To reproduce the shape of $V(x)$ correctly, the density $n(x)$ should vary as $(N/g)V(x)$. For this, the jump condition is given by

$$\frac{\psi_{1,2}(+)}{\psi_{1,2}(-)} = \lim_{N\to\infty}\left[\frac{1\mp(i/2)(g/N)}{1\pm(i/2)(g/N)}\right]^N \to e^{\mp ig} \quad (16)$$

in agreement with Eq. (5). This then shows that a local, deltalike potential can be viewed as an infinite set of separable potentials, resolving the particular ambiguity noted by Sutherland and Mattis.[5]

In summary, Eq. (5) must be employed when a local delta function potential is used in the Dirac equation. Steps leading to the erroneous use of Eq. (4) have been explained. There is no inconsistency, as is sometimes mentioned in the literature, in regarding a local delta function potential as the limiting case of a local square well. In addition, the local delta function potential can also be viewed, as described in this article, as the superposition of an infinite sequence of nonlocal separable potentials. Both approaches yield Eq. (5). That a local delta function potential is not the limiting case of a *single* nonlocal separable potential, contrary to what one might first expect, also resolves the paradox pointed out in Ref. 5.

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[7]Our matrix representation is related to the more customary representation $\tilde{\alpha} = \sigma_x$, $\tilde{\beta} = \sigma_z$ by a unitary transformation $\alpha = u\tilde{\alpha}u^+$. $\beta = u\tilde{\beta}u^+$, and $\psi = u\tilde{\psi}$, with $u = (2)^{-1/2}\left[\begin{smallmatrix}1 & 1\\ 1 & -1\end{smallmatrix}\right]$. Our ψ_1 and ψ_2 are thus related to the usual $\tilde{\psi}_1$ and $\tilde{\psi}_2$ by $\tilde{\psi}_{1,2} = (\psi_1 \pm \psi_2)/\sqrt{2}$. Our choice of basis simplifies the calculation; the physics are of course independent of the representation. Note that Ref. 5 uses yet another representation: $\hat{\alpha} = \sigma_z$, $\hat{\beta} = -\sigma_x$.
[8]I. R. Lapidus, Am. J. Phys. 51, 1036 (1983).
[9]If $\theta(x) = \int_{-\infty}^{x} dx\, \delta(x)$, Eq. (3) is correct.
[10]In contrast, for the Schrödinger equation $\psi^{(2n)} \sim \epsilon^{-n}$, and Eq. (3) is correct.
[11]For two separable square wells of width 2ϵ centered on $\pm a$, $v_{1,2}(x) = \Delta(x \pm a)$, and $f_{12} = (a/\epsilon)(2 - a/\epsilon)$ for $0 < a < \epsilon$. f_{12} is the fraction of the area of each square not overlapping the other.

Does mass really depend on velocity, dad?

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Relativistic mass is discussed in both a pedagogical and historical context. It is pointed out that its introduction into the theory of special relativity was much in the way of a historical accident. Gaining widespread use initially in instruction, the use of relativistic mass is showing signs of progressive disfavor. An analysis and criticism of the various ways relativistic mass is used in relativity is detailed and special attention is given to the frequent misuse of relativistic mass as an inertia.

I. INTRODUCTION

The title of this article is a question my son asked me after his first day of high school physics. My answer, "No!" "Well, yes..." "Actually, no, but don't tell your teacher." The next day my son dropped physics. The confusion obvious in my answer is a result of a dramatic change in the role the concept of relativistic mass has played in the understanding of the special theory of relativity. An example of the impact of this change can be found in recent editions of the classic text, *University Physics*, by Sears and Zemansky[1]: "...The increase of mass with velocity is of the greatest importance when dealing with atomic and subatomic particles..."[2] "...There is ample experimental evidence that it (mass) is actually a function of the velocity of the body, increasing with increasing velocity according to the relation $m = m_0/(1 - v^2/c^2)^{1/2}$. This equation was predicted by Lorentz and Einstein on theoretical grounds based on relativity considerations and it is directly verified by experiments."[3] "Although the concept of relativistic mass increase is widely used in the literature, it can be misleading. In any event, it is not necessary, and will not be used in this book."[4] "It is sometimes useful to introduce the concept of a variable, velocity-dependent relativistic mass. This concept is not needed in the present analysis, and it is not used in this discussion."

Examples similar (although not as dramatic) to the one cited above can be found in the evolution of other texts. Furthermore, some textbooks have very negative discussions of the relativistic mass concept,[5] some ignore it,[6] and others give a standard treatment.[7] Depending on which textbook or even in some cases on which edition of a textbook a student has used he/she could regard the concept of relativistic mass as "fundamental or archaic," "useful or misleading," or "necessary or unnecessary." In some cases, students taking "college physics" might well talk to students taking "university physics" at the same university and discover that while the one group regards the concept of relativistic mass as a fundamental part of the theory, the other group has never heard of it. The potential for confusion is obvious. The solution is for physics teachers to understand that relativistic mass is a concept in turmoil. If they choose to use it in their course, they should caution the students to this effect.

II. RELATIVISTIC MASS AND ITS USE

There are actually several different relativistic masses. By far the most common use of relativistic mass is in the defining equations for momentum. Namely, in relativity, momentum can be defined as

$$P = m_r v, \tag{1}$$

where

$$m_r = m/(1 - v^2/c^2)^{1/2} \tag{2}$$

is the relativistic mass and m is the rest mass. In this context, relativistic mass is no more than that quantity which multiplies velocity to produce momentum. It is not an inertia in the classical sense. Most textbooks use the notation $m = m_0/(1 - v^2/c^2)^{1/2}$ instead of that given by Eq. (2). To be consistent with the common textbooks, I will adopt this latter notation.

Under the special case when a force is applied perpendicular to a particle's motion, a transverse relativistic inertial mass can be defined by $m_t = m_0/(1 - v^2/c^2)^{1/2}$ and used in the equation

$$F_t = m_t a, \tag{3a}$$

where a is the acceleration in the direction of the force. Similarly, if the force is parallel to the particles motion, then a longitudinal inertial mass can be defined by $m_l = m_0/(1 - v^2/c^2)^{3/2}$ and used in the equation

$$F = m_l a. \tag{3b}$$

In either of the situations described above, the relativistic mass function can be thought of as an inertia since it represents a proportionality between force and acceleration. Problems with using relativistic mass as an inertia will be discussed in the next section.

In addition to momentum mass, transverse mass, and longitudinal mass one can occasionally find a gravitational relativistic mass. Some authors assume[8] or derive[9] a gravitational relativistic mass of the form $m_g = m_0/(1 - v^2/c^2)^{1/2}$ to be used in describing the falling body problem. The relevant equation in this case would presumably be of the form

$$\frac{d}{dt} mv = -m_g g. \tag{4}$$

The derivation[9] of this formula, however, is flawed because it utilizes special relativity and a noninertial observer to arrive at its result. General relativity can be used to derive[10] a "gravitational mass" and the result is

$$m_g = m_0/[1 + (2\Phi/c^2) - u^2/c^2]^{1/2}, \tag{5}$$

where Φ is the gravitational potential. Furthermore, Eq. (5) can be put into an equation that looks very much like Eq. (4). However, caution must be used, since, even though u is a velocity in Eq. (5) it is not a coordinate velocity, nor is the derivative that is present in the analogous equation to Eq. (4) a normal derivative (it is a covariant derivative). These complications prevent the use of gravitational relativistic mass in introductory treatments of special relativity. (It should also be pointed out that there is no real reason to introduce relativistic mass in General Relativity.)

In fact, most of the elementary textbooks that I am aware of that treat special relativity usually only introduce m_r in the form $m = m_0/(1 - v^2/c^2)^{1/2}$. Having introduced relativistic mass in the context of momentum, they occasionally proceed to use it as an inertia. For example, one current text follows the introduction of relativistic mass with the discussion[11]

"That the speed of light is a natural speed limit in the Universe can be seen ... from the mass-increase formula $m = m_0/(1 - v^2/c^2)^{1/2}$. As an object is accelerated to greater and greater speeds the mass becomes larger and larger."

This clearly implies that mass as defined by $m = m_0/(1 - v^2/c^2)^{1/2}$ (which was introduced by the author in the momentum equation) is a resistance to acceleration. Of course, this is not correct. It could be argued that m and m_t have the same functional form, that is, they both equal $m_0/(1 - v^2/c^2)^{1/2}$, and therefore, it is all right for the author to slide from m (the coefficient of velocity in the momentum definition) to m_t (the resistance to a transverse acceleration). However, the author is discussing a situation in which the speed is increasing and, therefore, the force cannot be purely perpendicular to velocity (i.e., it cannot be a transverse force) because in that case the speed would be a constant. Thus, neither m_r nor m_t can be correctly used to explain the speed limit of c for a material particle.

III. RELATIVISTIC MASS AND INERTIA

In the previous section the idea of transverse mass and longitudinal mass were introduced; and it was pointed out that they, under the circumstances in which they are defined, can be considered to be an inertia. However, inertia used in this way is still considerably different from inertia as encountered in classical physics.

In Newtonian mechanics, inertial mass is a property of a body and, as such, it is independent of the circumstances affecting the body. This is not the case with longitudinal or transverse mass. Used as an inertial mass, either one of these is no longer a property of the body; rather it is a "property" (through m_0) of the body and (through v) of the observer.

If the applied force is not either parallel or perpendicular to the bodies' motion, the situation is more complicated. In fact, in that case the acceleration $\mathbf{a}$ is no longer in the direction of the force $\mathbf{F}$; and therefore, no simple inertial concept can be developed. If one still wishes to define the inertia of a moving point particle by the relationship $I = |\mathbf{F}|/|\mathbf{a}|$ one would obtain

$$I = m_t/(1 - v^2 \cos^2 \theta / c^2), \tag{6}$$

where θ is the angle between the force and the velocity. The "inertia" so defined is now a "property" of the body, the observer, and (through θ) the agent which applies the force. This I take to be an unsatisfactory situation and I would argue that it is better not to introduce the concept of an inertial relativistic mass. I realize that this seems to contradict conventional approaches. For example, in *Einstein: A Centenary Volume*, Bondi writes[12]

"After all, the mass of a moving body can be taken to be either its rest mass or its total mass which includes the mass of its kinetic energy... For inertia there is no doubt: It is a well tested part of special relativity that inertia is given by the total mass."

This I take to be common view, but I believe it should be read with the word "internal" understood to precede kinetic energy. In one of his last papers[13] on this subject, Einstein writes

"Every system can be looked upon as a material point as long as we consider no processes other than changes in its translational velocity as a whole. It has a clear meaning, however, to consider changes in the rest-energy in case changes are to be considered other than *mere changes of translational velocity*. The above interpretation asserts, then, that in such a transformation of a material point *its inertial mass changes as its rest-energy...*" (emphasis mine).

In fact, for a body the only generally well-defined inertial mass is the rest mass. This is its inertia as seen from a comoving observer. For a composite body viewed by a comoving observer (i.e., one moving with the velocity of a center of momentum), the inertial mass (or rest mass) is the sum of the rest masses and kinetic energies of its constituents.[14] This is the sense in which I believe Bondi's statement quoted above should be read. It is internal kinetic energy that counts toward inertia not (to paraphrase Einstein] *mere translational kinetic energy of the body as a whole.*

In the passage quoted above, Einstein equated inertial mass with rest energy. Nowhere in the paper from which the passage is taken does he discuss an increase in inertia with translational energy. However, in a popular book[15] written along with Infield at about the same time he does say

"A moving body has both mass and kinetic energy. It resists charge of velocity more strongly than the resting body. It seems as though the kinetic energy of the moving body increases its resistance. If two bodies have the same rest mass, the one with the greater kinetic energy resists the action of an external force more strongly."

Two things should be noted in the context of this passage. First, it does not associate the increased "resistance" of a moving body with an "inertial mass." It simply states what is true; i.e., a body with kinetic energy seems to have more resistance to a change in velocity than the same body at rest. Second, Einstein goes on in the passages that follow this one to use this idea in the context of a body composed of randomly moving (i.e., zero total momentum) particles each having kinetic energy and each contributing this energy to the composite mass of the parent body. This, of course, is correct and as argued above presents a well defined usage of the concept of inertial mass. Elsewhere Einstein states explicitly[16] that the mass of a body is nothing else than the energy possessed by the body *as judged from a coordinate system moving with the body.*

Finally, I wish to address the use of the word "seems" in the quoted passage. This word is well advised because the apparent increase in "resistance" with velocity of a moving particle is an illusion. It comes about simply because of time dilation. An applied force on a moving particle of a given rest mass apparently takes a longer time to accelerate it when the particle is moving faster than the same force applied to the same particle when moving slowly. Thus the particle appears to have more resistance to acceleration at high speeds. In reality, measured by a clock instantaneously traveling with the particle, the same force always produces the same effect in the same time interval. From our point of view, however, the time kept by a rapidly moving particle is dilated and hence, as the particle's speed increases, apparently greater time intervals are taken to produce the same effect, thus the apparent increase in resistance.

However, for a composite body viewed by a co-moving observer, the mass increase produced by the kinetic energy of its constituents is real. Any energy possessed internally by the body changes its mass. If this energy is possessed by randomly moving particles, then the mass of the composite body increases to the extent of the energy of these particles. The increase in mass of the composite body should be viewed as due to the energy, i.e., $m_0c^2/(1-v^2/c^2)^{1/2}$, of its constituent particles, not to the fact that they have themselves an increased relativistic mass.

IV. HISTORICAL PERSPECTIVE

The electromagnetic world view that occupied much of the first quarter of this century has been extensively and elegantly discussed elsewhere.[17] The general idea was to construct an electromagnetic model of the extended, as opposed to the point, electron. The properties derived in that way were assumed to be extendable to bodies other than the electron.[18] One result of this work was to predict a velocity-dependent mass. Extensive experiments were done to determine the functional form of this dependence for the electron. The experimental evidence available during the beginning of this century for the velocity dependence of this mass was convincing. The interest in the problem was strong. Many prominent physicists were involved in the inquiry. The list of names would include: Abraham, Bucherer, Fermi, Kaufman, Langevin, von Laue, Lorentz, Planck, Poincare, among others. In view of this great interest, it certainly should not be surprising that Einstein would choose to relate his special theory of relativity to a topic such as this one that was already under intensive investigation both experimentally and theoretically. However, relativistic mass, in particular, and dynamics, in general, was not prominent in Einstein's original paper. The whole discussion of dynamics arises in the last section of the paper[19] and accounts for only about 15% of the total paper. It is clear that in this paper Einstein regarded the electron as being unstructured and having a mechanical, as opposed to electromagnetic, "rest mass." Einstein's relativistic mass had its origin in the kinetics of his special theory and not in the structure of the particle. In fact he observes that "with a different definition of force and acceleration we should naturally obtain other values for the masses (meaning, longitudinal and transverse masses)."[20]

I take this to mean that he wanted to indicate that the idea of relativistic mass is created in special relativity "by definition" and is not fundamental to the theory. Furthermore, in a later paper[21] also published in 1905, "Does the Inertia of a Body Depend Upon its Energy Content?" Einstein does not use the idea of relativistic mass even though he is discussing the change in the mass of a moving object resulting from the emission of energy. Nonetheless, he evidently realized at last the societal importance of the topic since in

1906 he published[22] "a proposed experiment" that could be used to distinguish between his formulas for relativistic mass and those of competing theories. It should be noted that Einstein's original formula for transverse mass was incorrect. It was corrected by Planck in 1906.[23] Planck was the first to introduce the formula $P = m_0 v/(1 - v^2/c^2)^{1/2}$. However, Lewis and Tolman[24] in 1909 were the first to introduce a purely mechanical derivation of m_r done much in the manner that it is done today.

Einstein also dealt with this topic in at least one other paper[25] published at around the same time. The difference in Einstein's (corrected) relativistic mass formulas and those of competing theories, such as Lorentz's theory (which predicted mathematically identical formulas) is that Einstein's mass formulas are artifacts of the kinematical transformation of space and time whereas, for example, the Lorentz formulas result from "real" changes in the structure of the electron.

Whatever Einstein's precise early views were on the subject, his view in later life appears clear. In a 1948 letter to Lincoln Barnett, he wrote

"It is not good to introduce the concept of the mass $M = m/(1 - v^2/c^2)^{1/2}$ of a body for which no clear definition can be given. It is better to introduce no other mass than 'the rest mass' m. Instead of introducing M, it is better to mention the expression for the momentum and energy of a body in motion."[26]

The question naturally arises as to what motivated Einstein to this new view given his earlier use of the concept. The answer, I believe, is that by at least 1922 he had adopted[27] Minkowski's 1908 space-time (four-vector) approach to special relativity. Anyone who has tried to teach special relativity using the four-vector space-time approach knows that relativistic mass and four-vectors make for an ill-conceived marriage.

In fact, most of the recent criticism of relativistic mass is presented in the context of the four-vector formulation of special relativity. Three sources in particular that I would cite are Goldstein[28] in his very influential text *Classical Mechanics* (1954), Taylor and Wheeler[29] in *Spacetime Physics* (1964), and Robert Brehne[30] in an article published (1968) in *The American Journal of Physics*. Given the timing of the changes in the treatment of relativistic mass in some textbooks, it might be suspected that the last two references were particularly influential. In any case, all three of the above sources give similar arguments against using relativistic mass. Most of the points they raise address the issue that relativistic mass obscures the elegance and clarity brought to relativity by a four-vector formulation. Other arguments are also presented by these authors, for example, the use of relativistic mass can mislead students into believing that the structure of moving objects is actually affected by their motion as was actually the case in the Lorentz theory when, in fact, the observed effects are due to an alteration in space-time.

V. PROS AND CONS

I am confident that almost everyone using four-vectors to teach relativity will be led to an approach that excludes serious use of the relativistic mass concept. But the truth is that almost all introductions to relativistic mass do not use the four-vector approach.[31] Therefore, the question arises, "Why not use relativistic mass in simple introductions to the subject?" One knowledgeable correspondent[32] on this issue cited three reasons for using it in introductory courses. The points raised by the correspondent were

(1) "If one understands mass to be a measure of inertia, it is easy to use relativistic mass to explain why it takes so much energy to accelerate an atomic particle to 0.999c or the like."

(2) "Most students are aware of $E = mc^2$ if they are aware of anything connected to relativity. Doing away with relativistic mass does away with their one point of initial understanding."

(3) "Relativistic mass greatly aids in the teaching of the gravitational redshift, etc."

In answering point 1, I would raise the issue discussed earlier. Unless one goes all the way and introduces m_t and m_l in addition to m, you are actually "tricking" the students into believing that they have an understanding of the subject when in fact their "understanding" is predicated on the misconception than $m = m_0/(1 - v^2/c^2)^{1/2}$ can be considered as inertia when, in fact, it usually cannot. Instead, why not simply say that since $E = mc^2/(1 - v^2/c^2)^{1/2}$, it follows that progressively larger increments of energy are necessary to accelerate a particle as its velocity approaches c. This is the approach taken in the classic instructional film, *The Ultimate Speed*.[33]

To respond to the second point, I would raise the question: Does E really equal mc^2 in the sense raised by my correspondent? Einstein in his early work treats this issue in two ways.[34,35] First, in the sense of $E = \Delta mc^2$, where E is energy lost or gained from a body of mass m, and Δm is the resulting mass change in the *rest mass* of the body; and, also $M = E/c^2$, where M is a mass to be associated with radiation of energy E emitted or absorbed by a body. In 1935 in a paper[36] referred to earlier entitled, "Elementary Derivation of the Equivalence of Mass and Energy," Einstein is clearly interested in the relationship $E_0 = m_0c^2$. Nowhere does he speak of the relationship of "total mass" and "total energy." This does not stop one from using $E = mc^2$ in this way. However, what is clearly significant in the relationship between energy and mass is that the internal energy of a body affects its mass and, furthermore, that (rest) mass itself is a form of energy. This is a profound truth. The relationship $E = mc^2$ in which $m = m_0/(1 - v^2/c^2)^{1/2}$ is by way of contrast, merely a definition of m and should not be put on the same level of importance as $E_0 = m_0c^2$.

The third point is a more difficult one to which to respond. Of course, for photons one does not use relativistic mass in the sense of $m = m_0/(1 - v^2/c^2)^{1/2}$. Rather, one uses $E = mc^2$ (presumably previously established in the typical relativistic mass approach as being "true" for moving material particles) to motivate the use of E/c^2 for the mass of a photon. Having done this, it is easy to show the desired effect of gravity on frequency. In fact, I would agree that this would seem to be the easiest way to proceed. Any teacher who wishes to discuss this topic and has not introduced relativistic mass and the ancillary relationship $E = mc^2$ (instead of $E_0 = m_0c^2$) has two other primary alternatives. Either use the approach to this topic which hinges on the slowing down of time in a gravitational field, or take recourse in the relationship $M = E_e/c^2$, where M is the change in the rest mass of a body which emits electromagnetic energy E_e, to motivate the idea of an associated mass for emitted photons given by their energy divided by c^2.

VI. CONCLUSION

The use of relativistic mass as an aid to teaching special relativity appears to offer few advantages. Its origin is in the elegant but abandoned prerelativity electromagnetic mass theories of Lorentz and Poincare. Its role in special relativity as developed by Einstein is that of an artifact. In its simplest form, i.e., "$m = m_0/(1 - v^2/c^2)^{1/2}$," it is of no legitimate use in most discussions of inertia. In this context it is probably more abused than used. Relativistic mass does allow for the use of $E = mc^2$ (as opposed to $E_0 = m_0c^2$), which some might find to be an advantage. Perhaps there are also other advantages with which I am not familiar. On balance, some teachers might find it useful, but if it is used, its status as a "convenience" should be made clear and it should not be presented as, in any sense, fundamental to the special theory of relativity.

[1]F. W. Sears and M. W. Zemansky, *University Physics* (Addison-Wesley, Reading MA, 1963), 3rd ed., p. 207.

[2]F. W. Sears and M. W. Zemansky, *University Physics* (Addison-Wesley, Reading MA, 1970), 4th Ed., p. 107.

[3]F. W. Sears, M. W. Zemansky, and H. D. Young, *University Physics* (Addison-Wesley, Reading MA, 1976), 5th Ed., p. 131.

[4]F. W. Sears, M. W. Zemansky, and H. D. Young, *University Physics* (Addison-Wesley, Reading MA, 1982), 6th Ed., p. 834.

[5]H. Semat and P. Baumel, *Fundamentals of Physics* (Holt, Rinehart and Winston, New York, 1974), 5th ed., p. 157.

[6]J. P. Hurley and C. Garrod, *Principles of Physics* (Houghton Mifflin, Boston, 1978), Chap. 35.

[7]S. H. Radin and R. T. Folk, *Physics for Scientists and Engineers* (Prentice-Hall, Englewood Cliffs, NJ, 1982), Chap. 19.

[8]I. R. Lapidus, Am. J. Phys. **40**, 984 (1972).

[9]L. Tsai, Am. J. Phys. **54**, 340 (1986).

[10]C. Moller, *Theory of Relativity* (Oxford U. P., Oxford, 1972), p. 291. See also I. R. Lapidus, Am. J. Phys. **40**, 1509 (1972).

[11]The identity of this particular textbook is unimportant; arguments like the one presented are all too common. In fact, specious arguments like this one can even find their way into high level works, e.g., V. Fock, *The Theory of Spacetime and Gravitation*, 2nd revised ed. (MacMillan, New York, 1964), p. 76.

[12]*Einstein: A Centenary Volume*, edited by A. P. French (Harvard U. P., Cambridge, MA, 1979), p. 127.

[13]A. Einstein, Bull. Am. Math. Soc. **41**, 223 (1935).

[14]M. A. B. Whitaker, Phys. Ed. **11**, 55 (1986).

[15]A. Einstein and L. Infield, *The Evolution of Physics: The growth of ideas from early concepts to relativity and quanta* (Simon and Schuster, New York, 1938), p. 207.

[16]A. Einstein, *Relativity: The Special and the General Theory*, (Methuen, London, 1954), 15th ed., pp. 45–47.

[17]J. T. Cushing, Am. J. Phys. **49**, 1131 (1981); A. Miller, *Albert Einstein's Special Theory of Relativity: Emergence* (1905) *and early interpretation* (1905–1911) (Addison-Wesley, Reading, MA, 1981); A. Miller, Arch. Hist. Exact Sci. **10**, 207 (1973); A. Miller, Am. J. Phys. **44**, 912 (1976).

[18]"...if we suppose *that the masses of all particles are influenced by a translation to the same degree as the electromagnetic masses of the electrons*"—H. A. Lorentz in "Electromagnetic Phenomena in A System Moving With any Velocity Less Than That of Light." Reprinted in *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity* (Dover, New York, 1952), p.30.

[19]Reference 18, pp. 61–65.

[20]Reference 18, p. 63.

[21]Reference 18, pp. 67–71.

[22]A. Einstein, Ann. Phys. **21**, 583 (1906).

[23]M. K. L. Planck, Verh. Phys. Ges. **4**, 136 (1906).

[24]G. N. Lewis and P. C. Tolman, Philos. Mag. **18**, 510 (1909).

[25]A. Einstein, Jahrb. Radioakt. **4**, 411 (1907).

[26]My thanks to John Stachel from the Einstein Project who called my attention to this previously unpublished passage; and to the Hebrew University of Jerusalem, Israel, for the permission to publish it.

[27]A. Einstein, *The Meaning of Relativity* (Princeton U. P., Princeton, NJ, 1955). This contains the text of Einstein's Stafford Little Lectures on Relativity given in May 1921. The approach taken by Einstein is very similar to that adopted by Minkowski (see Ref. 18, pp. 75–96) in his "Space and Time" paper. Minkowski explicitly deals with "The constant mechanical mass" and in his approach the $1/(1 - v^2/c^2)^{1/2}$ sometimes associated with mass clearly arises from the difference between observer time and proper time.

[28]H. Goldstein, *Classical Mechanics* (Addison-Wesley, Reading, MA, 1959).

[29]E. F. Taylor and J. A. Wheeler, *Spacetime Physics* (Freeman, San Francisco, 1966).

[30]R. W. Brehme, Am. J. Phys. **36**, 896 (1968).

[31]For an exception, however, see Hurley and Garrod, Ref. 8.

[32]The correspondent is an author of a Modern Physics Book. Since I have decided to paraphrase his arguments, I don't believe it is appropriate to identify him.

[33]Produced by the Education Development Center, Newton, MA.

[34]A. Einstein, Ann. Phys. **18**, 639 (1905).

[35]A. Einstein, Ann. Phys. 20, 627 (1906).

[36]Reference 13.

PROBLEM: OSCILLATORY MOTION OF A BODY ON A FREELY SLIDING PARABOLIC BASE

A small block of mass m rests on a parabolic base of mass M which, in turn, rests on a horizontal table, as shown in Fig. 1. The shape of the base is given by

$$\eta = \tfrac{1}{2} k\xi^2, \quad -a \leqslant \xi \leqslant a$$

in the coordinate system (ξ, n) fixed to the base. All surfaces are frictionles. If the system starts at rest with point P of the block at a distance $y_0 = \frac{1}{2} ka^2$ above the table, find the trajectory of the block and the period of the oscillatory motion of the system. (Solution is on p. 763.)