Introduction

**Preparation:** Before coming to lab, read this lab handout and the suggested reading in Giancoli (through Chapter 7). Some of the questions that appear in this lab handout are pre-lab questions and the answers are to be handed in at the beginning of your lab — the questions are compiled on a separate sheet at the end of this handout. *Be sure to bring your completed Pre-Lab Questionnaire to lab along with this handout, writing paper, a calculator, and your copy of the Lab Companion.*

**Post Lab Questions:** At the beginning of each lab section, you will be given an additional handout with a series of questions to be answered and handed in at the end of the experiment. Try to answer these questions with one or two concise sentences. To answer the post lab questions satisfactorily you should pay special attention to all text that appears in italics in this handout.

**Background:** The conservation of momentum principle may be applied to find the speed of a projectile by simple, direct measurement. The apparatus (still used in modern ordinance laboratories) was invented by Benjamin Robins in 1742 and bears the descriptive name Ballistic Pendulum. In its simplest form, the ballistic pendulum is a block of wood, hanging freely on ropes or wires and initially at rest. When a projectile is shot into the wood, the block begins to swing in a pendulum-like motion. The height to which the block (plus embedded projectile) rises is a measure of the speed achieved during the collision and that, in turn, is a measure of the speed of the projectile just before the collision.

**Overview and Objective:** You will fire a paintball into a pendulum bob and measure how far it moves horizontally. Knowing that the pendulum swings in a circular arc allows you to calculate to what height it rose.

From the maximum height to which it rose, you can then deduce the velocity it had at the bottom of the swing (using conservation of mechanical energy).

The velocity at the bottom of the swing is the velocity *after* the collision of the paintball with the pendulum bob. Next you invoke conservation of momentum to figure out the velocity of the paintball *before* the collision — and that’s the objective.

You will also measure the velocity of the paintball *independently* (using a dual photogate timer) to compare your mechanical measurements with the electronic measurements. This comparison will give you further ammunition in deciding whether or not the conservation laws were justifiably and correctly applied in this experiment.
Experimental Setup: The apparatus shown in the figure consists of a pendulum bob (labeled canister) and a paintball marker mounted inside a wooden box. The pendulum bob is suspended from the ceiling. A paintball is propelled from a paintball marker by high pressure CO₂ gas.

Theory

The total linear momentum, \( \vec{p} \), of a system is conserved if no net external force acts on the system:

\[
\vec{p}_{\text{initial}} = \vec{p}_{\text{final}} \tag{1}
\]

This holds regardless of any particular properties of the bodies such as elasticity or hardness; the final momenta depend only upon the initial (vector) momenta. The eight questions on pp. 2 and 4 are repeated at the end as the pre-lab questionnaire.

1→Define the paintball plus pendulum bob as a system at the instant that the paintball collides with the bob. Do any external forces act on this system? Illustrate your answer by drawing a free body diagram.

2→Will momentum be conserved during the collision? Is any approximation involved? Explain.

3→Will momentum to be conserved while the pendulum is swinging? Explain.

4 → Since the velocity and therefore the momentum of the pendulum and ball are zero at the height h, how can we say that momentum is conserved in the experiment? Explain.

5→Comment in a couple of lines upon the following statement: “The momentum of the paintball and bob could not be conserved in this experiment because of the large frictional forces between the bob and the entering paintball.”

Apply conservation of momentum to the collision: Let \( m_{\text{ball}} \) and \( v_{\text{ball}} \) designate the mass and speed of the paintball just before the paintball strikes the stationary pendulum bob. Let \( m_{\text{pend}} \) and \( v_{\text{pend}} \) represent the mass and speed of the pendulum bob just after the collision. If we assume that the suspension wires have very little mass and so carry no
significant momentum after the collision, we can write

\[ m_{\text{ball}}v_{\text{ball}} = (m_{\text{pend}} + m_{\text{ball}}) v_{\text{pend}} \]  

[2]

The vector Equation [1] may be written as a scalar Equation [2] because all velocities are in one dimension and may therefore be added algebraically. Equation [2] relates the velocity of the paintball before the collision to the masses of the paintball and the bob, and to the velocity of the bob just after the paintball has struck. The mass of the paint ball and the mass of the pendulum may be measured easily and accurately, but \( v_{\text{pend}} \) is best found by invoking the Law of Conservation of Energy, a principle entirely independent of the Law of Conservation of Momentum.

Energy is conserved when no nonconservative forces act on the system. We will apply conservation of mechanical energy to the pendulum as it swings, temporarily neglecting air resistance and friction in the suspension. The kinetic energy at the lowest point of the pendulum arc is converted entirely into potential energy at the upper-most point of the swing. Let \( h \) denote the vertical displacement of the bob from collision to the uppermost point of the arc. Then,

\[ KE_{\text{bottom}} = PE_{\text{top}} \]  

[3]

\[ \frac{1}{2} (m_{\text{pend}} + m_{\text{ball}}) v_{\text{pend}}^2 = (m_{\text{pend}} + m_{\text{ball}}) gh \]  

[4]

Therefore,

\[ v_{\text{pend}} = \sqrt{2gh} \]  

[5]

The difficult-to-measure \( v_{\text{pend}} \) is thus expressed in terms of the height \( h \), which is really not much easier to measure directly (because it’s such a tiny \( h \)). The quantity that is easy to measure is the horizontal displacement \( d \) of the bob after the collision. We will do this by noting how far the pendulum bob pushes a light metal rider (shown in figure 1) as the bob swings. Referring to figure 2 on the next page, note that \( h \) and \( d \) are related (Pythagorean theorem) by \( L^2 = (L - h)^2 + d^2 \), where \( L \) is the length of the suspension. Then \( 0 = h^2 - 2Lh + d^2 \) which can be solved for \( h \) by applying the quadratic formula:

\[ h = \frac{2L \pm \sqrt{(-2L)^2 - 4d^2}}{2} = \frac{L \pm \sqrt{L^2 - d^2}}{2} = L \pm L \sqrt{1 - \frac{d^2}{L^2}}. \]  

[6]
From the two solutions for \( h \), we choose the solution with the negative sign before the radical, as it corresponds to our physical situation \( \text{(why?)} \). Also, since \( d \ll L \), then \( \frac{d^2}{L^2} \ll 1 \), and equation [6] can be simplified by using the binomial expansion \( \text{(Giancoli, Appendix A-5)} \)

\[
\begin{align*}
  h &= L - L \sqrt{1 - \frac{d^2}{L^2}} \\
  &= L \left( 1 - \left[ 1 + \left( -\frac{d^2}{L^2} \right) \right]^{\frac{1}{2}} \right) \\
  &= L \left( 1 - \left[ 1 - \left( \frac{d^2}{2L^2} \right) \right] \right) \\
  &= L \left( 1 - \left[ 1 - \frac{d^2}{2L} \right] \right) = \frac{d^2}{2L}.
\end{align*}
\]

This approximation gives us a simpler expression for \( h \) and will save you some computation time:

\[
   h = \frac{d^2}{2L}.
\]  \[7\]

You can verify that this is an excellent approximation by solving both Equations [6] and [7] with some typical values. Try calculating \( h \) and \( v_{\text{pend}} \) to six significant figures when \( L = 200 \text{ cm} \) and \( d = 5 \text{ cm} \).

6→ What percentage error is produced in \( v_{\text{pend}} \) by using this approximate relationship?  

7→ Does it increase or decrease the calculated value of \( v_{\text{pend}} \)?

8→ Combine Equations [2], [5] and [7] to obtain a single expression for the velocity of the paintball, \( v_{\text{ball}} \), in terms of \( d, L, g, m_{\text{ball}} \) and \( m_{\text{pend}} \). Referring to the section on combination or compounding of errors (pages 9 & 10) in the Lab Companion, one might estimate an error of 3 to 4% in the value of the ball’s velocity — this assumes that the measured values of \( d, L, \) and \( m \) are good to 1%, 0.2%, and 1%, respectively.
Procedure

The Pendulum

The paintball marker fires a ball into one end of the pendulum bob, which catches it. The pendulum bob consists of a hollow steel cylinder with one end covered by aluminum foil and the other end covered by a steel plate. The bob hangs from a double bifilar suspension that prevents the bob from rotating as it rises after being struck by the paintball. The suspension assembly has been carefully adjusted so that the canister can swing freely inside the safety box. Do not yank on the suspension system or lean your weight on the hanging canister! Consult your TF if the pendulum bob rubs against the plastic lid of the box.

The bob is enclosed in a wooden box that has an open channel along the top. A small metal rider sits in this channel and registers any movement of the pendulum bob. The box also secures and aligns the paintball marker correctly. A baffle or blast screen is located between the end of the marker barrel and the bob. This screen prevents muzzle gases from imparting any additional momentum to the bob.

Caution → The paintball marker should never be loaded while the pendulum bob is removed, or while any adjustments or measurements are being made.

1. Carefully remove the canister from the suspension. To make cleanup at the end of the lab easier, line the inside of your canister with one of the supplied plastic bags prior to starting the experiment. Cover the end of the canister with aluminum foil and fix it in place with the ring end-cap. Weigh the canister+bag+aluminum foil and record their combined mass. Your lab TF will tell you the weight of the paintballs. Record these values along with uncertainties.

2. Carefully click the canister back onto the two bifilar suspensions hanging from the ceiling. The assembly has been adjusted so that the canister can swing freely inside the safety assembly. Make certain that the pendulum swings freely without rotating about the vertical axis and isn’t rubbing against anything. Please seek help from your TF when performing these operations.

3. With the Plexiglas lid of the safety box pushed out of the way, measure the vertical length of the suspension, \( L \), using the two-meter stick; record your value along with uncertainty.

4. Slide the Plexiglas lid into place and position the light metal rider into the grove of the lid so that it is butted up against the strings of the bifilar suspension. Record its location.

The Photogate

Each ballistic pendulum apparatus is equipped with a set of two photogate timers that are separated by a distance of 0.2 meters. As the paint ball passes through each of the photogates, an electrical pulse is sent to a CPO Timer™. The timer, when set to the Interval Mode, measures the amount of time that it takes for the paint ball to travel from one photogate to the other. If we know the separation distance \( x \), and the time of flight \( t \), we can calculate the ball's average velocity over that distance.
Operation instructions for the CPO Timer™

You should familiarize yourself with the operation before the safety lid is in place. Turn on the timer with the small black switch on the left side of the clock unit. Toggle the yellow function switch until a small green light shows above the interval setting. Next, press the yellow buttons labeled A and B — a green light should show above each letter. You are now ready to make an interval measurement. Try it. Pass your finger through photogate A and then through photogate B. The display shows the elapsed time. To reset the timer, press the red button.

(5) Use the CPO Timer™ to measure the velocity of each paintball that you fire.

Ready To Fire

(6) You are now ready to load and fire a paintball. **PUT ON YOUR SAFETY GLASSES. ASK YOUR TF TO SHOW YOU HOW TO LOAD THE MARKER AND FIRE A PAINTBALL.**

(7) Having fired, record the new position of the rider and determine the horizontal distance \(d\) through which the bob has been displaced. Allow the pendulum to come to rest and reset the rider. Also record the time interval indicated on the CPO Timer. Fire at least two more paintballs, again determining the horizontal distance \(d\). **For each shot, the bob will be heavier by the mass of one paintball.**

Systematic Error

So far, our analysis of the ballistic pendulum has been highly idealistic. We have neglected some obvious systematic errors in measuring \(d\). For example, we have neglected friction in the wires, air resistance, the mass of the wires, and rider friction. The systematic error due to wire friction, the mass of the wires, and air resistance can be measured by setting the bob swinging with a horizontal amplitude \(d\) and then noting how much smaller each successive swing becomes. The loss in one bob swing can then be determined. The error in \(d\) would be one quarter of this difference.

(8) Make this measurement and record the result.

(9) Now design and carry out an experiment to determine the error in \(d\), due to rider friction. Take a hint from the foregoing procedure. You can now determine a value for \(d\) that has been corrected for systematic errors. **Use this corrected value of \(d\) for all the following calculations.**

(10) When finished, remove the pendulum canister, clean it out, and carefully hang the assembly again.

Analysis

(1) **Calculate the velocity of the paintball just before impact for each shot, keeping in mind that the uncertainty is somewhere between 3 and 4%. Are the values reasonable? If not, check your formula for \(v_{ball}\).**

(2) **Calculate the ball's average velocity for each shot using the time-of-flight measured with the dual photogate. Recall that the two photogates are separated by 0.2 meter.**
(3) Then choose one shot only and calculate the following:

- The momentum of the paintball plus pendulum just after impact.
- The momentum of the paintball just before impact.
- The kinetic energy of the paintball just before impact.
- The kinetic energy of the paintball plus pendulum just after impact.

Appendix: Optional Experiment

The pair of photogates can be moved to different locations inside apparatus. You may place them near the blast shield, or closer to the marker, or anywhere in between. You should be able to determine if the horizontal speed of the ball changes with distance from the maker by making measurements at different locations.

Try a couple of different locations. Can you detect a change in the average velocity of the ball depending on the position of the photogates? If there is a change, what do you think is causing it?
1. Define the paintball plus pendulum bob as a system at the instant that the paintball collides with the bob. Do any external forces act on this system? Illustrate your answer using a free body diagram.


3. Will momentum to be conserved while the pendulum is swinging? Explain.

4. Since the velocity and therefore the momentum of the pendulum and ball are zero at the height h, how can we say that momentum is conserved in the experiment? Explain.
5→Comment in a couple of lines upon the following statement: “The momentum of the paintball and bob could not be conserved in this experiment because of the large frictional forces between the bob and the entering paintball.”

6→ What percentage error is produced in \( v_{pend} \) by using the approximate relationship for \( h \) obtained from the binomial expansion?

7→ Does it increase or decrease the calculated value of \( v_{pend} \)?

8→ Combine Equations [2], [5] and [7] to obtain a single expression for the velocity of the paintball, \( v_{ball} \), in terms of \( d, L, g, m_{ball} \) and \( m_{pend} \).