a collection of data with two main operations:
  - *search* for an item (and possibly delete it)
  - *insert* a new item

- If we use a *sorted* list to implement it, efficiency = \(O(n)\).

<table>
<thead>
<tr>
<th>data structure</th>
<th>searching for an item</th>
<th>inserting an item</th>
</tr>
</thead>
<tbody>
<tr>
<td>a list implemented using an array</td>
<td>(O(\log n)) using binary search</td>
<td>(O(n)) because we need to shift items over</td>
</tr>
<tr>
<td>a list implemented using a linked list</td>
<td>(O(n)) using linear search</td>
<td>(O(n)) ((O(1)) to do the actual insertion, but (O(n)) to find where it belongs)</td>
</tr>
</tbody>
</table>

- In the next few lectures, we’ll look at how we can use a *tree* for a data dictionary, and we’ll try to get better efficiency.
- We’ll also look at other applications of trees.
What Is a Tree?

- A tree consists of:
  - a set of nodes
  - a set of edges, each of which connects a pair of nodes

- Each node may have one or more data items.
  - each data item consists of one or more fields
  - key field = the field used when searching for a data item
  - data items with the same key are referred to as duplicates

- The node at the "top" of the tree is called the root of the tree.

Relationships Between Nodes

- If a node N is connected to nodes directly below it in the tree:
  - N is referred to as their parent
  - they are referred to as its children.
    - example: node 5 is the parent of nodes 10, 11, and 12

- Each node is the child of at most one parent.

- Nodes with the same parent are siblings.
A node’s \textit{ancestors} are its parent, its parent’s parent, etc.
\begin{itemize}
  \item example: node 9’s ancestors are 3 and 1
\end{itemize}

A node’s \textit{descendants} are its children, their children, etc.
\begin{itemize}
  \item example: node 1’s descendants are all of the other nodes
\end{itemize}

\begin{itemize}
  \item A \textit{leaf node} is a node without children.
  \item An \textit{interior node} is a node with one or more children.
\end{itemize}
A Tree is a Recursive Data Structure

• Each node in the tree is the root of a smaller tree!
  • refer to such trees as subtrees to distinguish them from the tree as a whole
  • example: node 2 is the root of the subtree circled above
  • example: node 6 is the root of a subtree with only one node

• We’ll see that tree algorithms often lend themselves to recursive implementations.

Path, Depth, Level, and Height

• There is exactly one path (one sequence of edges) connecting each node to the root.

• depth of a node = # of edges on the path from it to the root

• Nodes with the same depth form a level of the tree.

• The height of a tree is the maximum depth of its nodes.
  • example: the tree above has a height of 2
Binary Trees

- In a *binary tree*, nodes have *at most two* children.
  - distinguish between them using the direction *left* or *right*

- Example:

- Recursive definition: a binary tree is either:
  1) empty, or
  2) a node (the root of the tree) that has:
    - one or more pieces of data (the key, and possibly others)
    - a *left subtree*, which is itself a binary tree
    - a *right subtree*, which is itself a binary tree

---

Which of the following is/are **not** true?

- A. This tree has a height of 4.
- B. There are 3 leaf nodes.
- C. The 38 node is the right child of the 32 node.
- D. The 12 node has 3 children.
- E. more than one of the above are **not** true (which ones?)
Representing a Binary Tree Using Linked Nodes

```java
public class LinkedTree {
    private class Node {
        private int key;      // limit ourselves to int keys
        private LLList data;  // list of data for that key
        private Node left;    // reference to left child
        private Node right;   // reference to right child
    }
    private Node root;
}
```
Traversing a Binary Tree

- Traversing a tree involves visiting all of the nodes in the tree.
  - visiting a node = processing its data in some way
    - example: print the key

- We'll look at four types of traversals.
  - each visits the nodes in a different order

- To understand traversals, it helps to remember that every node is the root of a subtree.

1: Preorder Traversal

- preorder traversal of the tree whose root is N:
  1) visit the root, N
  2) recursively perform a preorder traversal of N's left subtree
  3) recursively perform a preorder traversal of N's right subtree

- preorder because a node is visited before its subtrees
- The root of the tree as a whole is visited first.
Implementing Preorder Traversal

```java
public class LinkedTree {
    private Node root;
    public void preorderPrint() {
        if (root != null) {
            preorderPrintTree(root);
        }
        System.out.println();
    }
    private static void preorderPrintTree(Node root) {
        System.out.print(root.key + " ");
        if (root.left != null) {
            preorderPrintTree(root.left);
        }
        if (root.right != null) {
            preorderPrintTree(root.right);
        }
    }
}
```

- `preorderPrintTree()` is a static, recursive method that takes
the root of the tree/subtree that you want to print.
- `preorderPrint()` is a non-static "wrapper" method that makes
the initial call. It passes in the root of the entire tree.

Tracing Preorder Traversal

```java
void preorderPrintTree(Node root) {
    System.out.print(root.key + " ");
    if (root.left != null) {
        preorderPrintTree(root.left);
    }
    if (root.right != null) {
        preorderPrintTree(root.right);
    }
}
```

base case, since
neither recursive
call is made!
we go back
up the tree
by returning!
Using Recursion for Traversals

void preorderPrintTree(Node root) {
    System.out.print(root.key + " ");
    if (root.left != null) {
        preorderPrintTree(root.left);
    }
    if (root.right != null) {
        preorderPrintTree(root.right);
    }
}

- Using recursion allows us to easily go back up the tree.
- Using a loop would be harder. Why?

2: Postorder Traversal

- postorder traversal of the tree whose root is N:
  1) recursively perform a postorder traversal of N's left subtree
  2) recursively perform a postorder traversal of N's right subtree
  3) visit the root, N

- postorder because a node is visited after its subtrees
- The root of the tree as a whole is visited last.
Implementing Postorder Traversal

```java
public class LinkedTree {
    private Node root;
    public void postorderPrint() {
        if (root != null) {
            postorderPrintTree(root);
        }
        System.out.println();
    }

    private static void postorderPrintTree(Node root) {
        if (root.left != null) {
            postorderPrintTree(root.left);
        }
        if (root.right != null) {
            postorderPrintTree(root.right);
        }
        System.out.print(root.key + " ");
    }
}
```

• Note that the root is printed after the two recursive calls.

Tracing Postorder Traversal

```java
void postorderPrintTree(Node root) {
    if (root.left != null) {
        postorderPrintTree(root.left);
    }
    if (root.right != null) {
        postorderPrintTree(root.right);
    }
    System.out.print(root.key + " ");
}
```
3: Inorder Traversal

• inorder traversal of the tree whose root is N:
  1) recursively perform an inorder traversal of N's left subtree
  2) visit the root, N
  3) recursively perform an inorder traversal of N's right subtree

• The root of the tree as a whole is visited between its subtrees.

• We'll see later why this is called in order traversal!

Implementing Inorder Traversal

```
public class LinkedTree {
    private Node root;
    public void inorderPrint() {
        if (root != null) {
            inorderPrintTree(root);
        }
        System.out.println();
    }
    private static void inorderPrintTree(Node root) {
        if (root.left != null) {
            inorderPrintTree(root.left);
        }
        System.out.print(root.key + " ");
        if (root.right != null) {
            inorderPrintTree(root.right);
        }
    }
}
```

• Note that the root is printed between the two recursive calls.
Tracing Inorder Traversal

```java
void inorderPrintTree(Node root) {
    if (root.left != null) {
        inorderPrintTree(root.left);
    }
    System.out.print(root.key + " ");
    if (root.right != null) {
        inorderPrintTree(root.right);
    }
}
```

Order in which nodes are visited:

```
root: 4, print 4
root: 8, print 8
root: 9, print 9
root: 7
root: 6, print 6
root: 2
root: 5
root: 9
```

time

Level-Order Traversal

- Visit the nodes one level at a time, from top to bottom and left to right.

```
Level-order traversal of the tree above: 7 9 5 8 6 2 4
```

- We can implement this type of traversal using a queue.
Tree-Traversals Summary

- **preorder**: root, left subtree, right subtree
- **postorder**: left subtree, right subtree, root
- **inorder**: left subtree, root, right subtree
- **level-order**: top to bottom, left to right

- Perform each type of traversal on the tree below:

![Tree Diagram](image)

Tree Traversal Puzzle

- **preorder traversal**: A M P K L D H T
- **inorder traversal**: P M L K A H T D
- Draw the tree!

- What's one fact that we can easily determine from one of the traversals?
Using a Binary Tree for an Algebraic Expression

- We'll restrict ourselves to fully parenthesized expressions using the following binary operators: +, −, *, /

- Example: \((a + (3 \times c)) - (d / 2)\)

- Leaf nodes are variables or constants.
- Interior nodes are operators.
  - their children are their operands

Traversing an Algebraic-Expression Tree

- Inorder gives conventional algebraic notation.
  - print '(' before the recursive call on the left subtree
  - print ')' after the recursive call on the right subtree
  - for tree at right: \(((a + (b \times c)) - (d / e))\)

- Preorder gives functional notation.
  - print '('s and ')'s as for inorder, and commas after the recursive call on the left subtree
  - for tree above: \(\text{subtr}(\text{add}(a, \text{mult}(b, c)), \text{divide}(d, e))\)

- Postorder gives the order in which the computation must be carried out on a stack/RPN calculator.
  - for tree above: push a, push b, push c, multiply, add,...
Fixed-Length Character Encodings

• A character encoding maps each character to a number.

• Computers usually use fixed-length character encodings.
  • ASCII - 8 bits per character
    example: "bat" is stored in a text file as the following sequence of bits:
    01100010 01100001 01110100
  • Unicode - 16 bits per character
    (allows for foreign-language characters; ASCII is a subset)

• Fixed-length encodings are simple, because:
  • all encodings have the same length
  • a given character always has the same encoding

<table>
<thead>
<tr>
<th>char</th>
<th>dec</th>
<th>binary</th>
</tr>
</thead>
<tbody>
<tr>
<td>'a'</td>
<td>97</td>
<td>01100001</td>
</tr>
<tr>
<td>'b'</td>
<td>98</td>
<td>01100010</td>
</tr>
<tr>
<td></td>
<td></td>
<td>...</td>
</tr>
<tr>
<td>'t'</td>
<td>116</td>
<td>01110100</td>
</tr>
</tbody>
</table>

A Problem with Fixed-Length Encodings

• They tend to waste space.

• Example: an English newspaper article with only:
  • upper and lower-case letters (52 characters)
  • spaces and newlines (2 characters)
  • common punctuation (approx. 10 characters)
  • total of 64 unique characters ➔ only need ___ bits

• We could gain even more space if we:
  • gave the most common letters shorter encodings (3 or 4 bits)
  • gave less frequent letters longer encodings (> 6 bits)
Variable-Length Character Encodings

- Variable-length encodings *compress* a text file by:
  - using encodings of different lengths for different characters
  - assigning shorter encodings to frequently occurring characters

- Example: if we had only four characters

<table>
<thead>
<tr>
<th>char</th>
<th>encoding</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>01</td>
</tr>
<tr>
<td>o</td>
<td>100</td>
</tr>
<tr>
<td>s</td>
<td>111</td>
</tr>
<tr>
<td>t</td>
<td>00</td>
</tr>
</tbody>
</table>

"test" would be encoded as

00 01 111 00 \(\rightarrow\) 000111100

- Challenge: when reading a document, how do we determine the boundaries between characters?
  - how do we know how many bits the next character has?
- One requirement: no character's encoding can be the prefix of another character's encoding (e.g., couldn't have 00 and 001).

Huffman Encoding

- One type of variable-length encoding

- Based on the actual character frequencies in a given document
  - different documents have different encodings

- Huffman encoding uses a binary tree:
  - to determine the encoding of each character
  - to *decode / decompress* an encoded file
  - putting it back into ASCII
Huffman Trees

- Example for a text with only six characters:

- Left branches are labeled with a 0, right branches with a 1.
- Leaf nodes are characters.
- To get a character's encoding, follow the path from the root to its leaf node.
  - example: i = ?

Building a Huffman Tree

1) Begin by reading through the text to determine the frequencies.

2) Create a list of nodes containing (character, frequency) pairs for each character in the text – sorted by frequency.

3) Remove and "merge" the nodes with the two lowest frequencies, forming a new node that is their parent.
   - left child = lowest frequency node
   - right child = the other node
   - frequency of parent = sum of the frequencies of its children
     - in this case, 11 + 23 = 34
Building a Huffman Tree (cont.)

4) Add the parent to the list of nodes (maintaining sorted order):

5) Repeat steps 3 and 4 until there is only a single node in the list, which will be the root of the Huffman tree.

Completing the Huffman Tree Example I

- Merge the two remaining nodes with the lowest frequencies:
Completing the Huffman Tree Example II

- Merge the next two nodes:

![Huffman Tree Diagram](image)

- Merge again:

![Huffman Tree Diagram](image)
Completing the Huffman Tree Example IV

- The next merge creates the final tree:

- Characters that appear more frequently end up higher in the tree, and thus their encodings are shorter.

The Shape of the Huffman Tree

- The tree on the last slide is fairly symmetric.

- This won't always be the case!
  - depends on the character frequencies

- For example, changing the frequency of 'o' from 11 to 21 would produce the tree shown below:

- This is the tree that we'll use in the remaining slides.
Huffman Encoding: Compressing a File

1) Read through the input file and build its Huffman tree.
2) Write a file header for the output file.
   • include the character frequencies so the tree can be rebuilt when the file is decompressed
3) Traverse the Huffman tree to create a table containing the encoding of each character:

4) Read through the input file a second time, and write the Huffman code for each character to the output file.

Huffman Decoding: Decompressing a File

1) Read the frequency table from the header and rebuild the tree.
2) Read one bit at a time and traverse the tree, starting from the root:
   when you read a bit of 1, go to the right child
   when you read a bit of 0, go to the left child
   when you reach a leaf node, record the character, return to the root, and continue reading bits

   *The tree allows us to easily overcome the challenge of determining the character boundaries*

example: 10111110000111100
first character = i
What are the next three characters?

1) Read the frequency table from the header and rebuild the tree.
2) Read one bit at a time and traverse the tree, starting from the root:
   - when you read a bit of 1, go to the right child
   - when you read a bit of 0, go to the left child
   - when you reach a leaf node, record the character,
     return to the root, and continue reading bits

The tree allows us to easily overcome the challenge of determining the character boundaries!

example: 10111110000111100
first character = i (101)
Huffman Decoding: Decompressing a File

1) Read the frequency table from the header and rebuild the tree.
2) Read one bit at a time and traverse the tree, starting from the root:
   - when you read a bit of 1, go to the right child
   - when you read a bit of 0, go to the left child
   - when you reach a leaf node, record the character,
     return to the root, and continue reading bits

The tree allows us to easily overcome the challenge of determining the character boundaries!

example: 1011111100001111

00 = left,left = t
01 = left,right = e
11 = right,right,right= s
10 = right,right,left = a
111 = right,right,right= s
110 = right,right,left = a
00 = left,left = t
01 = left,right = e
111 = right,right,right= s
110 = right,right,left = a
00 = left,left = t
Search Trees

Computer Science S-111
Harvard University
David G. Sullivan, Ph.D.

Binary Search Trees

- Search-tree property: for each node $k$ ($k$ is the key):
  - all nodes in $k$’s left subtree are $< k$
  - all nodes in $k$’s right subtree are $\geq k$

- Our earlier binary-tree example is a search tree:

- With a search tree, an inorder traversal visits the nodes in order!
  - in order of increasing key values
Searching for an Item in a Binary Search Tree

- Algorithm for searching for an item with a key $k$:
  - if $k ==$ the root node’s key, you’re done
  - else if $k <$ the root node’s key, search the left subtree
  - else search the right subtree

- Example: search for 7

Implementing Binary-Tree Search

```java
public class LinkedTree {    // Nodes have keys that are ints
    private Node root;
    public LLList search(int key) {        // "wrapper method"
        Node n = searchTree(root, key);        // get Node for key
        if (n == null) {
            return null;            // no such key
        } else {
            return n.data;            // return list of values for key
        }
    }

    private static Node searchTree(Node root, int key) {
        if ( ) {
            } else if ( ) {
            } else if (               ) {
            } else {
            }
    }
```
Inserting an Item in a Binary Search Tree

- public void insert(int key, Object data)
  will add a new (key, data) pair to the tree

- Example 1: a search tree containing student records
  - key = the student's ID number (an integer)
  - data = a string with the rest of the student record
  - we want to be able to write client code that looks like this:
    ```java
    LinkedTree students = new LinkedTree();
    students.insert(23, "Jill Jones,sophomore,comp sci");
    students.insert(45, "Al Zhang,junior,english");
    ```

- Example 2: a search tree containing scrabble words
  - key = a scrabble score (an integer)
  - data = a word with that scrabble score
    ```java
    LinkedTree tree = new LinkedTree();
    tree.insert(4, "lost");
    ```

Inserting an Item in a Binary Search Tree (cont.)

- To insert an item \((k, d)\),
  we start by searching for \(k\).

- If we find a node with key \(k\), we add \(d\) to the list of data values for that node.
  - example: `tree.insert(4, "sail")`

- If we don't find \(k\), the last node seen in the search becomes the parent \(P\) of the new node \(N\).
  - if \(k < P\)'s key, make \(N\) the left child of \(P\)
  - else make \(N\) the right child of \(P\)

- Special case: if the tree is empty,
  make the new node the root of the tree.

- Important: The resulting tree is still a search tree!
Implementing Binary-Tree Insertion

- We'll implement part of the `insert()` method together.
- We'll use iteration rather than recursion.
- Our method will use two references/pointers:
  - `trav`: performs the traversal down to the point of insertion
  - `parent`: stays one behind `trav`
    - like the `trail` reference that we sometimes use when traversing a linked list

```java
public void insert(int key, Object data) {
    Node parent = null;
    Node trav = root;
    while (trav != null) {
        if (trav.key == key) {
            trav.data.addItem(data, 0);
            return;
        }
        // what should go here?
    }
    Node newNode = new Node(key, data);
    if (root == null) { // the tree was empty
        root = newNode;
    } else if (key < parent.key) {
        parent.left = newNode;
    } else {
        parent.right = newNode;
    }
}
```
Deleting Items from a Binary Search Tree

- Three cases for deleting a node $x$
  - **Case 1:** $x$ has no children.
    Remove $x$ from the tree by setting its parent's reference to null.

  ![](image1.png)

  ex: delete 4

  ![Tree Diagram](image2.png)

- **Case 2:** $x$ has one child.
  Take the parent's reference to $x$ and make it refer to $x$'s child.

  ![](image3.png)

  ex: delete 12

  ![Tree Diagram](image4.png)

Deleting Items from a Binary Search Tree (cont.)

- **Case 3:** $x$ has two children
  - we can't give both children to the parent. why?

  • instead, we leave $x$'s node where it is, and we replace its key and data with those from another node
  • the replacement must maintain the search-tree inequalities

  ex: delete 12

  ![Tree Diagram](image5.png)

  two options: which ones?
Deleting Items from a Binary Search Tree (cont.)

- **Case 3:** \( x \) has two children (continued):
  - replace \( x \)'s key and data with those from the smallest node in \( x \)'s right subtree—call it \( y \)
  - we then delete \( y \)
    - it will either be a leaf node or will have one right child. why?
  - thus, we can delete it using case 1 or 2

ex: delete 12

Which Nodes Could We Use To Replace 9?
Implementing Deletion

```java
public LLList delete(int key) {
    // Find the node and its parent.
    Node parent = null;
    Node trav = root;
    while (trav != null && trav.key != key) {
        parent = trav;
        if (key < trav.key) {
            trav = trav.left;
        } else {
            trav = trav.right;
        }
    }
    // Delete the node (if any) and return the removed items.
    if (trav == null) {   // no such key
        return null;
    } else {
        LLList removedData = trav.data;
        deleteNode(trav, parent); // call helper method
        return removedData;
    }
}
```

Implementing Case 3

```java
private void deleteNode(Node toDelete, Node parent) {
    if (toDelete.left != null && toDelete.right != null) {
        // Find a replacement - and
        // the replacement's parent.
        Node replaceParent = toDelete;
        // Get the smallest item
        // in the right subtree.
        Node replace = toDelete.right;
        // what should go here?
        toDelete.key = replace.key;
        toDelete.data = replace.data;
        // Recursively delete the replacement
        // item's old node. It has at most one
        // child, so we don't have to
        // worry about infinite recursion.
        deleteNode(replace, replaceParent);
    } else {
        ...
    }
}
```
Implementing Cases 1 and 2

```java
private void deleteNode(Node toDelete, Node parent) {
    if (toDelete.left != null && toDelete.right != null) {
        ...
    } else {
        Node toDeleteChild;
        if (toDelete.left != null)
            toDeleteChild = toDelete.left;
        else
            toDeleteChild = toDelete.right;
        // Note: in case 1, toDeleteChild
        // will have a value of null.
        if (toDelete == root)
            root = toDeleteChild;
        else if (toDelete.key < parent.key)
            parent.left = toDeleteChild;
        else
            parent.right = toDeleteChild;
    }
}
```

Recall: Path, Depth, Level, and Height

- There is exactly one path (one sequence of edges) connecting each node to the root.
- Depth of a node = # of edges on the path from it to the root
- Nodes with the same depth form a level of the tree.
- The height of a tree is the maximum depth of its nodes.
  - example: the tree above has a height of 2
Efficiency of a Binary Search Tree

• For a tree containing \( n \) items, what is the efficiency of any of the traversal algorithms?
  • you process all \( n \) of the nodes
  • you perform \( O(1) \) operations on each of them

• Search, insert, and delete all have the same time complexity.
  • insert is a search followed by \( O(1) \) operations
  • delete involves either:
    • a search followed by \( O(1) \) operations (cases 1 and 2)
    • a search partway down the tree for the item, followed by a search further down for its replacement, followed by \( O(1) \) operations (case 3)

Efficiency of a Binary Search Tree (cont.)

• Time complexity of searching:
  • best case:

  • worst case:
    • you have to go all the way down to level \( h \) before finding the key or realizing it isn't there
    • along the path to level \( h \), you process \( h + 1 \) nodes
  • average case:

• What is the height of a tree containing \( n \) items?
Balanced Trees

- A tree is balanced if, for each of its nodes, the node’s subtrees have the same height or have heights that differ by 1.
  - example:
    - 26: both subtrees have a height of 1
    - 12: left subtree has height 0, right subtree is empty (height = -1)
    - 32: both subtrees have a height of 0
    - all leaf nodes: both subtrees are empty
  - For a balanced tree with \( n \) nodes, height = \( O(\log n) \)
    - each time that you follow an edge down the longest path, you cut the problem size roughly in half!
  - Therefore, for a balanced binary search tree, the worst case for search / insert / delete is \( O(h) = O(\log n) \)
    - the "best" worst-case time complexity

What If the Tree Isn't Balanced?

- Extreme case: the tree is equivalent to a linked list
  - height = \( n - 1 \)
- Therefore, for an unbalanced binary search tree, the worst case for search / insert / delete is \( O(h) = O(n) \)
  - the "worst" worst-case time complexity
- We’ll look next at search-tree variants that take special measures to ensure balance.
2-3 Trees

- A 2-3 tree is a balanced tree in which:
  - all nodes have equal-height subtrees (perfect balance)
  - each node is either
    - a 2-node, which contains one data item and 0 or 2 children
    - a 3-node, which contains two data items and 0 or 3 children
  - the keys form a search tree

- Example:

```
2-node:              3-node:
     28 61             77 90
     10               80 87
     3 14 20 34 51     93 97
     68                

2-node:             3-node:
     k               k1 k2
     <k            <k1 >k1
     >k            >k2 >k2
```

Search in 2-3 Trees

- Algorithm for searching for an item with a key $k$:
  - if $k$ == one of the root node’s keys, you’re done
  - else if $k$ < the root node’s first key
    - search the left subtree
  - else if the root is a 3-node and $k$ < its second key
    - search the middle subtree
  - else
    - search the right subtree

- Example: search for 87

```
28 61
  10
  3 14 20
  34 51
  68 80 87
  93 97
```
Insertion in 2-3 Trees

- Algorithm for inserting an item with a key $k$:
  search for $k$, but don’t stop until you hit a leaf node
  let $L$ be the leaf node at the end of the search
  if $L$ is a 2-node
    add $k$ to $L$, making it a 3-node
  else if $L$ is a 3-node
    split $L$ into two 2-nodes containing the items with the smallest and largest of: $k$, $L$’s 1st key, $L$’s 2nd key
    the middle item is “sent up” and inserted in $L$’s parent

example: add 52

Example 1: Insert 8

- Search for 8:

- Add 8 to the leaf node, making it a 3-node:
Example 2: Insert 17

- Search for 17:

- Split the leaf node, and send up the middle of 14, 17, 20 and insert it the leaf node's parent:

Example 3: Insert 92

- In which node will we initially try to insert it?
Example 3: Insert 92

- Search for 92:

```
28  61
  /    
10    40
  /  
3    14 20
  /    
34   51
```

- Split the leaf node, and send up the middle of 92, 93, 97 and insert it the leaf node’s parent:

```
28 61
  /    
40    77 90
    /  
34 51
```

- In this case, the leaf node’s parent is also a 3-node, so we need to split it as well:

```
28 61
  /    
40    77 90
    /  
34 51
```

Example 3 (cont.)

- We split the [77 90] node and we send up the middle of 77, 90, 93:

```
28 61
  /    
40    77 90
    /  
34 51
```

- We try to insert it in the root node, but the root is also full!

```
28 61
  /    
40    77 90
    /  
34 51
```

- Then we split the root, which increases the tree’s height by 1, but the tree is still balanced.

```
28 61
  /    
40    77 93
    /  
34 51
```

- This is only case in which the tree’s height increases.
Efficiency of 2-3 Trees

- A 2-3 tree containing n items has a height \( h \leq \log_2 n \).
- Thus, search and insertion are both \( O(\log n) \).
  - search visits at most \( h + 1 \) nodes
  - insertion visits at most \( 2h + 1 \) nodes:
    - starts by going down the full height
    - in the worst case, performs splits all the way back up to the root
- Deletion is tricky – you may need to coalesce nodes! However, it also has a time complexity of \( O(\log n) \).
- Thus, we can use 2-3 trees for a \( O(\log n) \)-time data dictionary!

External Storage

- The balanced trees that we've covered don't work well if you want to store the data dictionary externally – i.e., on disk.
- Key facts about disks:
  - data is transferred to and from disk in units called blocks, which are typically 4 or 8 KB in size
  - disk accesses are slow:
    - reading a block takes \( \sim 10 \) milliseconds \( (10^{-3} \text{ sec}) \)
    - vs. reading from memory, which takes \( \sim 10 \) nanoseconds
    - in 10 ms, a modern CPU can perform millions of operations!
B-Trees

- A B-tree of order \( m \) is a tree in which each node has:
  - at most \( 2m \) entries (and, for internal nodes, \( 2m + 1 \) children)
  - at least \( m \) entries (and, for internal nodes, \( m + 1 \) children)
  - exception: the root node may have as few as 1 entry
  - a 2-3 tree is essentially a B-tree of order 1

- To minimize the number of disk accesses, we make \( m \) as large as possible.
  - each disk read brings in more items
  - the tree will be shorter (each level has more nodes), and thus searching for an item requires fewer disk reads

- A large value of \( m \) doesn't make sense for a memory-only tree, because it leads to many key comparisons per node.

- These comparisons are less expensive than accessing the disk, so large values of \( m \) make sense for on-disk trees.

Example: a B-Tree of Order 2

- \( m = 2 \): at most \( 2m = 4 \) items per node (and at most 5 children)
  - at least \( m = 2 \) items per node (and at least 3 children)
  - (except the root, which could have 1 item)

- The above tree holds the same keys this 2-3 tree:

- We used the same order of insertion to create both trees:
  51, 3, 40, 77, 20, 10, 34, 28, 61, 80, 68, 93, 90, 97, 87, 14
Search in B-Trees

• Similar to search in a 2-3 tree.

• Example: search for 87

![B-Tree diagram]

Insertion in B-Trees

• Similar to insertion in a 2-3 tree:
  search for the key until you reach a leaf node
  if a leaf node has fewer than $2m$ items, add the item to the leaf node
  else split the node, dividing up the $2m + 1$ items:
    the smallest $m$ items remain in the original node
    the largest $m$ items go in a new node
    send the middle entry up and insert it (and a pointer to the new node) in the parent

• Example of an insertion without a split: insert 13

![B-Tree diagram]
Splits in B-Trees

- Insert 5 into the result of the previous insertion:

\[ m = 2 \]

\[
\begin{array}{c}
3 \quad 5 \quad 10 \quad 13 \quad 14 \\
20 \quad 40 \quad 68 \quad 90 \\
28 \quad 34 \quad 51 \quad 61 \\
\end{array}
\]

\[
\begin{array}{c}
3 \quad 5 \quad 10 \quad 13 \quad 14 \\
20 \quad 40 \quad 68 \quad 90 \\
28 \quad 34 \quad 51 \quad 61 \\
\end{array}\]

- The middle item (the 10) is sent up to the root.

The root has no room, so it is also split, and a new root is formed:

\[
\begin{array}{c}
3 \quad 5 \quad 10 \quad 13 \quad 14 \\
20 \quad 40 \quad 68 \quad 90 \\
28 \quad 34 \quad 51 \quad 61 \\
\end{array}\]

\[
\begin{array}{c}
3 \quad 5 \quad 10 \quad 13 \quad 14 \\
20 \quad 40 \quad 68 \quad 90 \\
28 \quad 34 \quad 51 \quad 61 \\
\end{array}\]

- Splitting the root increases the tree’s height by 1, but the tree is still balanced. This is only way that the tree’s height increases.

- When an internal node is split, its \(2m + 2\) pointers are split evenly between the original node and the new node.

Analysis of B-Trees

- All internal nodes have at least \(m\) children (actually, at least \(m+1\)).

- Thus, a B-tree with \(n\) items has a height \(\leq \log_{m}n\), and search and insertion are both \(O(\log_{m}n)\).

- As with 2-3 trees, deletion is tricky, but it’s still logarithmic.
Search Trees: Conclusions

- Binary search trees can be $O(\log n)$, but they can degenerate to $O(n)$ running time if they are out of balance.

- 2-3 trees and B-trees are balanced search trees that guarantee $O(\log n)$ performance.

- When data is stored on disk, the most important performance consideration is reducing the number of disk accesses.

- B-trees offer improved performance for on-disk data dictionaries.
Priority Queue

- A *priority queue* (PQ) is a collection in which each item has an associated number known as a *priority*.
  - use a higher priority for items that are "more important"

- Example application: scheduling a shared resource like the CPU
  - give some processes/applications a higher priority, so that they will be scheduled first and/or more often

- Key operations:
  - *insert*: add an item (with a position based on its priority)
  - *remove*: remove the item with the highest priority

- One way to implement a PQ efficiently is using a type of binary tree known as a *heap*. 
Complete Binary Trees

- A binary tree of height $h$ is complete if:
  - levels 0 through $h - 1$ are fully occupied
  - there are no "gaps" to the left of a node in level $h$

- Complete:

- Not complete (○ = missing node):

Representing a Complete Binary Tree

- A complete binary tree has a simple array representation.

- The tree's nodes are stored in the array in the order given by a level-order traversal.
  - top to bottom, left to right

- Examples:
Navigating a Complete Binary Tree in Array Form

- The root node is in \( a[0] \)
- Given the node in \( a[i] \):
  - its left child is in \( a[2*i + 1] \)
  - its right child is in \( a[2*i + 2] \)
  - its parent is in \( a[(i - 1)/2] \) (using integer division)
- Examples:
  - the left child of the node in \( a[1] \) is in \( a[2*1 + 1] = a[3] \)
  - the left child of the node in \( a[2] \) is in \( a[2*2 + 1] = a[5] \)
  - the right child of the node in \( a[3] \) is in \( a[2*3 + 2] = a[8] \)
  - the right child of the node in \( a[2] \) is in ___________________
  - the parent of the node in \( a[4] \) is in \( a[(4 - 1)/2] = a[1] \)
  - the parent of the node in \( a[7] \) is in ________________

What is the left child of 24?

- Assume that the following array represents a complete tree:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>26</td>
<td>12</td>
<td>32</td>
<td>24</td>
<td>18</td>
<td>28</td>
<td>47</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>
Heaps

- Heap: a complete binary tree in which each interior node is greater than or equal to its children
  - examples:
    
    ![Tree Diagram]

  - The largest value is always at the root of the tree.
  - The smallest value can be in any leaf node - there’s no guarantee about which one it will be.
  - We’re using max-at-top heaps.
    - in a min-at-top heap, every interior node <= its children

Which of these is a heap?

- A. 
  ![Tree Diagram]

- B. 
  ![Tree Diagram]

- C. 
  ![Tree Diagram]

D. more than one (which ones?)

E. none of them
Removing the Largest Item from a Heap

- Remove and return the item in the root node.
- In addition, need to move the largest remaining item to the root, while maintaining a complete tree with each node >= children

Algorithm:
1. make a copy of the largest item
2. move the last item in the heap to the root
3. “sift down” the new root item until it is >= its children (or it’s a leaf)
4. return the largest item

Sifting Down an Item

- To sift down item x (i.e., the item whose key is x):
  1. compare x with the larger of the item’s children, y
  2. if x < y, swap x and y and repeat

Other examples:
sift down the 10:

sift down the 7:
Inserting an Item in a Heap

• Algorithm:
  1. put the item in the next available slot (grow array if needed)
  2. “sift up” the new item
     until it is <= its parent (or it becomes the root item)

• Example: insert 35
  put it in place:

```
  20
 /  \
16   12
 /    \
5     8
```

  sift it up:

```
  20
 /  \
16   12
 /    \
5     8
```

```
  20
 /  \
16   12
 /    \
5     8
```

```
  20
 /  \
16   12
 /    \
5     8
```

```
  20
 /  \
16   12
 /    \
5     8
```

Time Complexity of a Heap

• A heap containing n items has a height <= log₂n. Why?

• Thus, removal and insertion are both O(log n).
  • remove: go down at most log₂n levels when sifting down;
    do a constant number of operations per level
  • insert: go up at most log₂n levels when sifting up;
    do a constant number of operations per level

• This means we can use a heap for a O(log n)-time priority queue.
Using a Heap for a Priority Queue

- Recall: a *priority queue* (PQ) is a collection in which each item has an associated number known as a *priority*.
  - use a higher priority for items that are "more important"

- To implement a PQ using a heap:
  - order the items in the heap according to their priorities
    - every item in the heap will have a priority >= its children
    - the highest priority item will be in the root node
  - get the highest priority item by calling `heap.remove()`!

Using a Heap to Sort an Array

- Recall selection sort: it repeatedly finds the smallest remaining element and swaps it into place:

  0  1  2  3  4  5  6
  5 | 16 | 8 | 14 | 20 | 1 | 26
  0  1  2  3  4  5  6
  1 | 16 | 8 | 14 | 20 | 5 | 26
  0  1  2  3  4  5  6
  1 | 5 | 8 | 14 | 20 | 16 | 26

  ...

  - It isn’t efficient, because it performs a linear scan to find the smallest remaining element (O(n) steps per scan).

  - Heapsort is a sorting algorithm that repeatedly finds the *largest* remaining element and puts it in place.

  - It *is* efficient, because it turns the array into a heap.
    - it can find/remove the largest remaining in O(log n) steps!
Converting an Arbitrary Array to a Heap

- To convert an array (call it contents) with \( n \) items to a heap:
  1. Start with the parent of the last element:
     
     \[
     \text{contents}[i], \text{where} \quad i = \frac{(n - 1) - 1}{2} = \frac{n - 2}{2}
     \]
  2. Sift down contents[\( i \)] and all elements to its left

- Example:

  \[
  \begin{array}{cccccc}
  5 & 16 & 8 & 14 & 20 & 1 & 26 \\
  \end{array}
  \]

  Last element’s parent = contents[\( (7 - 2)/2 \)] = contents[2].
  Sift it down:

Converting an Array to a Heap (cont.)

- Next, sift down contents[1]:

- Finally, sift down contents[0]:

\[
\begin{array}{cccccc}
  5 & 16 & 8 & 14 & 20 & 1 & 26 \\
  \end{array}
\]
Heapsort

• Pseudocode:

```java
heapSort(arr) {
    // Turn the array into a max-at-top heap.
    heap = new Heap(arr);
    endUnsorted = arr.length - 1;
    while (endUnsorted > 0) {
        // Get the largest remaining element and put it
        // at the end of the unsorted portion of the array.
        largestRemaining = heap.remove();
        arr[endUnsorted] = largestRemaining;
        endUnsorted--;
    }
}
```

Heapsort Example

• Sort the following array:

```
0 1 2 3 4 5 6
13 6 45 10 3 22 5
```

• Here’s the corresponding complete tree:

```
13
   / \
  6   45
 / \  /
10 3 22 5
```

• Begin by converting it to a heap:
Heapsort Example (cont.)

• Here's the heap in both tree and array forms:

```
  45  
 /   \ 
10   22 
/ \   / 
6   3 13 5
```

<table>
<thead>
<tr>
<th>0 1 2 3 4 5 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 10 22  6 13 5</td>
</tr>
</tbody>
</table>

```
endUnsorted: 6
```

• We begin looping:

```
while (endUnsorted > 0) {
    // Get the largest remaining element and put it
    // at the end of the unsorted portion of the array.
    largestRemaining = heap.remove();
    arr[endUnsorted] = largestRemaining;
    endUnsorted--;
}
```

Heapsort Example (cont.)

• Here's the heap in both tree and array forms:

```
  45  
 /   \ 
10   22 
/ \   / 
6   3 13 5
```

<table>
<thead>
<tr>
<th>0 1 2 3 4 5 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 10 22  6 13 5</td>
</tr>
</tbody>
</table>

```
endUnsorted: 6
```

• Remove the largest item and put it in place:

```java
remove() copies 45; moves 5 to root

remove() sifts down 5; returns 45

heapSort() puts 45 in place; decrements endUnsorted

toRemove: 45

endUnsorted: 6
largestRemaining: 45
```
Heapsort Example (cont.)

### Heapsort Example (cont.)

1. **Copy 22; move 5 to root**
   - To Remove: 22
   - End Unsorted: 5
   - Largest Remaining: 22
   - Heapsort Example (cont.)

2. **Copy 13; move 3 to root**
   - To Remove: 13
   - End Unsorted: 4
   - Largest Remaining: 13
   - Heapsort Example (cont.)

3. **Copy 10; move 3 to root**
   - To Remove: 10
   - End Unsorted: 3
   - Largest Remaining: 10
   - Heapsort Example (cont.)

4. **Copy 6; move 5 to root**
   - To Remove: 6
   - End Unsorted: 2
   - Largest Remaining: 6

### Heapsort Example (cont.)

1. **Sift down 5; return 22**
   - Heapsort Example (cont.)

2. **Put 22 in place; decrement endUnsorted**
   - Heapsort Example (cont.)

3. **Copy 13; move 3 to root**
   - Heapsort Example (cont.)

4. **Sift down 3; return 13**
   - Heapsort Example (cont.)

5. **Put 13 in place; decrement**
   - Heapsort Example (cont.)

6. **Copy 6; move 5 to root**
   - Heapsort Example (cont.)

7. **Sift down 5; return 6**
   - Heapsort Example (cont.)

8. **Put 6 in place; decrement**
   - Heapsort Example (cont.)

### Heapsort Example (cont.)

1. **Copy 10; move 3 to root**
   - Heapsort Example (cont.)

2. **Sift down 10; return 3**
   - Heapsort Example (cont.)

3. **Put 10 in place; decrement**
   - Heapsort Example (cont.)

4. **Copy 6; move 5 to root**
   - Heapsort Example (cont.)

5. **Sift down 5; return 6**
   - Heapsort Example (cont.)

6. **Put 6 in place; decrement**
   - Heapsort Example (cont.)

### Heapsort Example (cont.)

1. **Copy 22; move 5 to root**
   - Heapsort Example (cont.)

2. **Sift down 5; return 22**
   - Heapsort Example (cont.)

3. **Put 22 in place; decrement endUnsorted**
   - Heapsort Example (cont.)

4. **Copy 13; move 3 to root**
   - Heapsort Example (cont.)

5. **Sift down 3; return 13**
   - Heapsort Example (cont.)

6. **Put 13 in place; decrement**
   - Heapsort Example (cont.)

7. **Copy 6; move 5 to root**
   - Heapsort Example (cont.)

8. **Sift down 5; return 6**
   - Heapsort Example (cont.)

9. **Put 6 in place; decrement**
   - Heapsort Example (cont.)
Heapsort Example (cont.)

- And now we terminate the loop:
  ```java
  while (endUnsorted > 0) {
    largestRemaining = heap.remove();
    arr[endUnsorted] = largestRemaining;
    endUnsorted--;
  }
  ```

Time Complexity of Heapsort

- Time complexity of creating a heap from an array?

- Time complexity of sorting the array?

- Thus, total time complexity = ?
How Does Heapsort Compare?

<table>
<thead>
<tr>
<th>algorithm</th>
<th>best case</th>
<th>avg case</th>
<th>worst case</th>
<th>extra memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>selection sort</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>insertion sort</td>
<td>$O(n)$</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Shell sort</td>
<td>$O(n \log n)$</td>
<td>$O(n^{1.5})$</td>
<td>$O(n^{1.5})$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>bubble sort</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>quicksort</td>
<td>$O(n \log n)$</td>
<td>$O(n \log n)$</td>
<td>$O(n^2)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>mergesort</td>
<td>$O(n \log n)$</td>
<td>$O(n \log n)$</td>
<td>$O(n \log n)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>heapsort</td>
<td>$O(n \log n)$</td>
<td>$O(n \log n)$</td>
<td>$O(n \log n)$</td>
<td>$O(1)$</td>
</tr>
</tbody>
</table>

- Heapsort matches mergesort for the best worst-case time complexity, but it has better space complexity.
- Insertion sort is still best for arrays that are almost sorted.
  - heapsort will scramble an almost sorted array before sorting it!
- Quicksort is still typically fastest in the average case.
Data Dictionary Revisited

- We've considered several data structures that allow us to store and search for data items using their key fields:

<table>
<thead>
<tr>
<th>data structure</th>
<th>searching for an item</th>
<th>inserting an item</th>
</tr>
</thead>
<tbody>
<tr>
<td>a list implemented using an array</td>
<td>O(log n) using binary search</td>
<td>O(n)</td>
</tr>
<tr>
<td>a list implemented using a linked list</td>
<td>O(n) using linear search</td>
<td>O(n)</td>
</tr>
<tr>
<td>binary search tree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>balanced search trees</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2-3 tree, B-tree, others)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- We'll now look at hash tables, which can do better than O(log n).
Ideal Case: Searching = Indexing

- We would achieve optimal efficiency if we could treat the key as an index into an array.

- Example: storing data about members of a sports team
  - key = jersey number (some value from 0-99).
  - class for an individual player's record:
    ```java
    public class Player {
        private int jerseyNum;
        private String firstName;
        ...
    }
    ```
  - store the player records in an array:
    ```java
    Player[] teamRecords = new Player[100];
    ```

- In such cases, search and insertion are $O(1)$:
  ```java
  public Player search(int jerseyNum) {
      return teamRecords[jerseyNum];
  }
  ```

Hashing: Turning Keys into Array Indices

- In most real-world problems, indexing is not as simple as the sports-team example. Why?
  -
  -
  -

- To handle these problems, we perform hashing:
  - use a hash function to convert the keys into array indices
    "Sullivan" \( \rightarrow \) 18
  - use techniques to handle cases in which multiple keys are assigned the same hash value

- The resulting data structure is known as a hash table.
Hash Functions

- A hash function defines a mapping from keys to integers.
- We then use the modulus operator to get a valid array index.

\[
\text{key value} \rightarrow \text{hash function} \rightarrow \text{integer} \mod \text{array length} = \text{integer in } [0, n - 1]
\]

- Here's a very simple hash function for keys of lower-case letters:
  \[h(key) = \text{ASCII value of first char} - \text{ASCII value of 'a'}\]
- examples:
  \[h("ant") = \text{ASCII for 'a'} - \text{ASCII for 'a'} = 0\]
  \[h("cat") = \text{ASCII for 'c'} - \text{ASCII for 'a'} = 2\]
- \(h(key)\) is known as the key's hash code.
- A collision occurs when items with different keys are assigned the same hash code.

Dealing with Collisions I: Separate Chaining

- Each position in the hash table serves as a bucket that can store multiple data items.
- Two options:
  1. each bucket is itself an array
     • need to preallocate, and a bucket may become full
  2. each bucket is a linked list
     • items with the same hash code are "chained" together
     • each "chain" can grow as needed
Dealing with Collisions II: Open Addressing

- When the position assigned by the hash function is occupied, find another open position.

- Example: "wasp" has a hash code of 22, but it ends up in position 23 because position 22 is occupied.

- We'll consider three ways of finding an open position – a process known as probing.

- We also perform probing when searching.
  - example: search for "wasp"
    - look in position 22
    - then look in position 23
    - need to figure out when to safely stop searching (more on this soon!)

Linear Probing

- Probe sequence: \( h(\text{key}) \), \( h(\text{key}) + 1 \), \( h(\text{key}) + 2 \), ..., wrapping around as necessary.

- Examples:
  - "ape" \( (h = 0) \) would be placed in position 1, because position 0 is already full.
  - "bear" \( (h = 1) \): try 1, 1 + 1, 1 + 2 – open!
  - where would "zebu" end up?

- Advantage: if there is an open cell, linear probing will eventually find it.

- Disadvantage: get "clusters" of occupied cells that lead to longer subsequent probes.
  - probe length = the number of positions considered during a probe
### Quadratic Probing

- **Probe sequence:**  \( h(key), h(key) + 1^2, h(key) + 2^2, h(key) + 3^2, \ldots \), wrapping around as necessary.

- **Examples:**
  - "ape" (\( h = 0 \)): try 0, 0 + 1 – open!
  - "bear" (\( h = 1 \)): try 1, 1 + 1, 1 + 4 – open!
  - "zebu"?

- **Advantage:** smaller clusters of occupied cells

- **Disadvantage:** may fail to find an existing open position. For example:

```
<table>
<thead>
<tr>
<th>Table Size</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>occupied</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>25</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trying to insert a key with ( h(key) = 0 )</td>
<td>1</td>
<td>x</td>
<td>1</td>
<td>81</td>
<td>6</td>
<td>x</td>
<td>16</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>offsets of the probe sequence in italics</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trying to insert a key with ( h(key) = 1 )</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trying to insert a key with ( h(key) = 2 )</td>
<td>4</td>
<td></td>
<td>x</td>
<td>4</td>
<td>64</td>
<td>9</td>
<td>x</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

### Double Hashing

- **Use two hash functions:**
  - \( h_1 \) computes the hash code
  - \( h_2 \) computes the increment for probing
  - **probe sequence:** \( h_1, h_1 + h_2, h_1 + 2h_2, \ldots \)

- **Examples:**
  - \( h_1 = \) our previous \( h \)
  - \( h_2 = \) number of characters in the string
  - "ape" (\( h_1 = 0, h_2 = 3 \)): try 0, 0 + 3 – open!
  - "bear" (\( h_1 = 1, h_2 = 4 \)): try 1 – open!
  - "zebu"?

- **Combines good features of linear and quadratic:**
  - reduces clustering
  - will find an open position if there is one, provided the table size is a prime number

```
<table>
<thead>
<tr>
<th>Table Size</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;ant&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;bear&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;cat&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;ape&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;emu&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;wolf&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;wasp&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;yak&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;zebra&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

CSCI S-111

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Removing Items Under Open Addressing

- Problematic example (using linear probing):
  - insert "ape" (h = 0): try 0, 0 + 1 – open!
  - insert "bear" (h = 1): try 1, 1 + 1, 1 + 2 – open!
  - remove "ape"
  - search for "ape": try 0, 0 + 1 – conclude not in table
  - search for "bear": try 1 – conclude not in table, but "bear" is further down in the table!

- To fix this problem, distinguish between:
  - removed positions that previously held an item
  - empty positions that have never held an item

- During probing, we don’t stop if we see a removed position. **ex: search for "bear": try 1 (removed), 1 + 1, 1 + 2 – found!**

- We can insert items in either empty or removed positions.

---

An Interface For Hash Tables

```java
public interface HashTable {
    boolean insert(Object key, Object value);
    Queue<Object> search(Object key);
    Queue<Object> remove(Object key);
}
```

- `insert()` takes a key-value pair and returns:
  - `true` if the key-value pair can be added
  - `false` if it cannot be added (referred to as overflow)

- `search()` and `remove()` both take a key, and return a queue containing all of the values associated with that key.
  - example: an index for a book
    - key = word
    - values = the pages on which that word appears
  - return `null` if the key is not found

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An Implementation Using Open Addressing

```java
public class OpenHashTable implements HashTable {
    private class Entry {
        private Object key;
        private LLQueue<Object> values;
    }
    private Entry[] table;
    private int probeType;
}
```

- We use a private inner class for the entries in the hash table.
- We use an LLQueue for the values associated with a given key.

Empty vs. Removed

- When we remove a key and its values, we:
  - leave the Entry object in the table
  - set the Entry object's key and values fields to null
  - example: after remove("ape"):

- Note the difference:
  - a truly empty position has a value of null in the table
    (example: positions 2, 3 and 4 above)
  - a removed position refers to an Entry object whose
    key and values fields are null (example: position 1 above)
Probing Using Double Hashing

private int probe(Object key) {
    int i = h1(key);    // first hash function
    int h2 = h2(key);   // second hash function

    // keep probing until we get an empty position or match
    while (table[i] != null && !key.equals(table[i].key)) {
        i = (i + h2) % table.length;
    }

    return i;
}

• It is essential that we:
  • check for table[i] != null first. why?
  • call the equals method on key, not table[i].key. why?

Avoiding an Infinite Loop

• The while loop in our probe method could lead to an infinite loop.

  while (table[i] != null && !key.equals(table[i].key)) {
      i = (i + h2) % table.length;
  }

• When would this happen?

• We can stop probing after checking n positions (n = table size),
  because the probe sequence will just repeat after that point.
  • for quadratic probing:
    \[
    (h1 + n^2) \mod n = h1 \mod n \\
    (h1 + (n+1)^2) \mod n = (h1 + n^2 + 2n + 1) \mod n = (h1 + 1) \mod n
    \]
  • for double hashing:
    \[
    (h1 + nh^2) \mod n = h1 \mod n \\
    (h1 + (n+1)h2) \mod n = (h1 + nh^2 + h2) \mod n = (h1 + h2) \mod n
    \]
private int probe(Object key) {
    int i = h1(key);    // first hash function
    int h2 = h2(key);   // second hash function
    int numChecked = 1;

    // keep probing until we get an empty position or a match
    while (table[i] != null && !key.equals(table[i].key)) {
        if (numChecked == table.length) {
            return -1;
        }
        i = (i + h2) % table.length;
        numChecked++;
    }
    return i;
}

public LLQueue<Object> search(Object key) {
    // throw an exception if key == null
    int i = probe(key);
    if (i == -1 || table[i] == null) {
        return null;
    } else {
        return table[i].values;
    }
}

public LLQueue<Object> remove(Object key) {
    // throw an exception if key == null
    int i = probe(key);
    if (i == -1 || table[i] == null) {
        return null;
    }
    LLQueue<Object> removedVals = table[i].values;
    table[i].key = null;
    table[i].values = null;
    return removedVals;
}
Insertion

- We begin by probing for the key.
- Several cases:
  1. the key is already in the table (we're inserting a duplicate) → add the value to the values in the key's entry
  2. the key is not in the table: three subcases:
     a. encountered 1 or more removed positions while probing → put the (key, value) pair in the first removed position seen during probing. why?
     b. no removed position; reached an empty position → put the (key, value) pair in the empty position
     c. no removed position or empty position → overflow; return false

Tracing Through Some Examples

- Start with the hash table at right with:
  - double hashing
  - our earlier hash functions h1 and h2
- Perform the following operations:
  - insert "bear" (h1 = 1, h2 = 4):
  - insert "bison" (h1 = 1, h2 = 5):
  - insert "cow" (h1 = 2, h2 = 3):
  - delete "emu" (h1 = 4, h2 = 3):
  - search "eel" (h1 = 4, h2 = 3):
  - insert "bee" (h1 = ____, h2 = _____):
Dealing with Overflow

- Overflow = can't find a position for an item

- When does it occur?
  - linear probing:
  - quadratic probing:
    -
    -
  - double hashing:
    - if the table size is a prime number: same as linear
    - if the table size is not a prime number: same as quadratic

- To avoid overflow (and reduce search times), grow the hash table when the % of occupied positions gets too big.
  - problem: we need to rehash all of the existing items. why?

Implementing the Hash Function

- Characteristics of a good hash function:
  1) efficient to compute
  2) uses the entire key
     - changing any char/digit/etc. should change the hash code
  3) distributes the keys more or less uniformly across the table
  4) must be a function!
     - a key must always get the same hash code

- In Java, every object has a hashCode() method.
  - the version inherited from Object returns a value based on an object's memory location
  - classes can override this version with their own
Hash Functions for Strings: version 1

- \( h_a = \) the sum of the characters’ ASCII values
  - example: \( h_a(“eat”) = 101 + 97 + 116 = 314 \)

- All permutations of a given set of characters get the same code.
  - example: \( h_a(“tea”) = h_a(“eat”) \)
  - could be useful in a Scrabble game
    - allow you to look up all words that can be formed from a given set of characters

- The range of possible hash codes is very limited.
  - example: hashing keys composed of 1-5 lower-case char’s (padded with spaces)
    - \( 26\times27\times27\times27\times27 = \) over 13 million possible keys
  - smallest code = \( h_a(“a “) = 97 + 4\times32 = 225 \)
  - largest code = \( h_a(“zzzzz“) = 5\times122 = 610 \)

\[
\begin{align*}
610 - 225 &= 385 \\
\end{align*}
\]

Hash Functions for Strings: version 2

- Compute a \textit{weighted} sum of the ASCII values:
  \[
  h_b = a_0b^{n-1} + a_1b^{n-2} + \ldots + a_{n-2}b + a_{n-1}
  \]

  where
  - \( a_i = \) ASCII value of the \( i \)th character
  - \( b = \) a constant
  - \( n = \) the number of characters

- Multiplying by powers of \( b \) allows the \textit{positions} of the characters to affect the hash code.
  - different permutations get different codes

- We may get arithmetic overflow, and thus the code may be negative. We adjust it when this happens.

- Java uses this hash function with \( b = 31 \) in the \texttt{hashCode()} method of the \texttt{String} class.
Hash Table Efficiency

- In the best case, search and insertion are $O(1)$.
- In the worst case, search and insertion are linear.
  - open addressing: $O(m)$, where $m$ = the size of the hash table
  - separate chaining: $O(n)$, where $n$ = the number of keys
- With good choices of hash function and table size, complexity is generally better than $O(\log n)$ and approaches $O(1)$.
- **load factor** = # keys in table / size of the table.
  To prevent performance degradation:
  - open addressing: try to keep the load factor < 1/2
  - separate chaining: try to keep the load factor < 1
- Time-space tradeoff: bigger tables have better performance, but they use up more memory.

Hash Table Limitations

- It can be hard to come up with a good hash function for a particular data set.
- The items are not ordered by key. As a result, we can't easily:
  - print the contents in sorted order
  - perform a range search (find all values between $v_1$ and $v_2$)
  - perform a rank search – get the kth largest item
  
We can do all of these things with a search tree.
Extra Practice

• Start with the hash table at right with:
  • double hashing
  • $h_1(\text{key}) = \text{ASCII of first letter} - \text{ASCII of 'a'}$
  • $h_2(\text{key}) = \text{key.length()}$
  • shaded cells are removed cells

• **What is the probe sequence for "baboon"?**
  (the sequence of positions seen during probing)

A. 1, 2, 5
B. 1, 6
C. 1, 7, 2
D. 1, 7, 3
E. 1, 7, 2, 8
Extra Practice

- Start with the hash table at right with:
  - double hashing
  - \( h_1(key) = \text{ASCII of first letter} - \text{ASCII of 'a'} \)
  - \( h_2(key) = \text{key.length()} \)
  - shaded cells are removed cells

- What is the probe sequence for "baboon"?
  \( (h_1 = 1, h_2 = 6) \)
  - try: \( 1 \mod 11 = 1 \)
  - \( (1 + 6) \mod 11 = 7 \)
  - \( (1 + 2 \times 6) \mod 11 = 2 \)
  - \( (1 + 3 \times 6) \mod 11 = 8 \)

  - A. 1, 2, 5
  - B. 1, 6
  - C. 1, 7, 2
  - D. 1, 7, 3
  - E. 1, 7, 2, 8

Extra Practice

- If we insert "baboon", in what position will it go?
  - A. 1
  - B. 7
  - C. 2
  - D. 8
Extra Practice

- Start with the hash table at right with:
  - double hashing
  - \( h_1(key) = \text{ASCII of first letter} - \text{ASCII of 'a'} \)
  - \( h_2(key) = \text{key.length()} \)
  - shaded cells are removed cells

- What is the probe sequence for "baboon"?
  \( (h_1 = 1, h_2 = 6) \) try:
    \[
    \begin{align*}
    1 \% 11 &= 1 \\
    (1 + 6) \% 11 &= 7 \\
    (1 + 2 \times 6) \% 11 &= 2 \\
    (1 + 3 \times 6) \% 11 &= 8
    \end{align*}
    \]

- If we insert "baboon", in what position will it go?
  A. 1  B. 7  C. 2  D. 8  
  the first *removed* position seen while probing
Graphs

Computer Science S-111
Harvard University
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Unit 10

What is a Graph?

• A graph consists of:
  • a set of vertices (also known as nodes)
  • a set of edges (also known as arcs), each of which connects a pair of vertices
Example: A Highway Graph

- Vertices represent cities.
- Edges represent highways.
- This is a weighted graph, with a cost associated with each edge.
  - in this example, the costs denote mileage
- We'll use graph algorithms to answer questions like “What is the shortest route from Portland to Providence?”

Relationships Among Vertices

- Two vertices are adjacent if they are connected by a single edge.
  - ex: c and g are adjacent, but c and i are not
- The collection of vertices that are adjacent to a vertex v are referred to as v’s neighbors.
  - ex: c’s neighbors are a, b, d, f, and g
A path is a sequence of edges that connects two vertices.

A graph is connected if there is a path between any two vertices.
- *ex:* the six vertices at right are part of a graph that is not connected

A graph is complete if there is an edge between every pair of vertices.
- *ex:* the graph at right is complete

A directed graph has a direction associated with each edge, which is depicted using an arrow:

Edges in a directed graph are often represented as ordered pairs of the form (start vertex, end vertex).
- *ex:* (a, b) is an edge in the graph above, but (b, a) is not.

In a path in a directed graph, the end vertex of edge i must be the same as the start vertex of edge i + 1.
- *ex:* \{ (a, b), (b, e), (e, f) \} is a valid path.
  \{ (a, b), (c, b), (c, a) \} is not.
**Cycles in a Graph**

- A *cycle* is a path that:
  - leaves a given vertex using one edge
  - returns to that same vertex using a different edge

- Examples: the highlighted paths below

- An *acyclic* graph has no cycles.

---

**Trees vs. Graphs**

- A tree is a special type of graph.
  - connected, undirected, and acyclic
  - we usually single out one of the vertices to be the root, but graph theory does *not* require this

- A graph that is *not* a tree, because it has cycles

- A tree using the same nodes

- Another tree using the same nodes
A spanning tree is a subset of a connected graph that contains:
- all of the vertices
- a subset of the edges that form a tree

Recall this graph with cycles from the previous slide:

The trees on that slide were spanning trees for this graph. Here are two others:

Representing a Graph: Option 1

Use an *adjacency matrix* — a two-dimensional array in which element \([r][c]\) = the cost of going from vertex \(r\) to vertex \(c\)

Example:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>54</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>54</td>
<td>39</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>83</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use a special value to indicate there’s no edge from \(r\) to \(c\):
- shown as a shaded cell above
- can’t use 0, because an edge may have an actual cost of 0

This representation:
- wastes memory if a graph is *sparse* (few edges per vertex)
- is memory-efficient if a graph is *dense* (many edges per vertex)
Representing a Graph: Option 2

- Use one *adjacency list* for each vertex.
  - a linked list with info on the edges coming from that vertex

- This representation uses less memory if a graph is sparse.
- It uses more memory if a graph is dense.
  - because of the references linking the nodes

Graph Class

```java
public class Graph {
    private class Vertex {
        private String id;
        private Edge edges;  // adjacency list
        private Vertex next;
        private boolean encountered;
        private boolean done;
        private Vertex parent;
        private double cost;...
    }

    private class Edge {
        private Vertex start;
        private Vertex end;
        private double cost;
        private Edge next;...
    }

    private Vertex vertices;...
}
```

The highlighted fields are shown in the diagram on the previous page.
Our Graph Representation

- Each vertex object (shown in blue) stores info. about a vertex.
  - including an adjacency list of edge objects (the purple ones)

- A graph object has a single field called vertices
  - a reference to a linked list of vertex objects
  - a linked list of linked lists!

Traversing a Graph

- Traversing a graph involves starting at some vertex and visiting all vertices that can be reached from that vertex.
  - visiting a vertex = processing its data in some way
  - if the graph is connected, all of its vertices will be visited

- We will consider two types of traversals:
  - **depth-first**: proceed as far as possible along a given path before backing up
  - **breadth-first**: visit a vertex
    - visit all of its neighbors
    - visit all unvisited vertices 2 edges away
    - visit all unvisited vertices 3 edges away, etc.

- Applications:
  - determining the vertices that can be reached from some vertex
  - web crawler (vertices = pages, edges = links)
Depth-First Traversal

- Visit a vertex, then make recursive calls on all of its yet-to-be-visited neighbors:
  
  ```
  dfTrav(v, parent) {
    visit v and mark it as visited
    v.parent = parent
    for each vertex w in v's neighbors
      if (w has not been visited)
        dfTrav(w, v)
  }
  ```

- Java method:
  ```java
  private static void dfTrav(Vertex v, Vertex parent) {
    System.out.println(v.id);    // visit v
    v.done = true;
    v.parent = parent;
    Edge e = v.edges;
    while (e != null) {
      Vertex w = e.end;
      if (!w.done)
        dfTrav(w, v);
      e = e.next;
    }
  }
  ```

Example: Depth-First Traversal from Portland

void dfTrav(Vertex v, Vertex parent) {
  System.out.println(v.id);    // visit v
  v.done = true;
  v.parent = parent;
  Edge e = v.edges;
  while (e != null) {
    Vertex w = e.end;
    if (!w.done)
      dfTrav(w, v);
    e = e.next;
  }
}
The edges obtained by following the parent references form a spanning tree with the origin of the traversal as its root.

From any city, we can get to the origin by following the roads in the spanning tree.

**Another Example:**
**Depth-First Traversal from Worcester**

- In what order will the cities be visited?
- Which edges will be in the resulting spanning tree?
To discover a cycle in an undirected graph, we can:
• perform a depth-first traversal, marking the vertices as visited
• when considering neighbors of a visited vertex, if we discover
  one already marked as visited, there must be a cycle

• If no cycles found during the traversal, the graph is acyclic.

This doesn't work for directed graphs:
• c is a neighbor of both a and b
• there is no cycle

Checking for Cycles in an Undirected Graph

Breadth-First Traversal

• Use a queue to store vertices we've seen but not yet visited:
  private static void bfsTrav(Vertex origin) {
    origin.encountered = true;
    origin.parent = null;
    Queue<Vertex> q = new LLQueue<Vertex>();
    q.insert(origin);
    while (!q.isEmpty()) {
      Vertex v = q.remove();         // Visit v.
      // Add v's unencountered neighbors to the queue.
      Edge e = v.edges;
      while (e != null) {
        Vertex w = e.end;
        if (!w.encountered) {
          w.encountered = true;
          w.parent = v;
          q.insert(w);
        }
        e = e.next;
      }
    }
  }
Example: Breadth-First Traversal from Portland

Evolution of the queue:

<table>
<thead>
<tr>
<th>remove</th>
<th>insert</th>
<th>queue contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland</td>
<td>Portland</td>
<td>Portland</td>
</tr>
<tr>
<td>Portland</td>
<td>Portsmouth, Concord</td>
<td>Portsmouth, Concord</td>
</tr>
<tr>
<td>Portland</td>
<td>Boston, Worcester</td>
<td>Concord, Boston, Worcester</td>
</tr>
<tr>
<td>Concord</td>
<td>none</td>
<td>Boston, Worcester</td>
</tr>
<tr>
<td>Boston</td>
<td>Providence</td>
<td>Worcester, Providence</td>
</tr>
<tr>
<td>Worcester</td>
<td>Albany</td>
<td>Providence, Albany</td>
</tr>
<tr>
<td>Providence</td>
<td>New York</td>
<td>Albany, New York</td>
</tr>
<tr>
<td>Albany</td>
<td>none</td>
<td>New York</td>
</tr>
<tr>
<td>New York</td>
<td>none</td>
<td>empty</td>
</tr>
</tbody>
</table>

Breadth-First Spanning Tree

**breadth-first spanning tree:**

- Portland
  - Portsmouth
    - Boston
      - Providence
      - New York
  - Concord
    - Worcester
    - Albany
    - New York

**depth-first spanning tree:**

- Portland
  - Portsmouth
    - Boston
    - Providence
  - Concord
    - Worcester
    - Albany
    - New York
Another Example: Breadth-First Traversal from Worcester

Evolution of the queue:
- remove
- insert
- queue contents

Time Complexity of Graph Traversals

- let $V =$ number of vertices in the graph
  $E =$ number of edges

- If we use an adjacency matrix, a traversal requires $O(V^2)$ steps.
  - why?

- If we use adjacency lists, a traversal requires $O(V + E)$ steps.
  - visit each vertex once
  - traverse each vertex's adjacency list at most once
    - the total length of the adjacency lists is at most $2E = O(E)$
  - for a sparse graph, $O(V + E)$ is better than $O(V^2)$
  - for a dense graph, $E = O(V^2)$, so both representations are $O(V^2)$

- In the remaining notes, we'll assume an adjacency-list implementation.
Minimum Spanning Tree

• A minimum spanning tree (MST) has the smallest total cost among all possible spanning trees.
  • example:

    - If all edges have unique costs, there is only one MST.
    - If some edges have the same cost, there may be more than one.

  • Example applications:
    - determining the shortest highway system for a set of cities
    - calculating the smallest length of cable needed to connect a network of computers

Building a Minimum Spanning Tree

• Claim: If you divide the vertices into two disjoint subsets A and B, the lowest-cost edge \((v_a, v_b)\) joining a vertex in A to a vertex in B must be part of the MST.

    - example: subset \(A = \text{unshaded}\)
    - subset \(B = \text{shaded}\)

    - The 6 bold edges each join a vertex in A to a vertex in B.
    - The one with the lowest cost (Portland to Portsmouth) must be in the MST.

Proof by contradiction:
1. Assume we can create an MST (call it T) that doesn't include \((v_a, v_b)\).
2. T must include a path from \(v_a\) to \(v_b\), so it must include one of the other edges \((v_a', v_b')\) that span A and B, such that \((v_a', v_b')\) is part of the path from \(v_a\) to \(v_b\).
3. Adding \((v_a, v_b)\) to T introduces a cycle.
4. Removing \((v_a', v_b')\) gives a spanning tree with a lower total cost, which contradicts the original assumption.
Prim’s MST Algorithm

- Begin with the following subsets:
  - $A$ = any one of the vertices
  - $B$ = all of the other vertices

- Repeatedly do the following:
  - select the lowest-cost edge $(v_a, v_b)$ connecting a vertex in $A$ to a vertex in $B$
  - add $(v_a, v_b)$ to the spanning tree
  - move vertex $v_b$ from set $B$ to set $A$

- Continue until set $A$ contains all of the vertices.

Example: Prim’s Starting from Concord

- Tracing the algorithm:

<table>
<thead>
<tr>
<th>edge added</th>
<th>set A</th>
<th>set B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Con, Wor)</td>
<td>{Con}</td>
<td>{Alb, Bos, NY, Ptl, Pts, Pro, Wor}</td>
</tr>
<tr>
<td>(Wor, Pro)</td>
<td>{Con, Wor}</td>
<td>{Alb, Bos, NY, Ptl, Pts, Pro}</td>
</tr>
<tr>
<td>(Wor, Bos)</td>
<td>{Con, Wor, Pro}</td>
<td>{Alb, Bos, NY, Ptl, Pts}</td>
</tr>
<tr>
<td>(Bos, Pts)</td>
<td>{Con, Wor, Pro, Bos}</td>
<td>{Alb, NY, Ptl}</td>
</tr>
<tr>
<td>(Pts, Ptl)</td>
<td>{Con, Wor, Pro, Bos, Pts}</td>
<td>{Alb, NY}</td>
</tr>
<tr>
<td>(Wor, Alb)</td>
<td>{Con, Wor, Pro, Bos, Pts, Ptl, Alb}</td>
<td>{NY}</td>
</tr>
<tr>
<td>(Pro, NY)</td>
<td>{Con, Wor, Pro, Bos, Pts, Ptl, Alb, NY}</td>
<td>{}</td>
</tr>
</tbody>
</table>
MST May Not Give Shortest Paths

- The MST is the spanning tree with the minimal total edge cost.
- It does not necessarily include the minimal cost path between a pair of vertices.
- Example: shortest path from Boston to Providence is along the single edge connecting them
  - that edge is not in the MST

Implementing Prim’s Algorithm

- We use the done field to keep track of the sets.
  - if v.done == true, v is in set A
  - if v.done == false, v is in set B
- We repeatedly scan through the lists of vertices and edges to find the next edge to add.
  \( \Rightarrow O(EV) \)
- We can do better!
  - use a heap-based priority queue to store the vertices in set B
  - priority of a vertex \( x = -1 \times \) cost of the lowest-cost edge connecting \( x \) to a vertex in set A
    - why multiply by \(-1?\)
  - somewhat tricky: need to update the priorities over time
  \( \Rightarrow O(E \log V) \)
The Shortest-Path Problem

- It's often useful to know the shortest path from one vertex to another – i.e., the one with the minimal total cost
  - example application: routing traffic in the Internet

- For an unweighted graph, we can simply do the following:
  - start a breadth-first traversal from the origin, v
  - stop the traversal when you reach the other vertex, w
  - the path from v to w in the resulting (possibly partial) spanning tree is a shortest path

- A breadth-first traversal works for an unweighted graph because:
  - the shortest path is simply one with the fewest edges
  - a breadth-first traversal visits cities in order according to the number of edges they are from the origin.

- Why might this approach fail to work for a weighted graph?

Dijkstra's Algorithm

- One algorithm for solving the shortest-path problem for weighted graphs was developed by E.W. Dijkstra.

- It allows us to find the shortest path from a vertex v (the origin) to all other vertices that can be reached from v.

- Basic idea:
  - maintain estimates of the shortest paths from the origin to every vertex (along with their costs)
  - gradually refine these estimates as we traverse the graph

- Initial estimates:
  
<table>
<thead>
<tr>
<th>path</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>the origin itself:</td>
<td>stay put!</td>
</tr>
<tr>
<td>all other vertices:</td>
<td>unknown</td>
</tr>
</tbody>
</table>

- Initial graph:
Dijkstra’s Algorithm (cont.)

- We say that a vertex \( w \) is *finalized* if we have found the shortest path from \( v \) to \( w \).

- We repeatedly do the following:
  - find the unfinalized vertex \( w \) with the lowest cost estimate
  - mark \( w \) as finalized (shown as a filled circle below)
  - examine each unfinalized neighbor \( x \) of \( w \) to see if there is a shorter path to \( x \) that passes through \( w \)
    - if there is, update the shortest-path estimate for \( x \)

**Example:**

\[
\begin{array}{c|c|c}
A & C & B \\
--- & --- & --- \\
(0) & C (inf) & B (inf) \\
\end{array}
\]

\[
\begin{array}{c|c|c}
A & C & B \\
--- & --- & --- \\
(0) & C (5) & B (14) \\
\end{array}
\]

\[
\begin{array}{c|c|c}
A & C & B \\
--- & --- & --- \\
(0) & C (5) & B (12) \\
\end{array}
\]

Another Example: Shortest Paths from Providence

- Initial estimates:
  - Boston: \( \text{infinity} \)
  - Worcester: \( \text{infinity} \)
  - Portsmouth: \( \text{infinity} \)
  - Providence: \( 0 \)

- Providence has the smallest unfinalized estimate, so we finalize it.

- We update our estimates for its neighbors:
  - Boston: \( 49 \) (\(<\) infinity)
  - Worcester: \( 42 \) (\(<\) infinity)
  - Portsmouth: \( \text{infinity} \)
  - Providence: \( 0 \)
• Worcester has the smallest unfinalized estimate, so we finalize it.
  • any other route from Prov. to Worc. would need to go via Boston,
    and since (Prov → Worc) < (Prov → Bos), we can't do better.

• We update our estimates for Worcester's unfinalized neighbors:
  Boston  49 (no change)
  Worcester  42
  Portsmouth  125 (42 + 83 < infinity)
  Providence  0

• Boston has the smallest unfinalized estimate, so we finalize it.
  • we'll see later why we can safely do this!

• We update our estimates for Boston's unfinalized neighbors:
  Boston  49
  Worcester  42
  Portsmouth  103 (49 + 54 < 125)
  Providence  0
Shortest Paths from Providence (cont.)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td>49</td>
</tr>
<tr>
<td>Worcester</td>
<td>42</td>
</tr>
<tr>
<td>Portsmouth</td>
<td>103</td>
</tr>
<tr>
<td>Providence</td>
<td>0</td>
</tr>
</tbody>
</table>

- Only Portsmouth is left, so we finalize it.

Finalizing a Vertex

- Let w be the unfinished vertex with the smallest cost estimate. Why can we finalize w, before seeing the rest of the graph?
- We know that w’s current estimate is for the shortest path to w that passes through only finalized vertices.
- Any shorter path to w would have to pass through one of the other encountered-but-unfinished vertices, but they are all further away from the origin than w is!
  - their cost estimates may decrease in subsequent stages, but they can’t drop below w’s current estimate!
Pseudocode for Dijkstra's Algorithm

dijkstra(origin)
    origin.cost = 0
    for each other vertex v
        v.cost = infinity;
    while there are still unfinalized vertices with cost < infinity
        find the unfinalized vertex w with the minimal cost
        mark w as finalized
        for each unfinalized vertex x adjacent to w
            cost_via_w = w.cost + edge_cost(w, x)
            if (cost_via_w < x.cost)
                x.cost = cost_via_w
                x.parent = w

• At the conclusion of the algorithm, for each vertex v:
  • v.cost is the cost of the shortest path from the origin to v
  • if v.cost is infinity, there is no path from the origin to v
  • starting at v and following the parent references yields the shortest path

Example: Shortest Paths from Concord

Evolution of the cost estimates (costs in bold have been finalized):

<table>
<thead>
<tr>
<th></th>
<th>inf</th>
<th>inf</th>
<th>197</th>
<th>197</th>
<th>197</th>
<th>197</th>
<th>197</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albany</td>
<td>inf</td>
<td>inf</td>
<td>197</td>
<td>197</td>
<td>197</td>
<td>197</td>
<td>197</td>
</tr>
<tr>
<td>Boston</td>
<td>inf</td>
<td>74</td>
<td>74</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concord</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New York</td>
<td>inf</td>
<td>inf</td>
<td>inf</td>
<td>inf</td>
<td>inf</td>
<td>290</td>
<td>290</td>
</tr>
<tr>
<td>Portland</td>
<td>inf</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portsmouth</td>
<td>inf</td>
<td>146</td>
<td>128</td>
<td>123</td>
<td>123</td>
<td>290</td>
<td>290</td>
</tr>
<tr>
<td>Providence</td>
<td>inf</td>
<td>105</td>
<td>105</td>
<td>105</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worcester</td>
<td>inf</td>
<td>63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that the Portsmouth estimate was improved three times!
Another Example: Shortest Paths from Worcester

Evolution of the cost estimates (costs in bold have been finalized):

<table>
<thead>
<tr>
<th>Albany</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concord</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New York</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portland</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portsmouth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Providence</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worcester</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Implementing Dijkstra's Algorithm

- Similar to the implementation of Prim's algorithm.
- Use a heap-based priority queue to store the unfinalized vertices.
  - priority = ?
- Need to update a vertex's priority whenever we update its shortest-path estimate.
- Time complexity = $O(E \log V)$
Topological Sort

• Used to order the vertices in a **directed acyclic graph** (a DAG).

• Topological order: an ordering of the vertices such that, if there is a directed edge from a to b, a comes before b.

• Example application: ordering courses according to prerequisites

  - a directed edge from a to b indicates that a is a prereq of b
  - There may be more than one topological ordering.

---

Topological Sort Algorithm

• A **successor** of a vertex v in a directed graph = a vertex w such that (v, w) is an edge in the graph

• Basic idea: find vertices with no successors and work backward.
  • there must be at least one such vertex. why?

• Pseudocode for one possible approach:

  ```plaintext
topolSort
  S = a stack to hold the vertices as they are visited
  while there are still unvisited vertices
    find a vertex v with no unvisited successors
    mark v as visited
    S.push(v)
  return S
  
  Popping the vertices off the resulting stack gives one possible topological ordering.
  ```
Topological Sort Example

Evolution of the stack:

**push**  stack contents (top to bottom)

- E-124   E-124
- E-162   E-162, E-124
- E-215   E-215, E-162, E-124
- E-104   E-104, E-215, E-162, E-124
- E-119   E-119, E-104, E-215, E-162, E-124
- E-10    E-10, E-160, E-119, E-104, E-215, E-162, E-124

one possible topological ordering

Another Topological Sort Example

Evolution of the stack:

**push**  stack contents (top to bottom)
Traveling Salesperson Problem (TSP)

• A salesperson needs to travel to a number of cities to visit clients, and wants to do so as efficiently as possible.

• A tour is a path that:
  • begins at some starting vertex
  • passes through every other vertex once and only once
  • returns to the starting vertex

• TSP: find the tour with the lowest total cost

TSP for Santa Claus

A “world TSP” with 1,904,711 cities.
The figure at right shows a tour with a total cost of 7,516,353,779 meters – which is at most 0.068% longer than the optimal tour.

• Other applications:
  • coin collection from phone booths
  • routes for school buses or garbage trucks
  • minimizing the movements of machines in automated manufacturing processes
  • many others

source: http://www.tsp.gatech.edu/world/pictures.html
Solving a TSP: Brute-Force Approach

- Perform an exhaustive search of all possible tours.
  - represent the set of all possible tours as a tree
    - The leaf nodes correspond to possible solutions.
      - for \( n \) cities, there are \((n - 1)!\) leaf nodes in the tree.
      - half are redundant (e.g., L-Cm-Ct-O-Y-L = L-Y-O-Ct-Cm-L)
    - Problem: exhaustive search is intractable for all but small \( n \).
      - example: when \( n = 14 \), \((n - 1)! / 2 = \) over 3 billion

Solving a TSP: Informed Search

- Focus on the most promising paths through the tree of possible tours.
  - use a function that estimates how good a given path is
- Better than brute force, but still exponential space and time.
Algorithm Analysis Revisited

- Recall that we can group algorithms into classes ($n =$ problem size):

<table>
<thead>
<tr>
<th>Name</th>
<th>Example Expressions</th>
<th>Big-O Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Time</td>
<td>$1, 7, 10$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Logarithmic Time</td>
<td>$3\log_{10}n, \log_2n + 5$</td>
<td>$O(\log n)$</td>
</tr>
<tr>
<td>Linear Time</td>
<td>$5n, 10n - 2\log_2n$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>Log $n$ Time</td>
<td>$4n \log_2n, n \log_2n + n$</td>
<td>$O(n \log n)$</td>
</tr>
<tr>
<td>Quadratic Time</td>
<td>$2n^2 + 3n, n^2 - 1$</td>
<td>$O(n^2)$</td>
</tr>
<tr>
<td>$n^c (c &gt; 2)$</td>
<td>$n^3 - 5n, 2n^3 + 5n^2$</td>
<td>$O(n^c)$</td>
</tr>
<tr>
<td>Exponential Time</td>
<td>$2^n, 5e^n + 2n^2$</td>
<td>$O(c^n)$</td>
</tr>
<tr>
<td>Factorial Time</td>
<td>$(n - 1)!/2, 3n!$</td>
<td>$O(n!)$</td>
</tr>
</tbody>
</table>

- Algorithms that fall into one of the classes above the dotted line are referred to as **polynomial-time** algorithms.

- The term **exponential-time algorithm** is sometimes used to include **all** algorithms that fall below the dotted line.
  - algorithms whose running time grows as fast or faster than $c^n$

Classifying Problems

- Problems that can be solved using a polynomial-time algorithm are considered “easy” problems.
  - we can solve large problem instances in a reasonable amount of time

- Problems that don’t have a polynomial-time solution algorithm are considered “hard” or "intractable" problems.
  - they can only be solved exactly for small values of $n$

- Increasing the CPU speed doesn't help much for intractable problems:

<table>
<thead>
<tr>
<th>CPU 1</th>
<th>CPU 2 (1000x faster)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max problem size for $O(n)$ alg:</td>
<td>$N$</td>
</tr>
<tr>
<td>$O(n^2)$ alg:</td>
<td>$N$</td>
</tr>
<tr>
<td>$O(2^n)$ alg:</td>
<td>$N$</td>
</tr>
</tbody>
</table>
Dealing With Intractable Problems

- When faced with an intractable problem, we resort to techniques that quickly find solutions that are "good enough".

- Such techniques are often referred to as heuristic techniques.
  - heuristic = rule of thumb
  - there's no guarantee these techniques will produce the optimal solution, but they typically work well

Take-Home Lessons

- Computer science is the science of solving problems using computers.

- Java is one programming language we can use for this.

- The key concepts transcend Java:
  - flow of control
  - variables, data types, and expressions
  - conditional execution
  - procedural decomposition
  - definite and indefinite loops
  - recursion
  - console and file I/O
  - memory management (stack, heap, references)
Take-Home Lessons (cont.)

• Object-oriented programming allows us to capture the abstractions in the programs that we write.
  • creates reusable building blocks
  • key concepts: encapsulation, inheritance, polymorphism

• Abstract data types allow us to organize and manipulate collections of data.
  • a given ADT can be implemented in different ways
  • fundamental building blocks: arrays, linked nodes

• Efficiency matters when dealing with large collections of data.
  • some solutions can be much faster or more space efficient
  • what’s the best data structure/algorithm for your workload?
    • example: sorting an almost sorted collection

Take-Home Lessons (cont.)

• Use the tools in your toolbox!
  • interfaces, generic data structures
  • lists/stacks/queues, trees, heaps, hash tables
  • recursion, recursive backtracking, divide-and-conquer

• Use built-in/provided collections/interfaces:
  • java.util.ArrayList<T> (implements List<T>)
  • java.util.LinkedList<T> (implements List<T> and Queue<T>)
  • java.util.Stack<T>
  • java.util.TreeMap<K, V> (a balanced search tree)
  • java.util.HashMap<K, V> (a hash table)
  • java.util.PriorityQueue<T> (a heap)

• But use them intelligently!
  • ex: LinkedList maintains a reference to the last node in the list
  • list.add(item, n) will add item to the end in O(n) time
  • list.addLast(item) will add item to the end in O(1) time!